

DE SOBRAL ÀS LENTES GRAVITACIONAIS: O UNIVERSO VISTO PELA GRAVIDADE

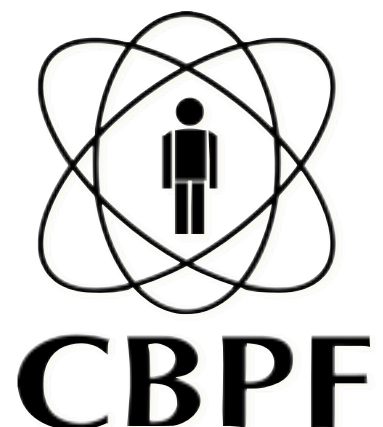
$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu}$$

DEEP INELASTIC SCATTERING
LIBERDADE ASSINTÓTICA

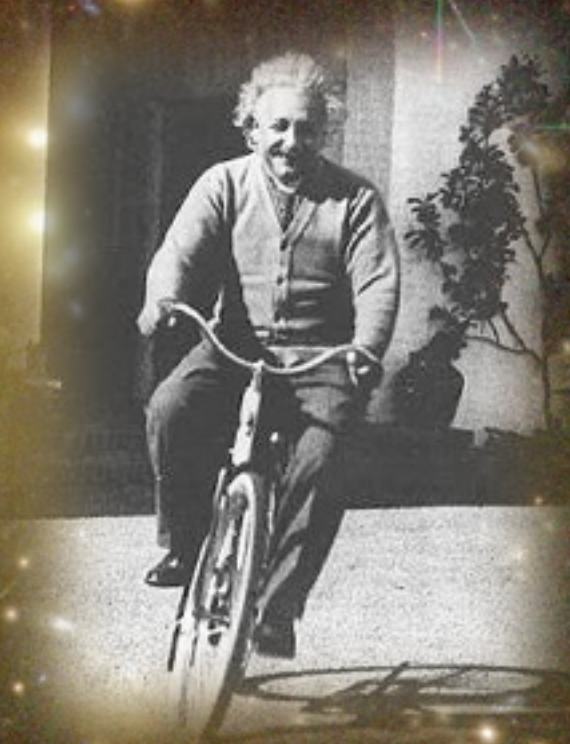
50 anos
1969 - 2019

MARTÍN MAKLER
CBPF

MCTI



DE SOBRAL ÀS LENTES GRAVITACIONAIS: O UNIVERSO VISTO PELA GRAVIDADE



MARTÍN MAKLER
CBPF

XVI Atividades Formativas de Verão/III Professor Global
26 de fevereiro a 1º de março de 2019, Rio de Janeiro

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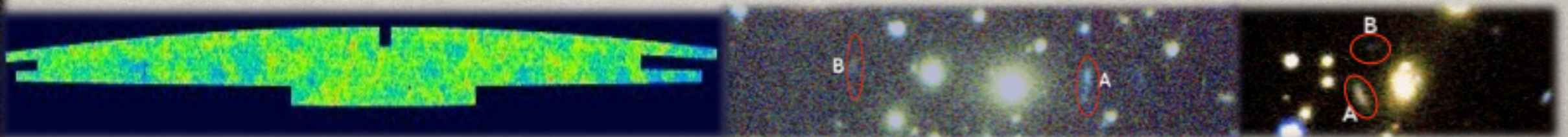


**“Não há perguntas bobas:
toda pergunta é um grito
para compreender o
mundo.”**

Carl Sagan

PROGRAMA PRELIMINAR

- *Overview* de lentes e formalismo básico
- Exemplos
- Arcos e efeito forte
- Efeito fraco de lentes
- Lentes e cosmologia (desvio temporal)
- Tópicos avançados:
 - Cosmologia (LSS + CMB)
 - Lentes e modelos de matéria escura
 - Testes de gravidade modificada



A scenic view of a beach with turquoise water and a clear blue sky. The text is overlaid on the image.

INTRODUÇÃO AO LENTEAMENTO GRAVITACIONAL

Uma nova visão do espaço-tempo

👁 Einstein 1905

A velocidade da luz (no vácuo) é uma constante fundamental da natureza e a mesma para todo observador!

Nova visão da mecânica (física dos movimentos, forças, etc.)

Revolução no conceito de espaço e de tempo



→ Mundo quadri-dimensional (Minkowsky)

$$x_0 = ct$$

$$\Delta S^2 = (\Delta x_0)^2 - (\Delta x_1)^2 - (\Delta x_2)^2 - (\Delta x_3)^2$$

👁 Equivalência massa-energia

$$E = mc^2$$

“As visões de espaço e de tempo que pretendo apresentar aqui provêm da física experimental e nisso reside a sua força.”

H. Minkowski

Vale em qualquer escala, mas ficou conhecida pela energia nuclear: estrelas, reatores, bombas...

Caminho para a relatividade geral

👁 1905 mudou nossas visões de espaço, tempo, massa, energia, luz... mas a maior contribuição individual de Einstein ainda estava por vir...

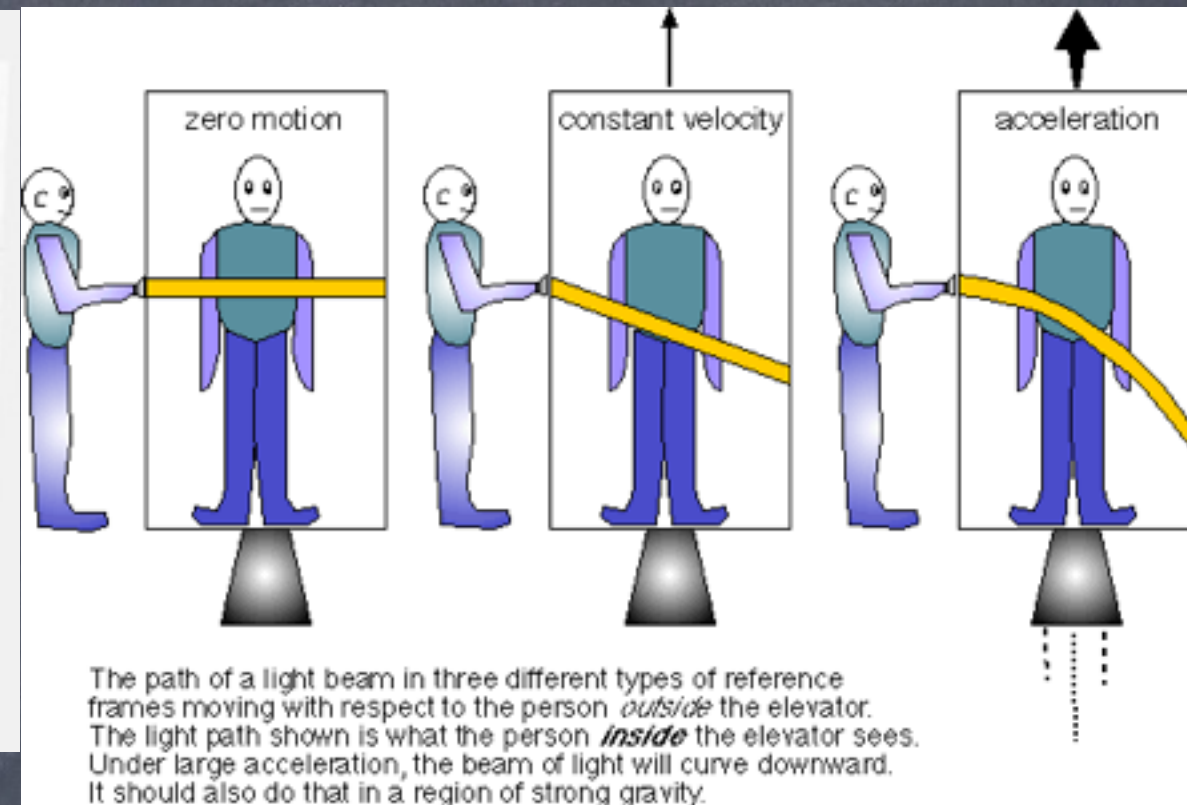
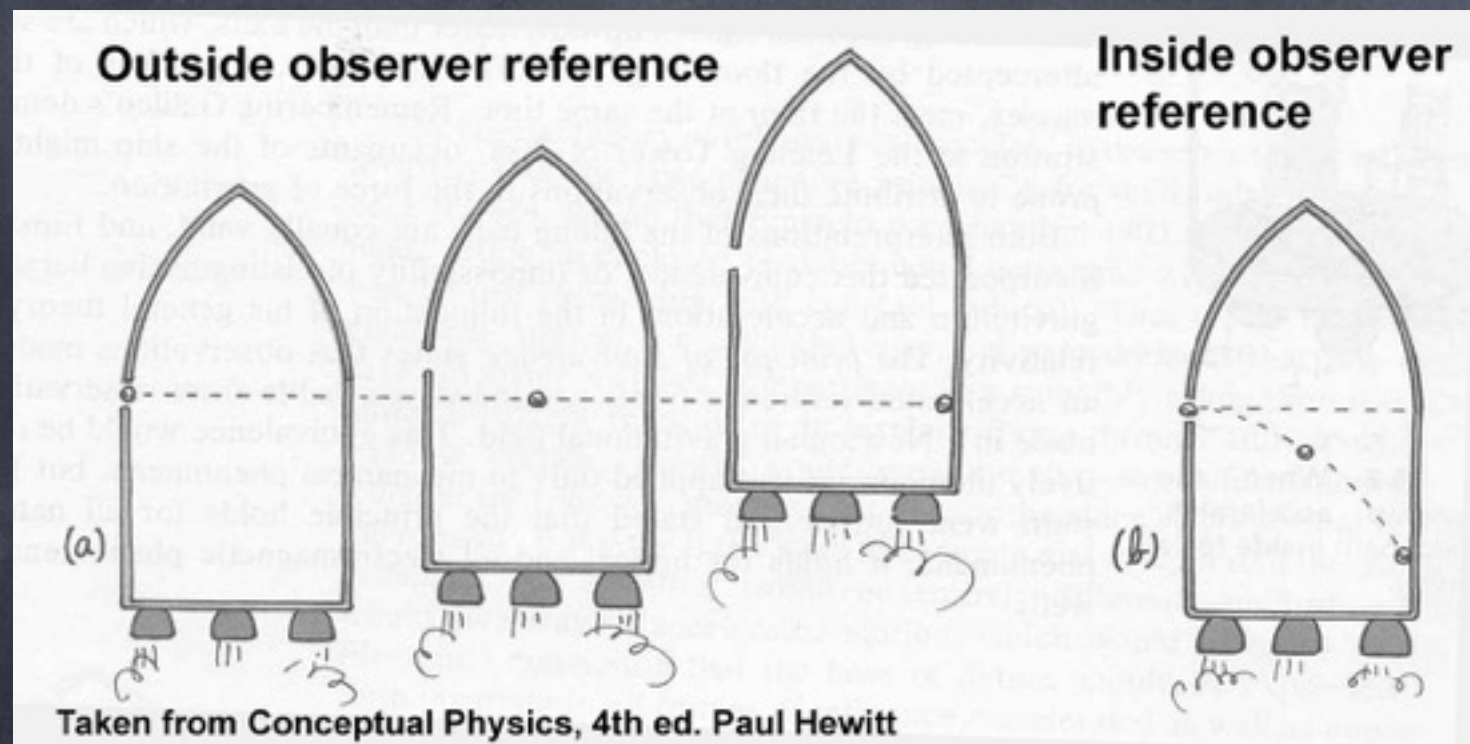
👁 1907: princípio de equivalência

Referencial acelerado “=” gravidade

Queda livre “=” sem gravidade



Caminho para a relatividade geral

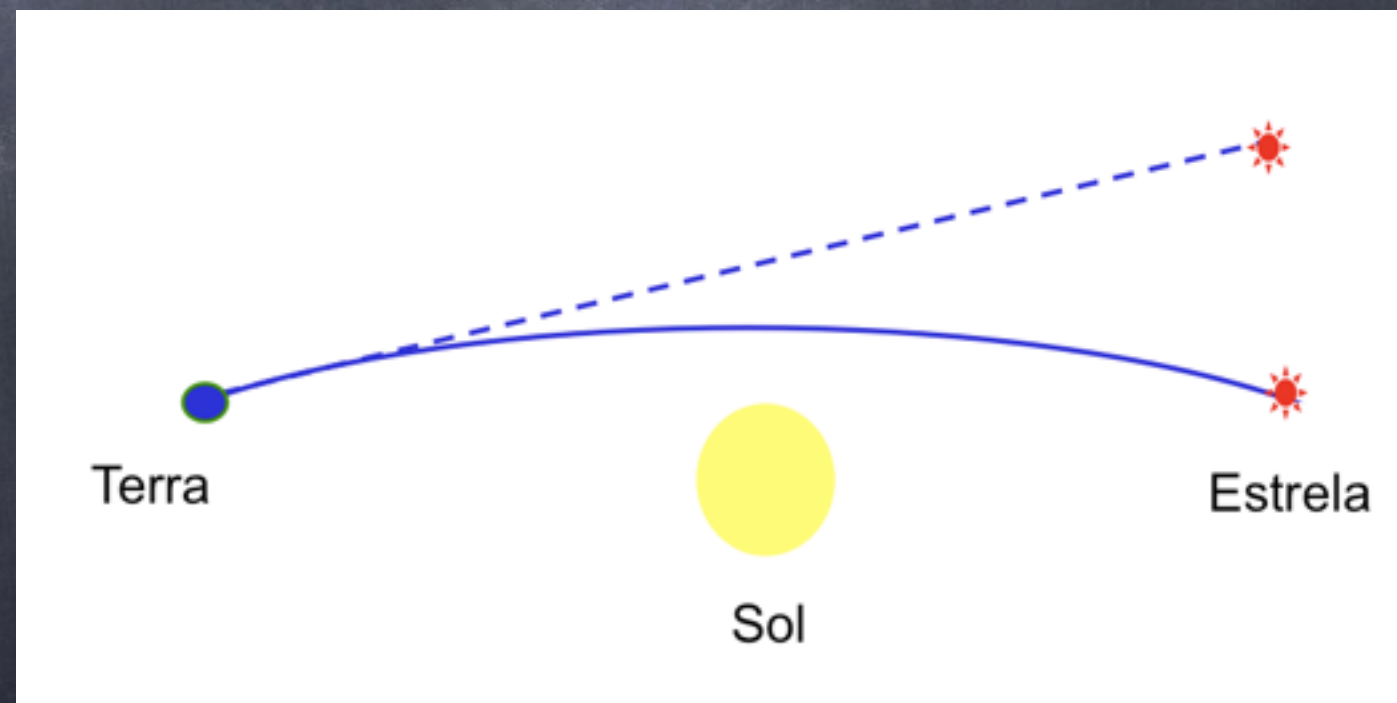


🌀 1907: princípio de equivalência

Referencial acelerado “=” gravidade
Queda livre “=” sem gravidade

🌀 1911: desvio da luz

🌀 1912: lentes gravitacionais (mas pensou que não seria observável)



A luz fazendo a curva

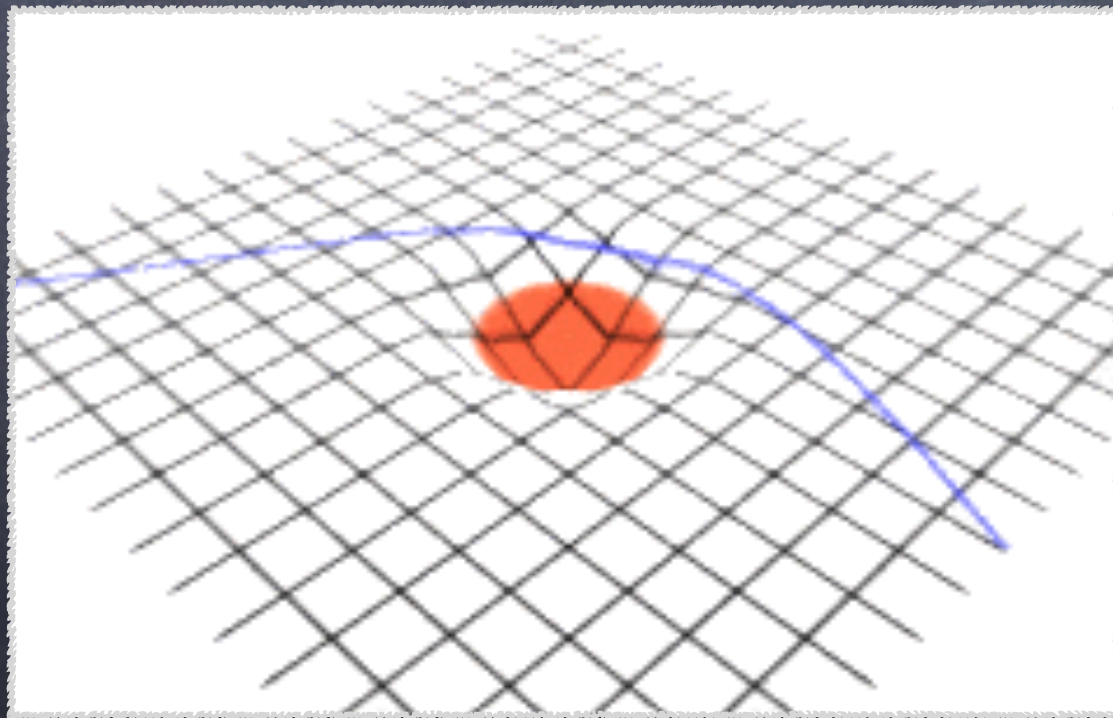
Propagação da luz e relatividade geral

1905: Einstein propõe a relatividade restrita

1911: Obtém o desvio da luz (igual ao newtoniano)

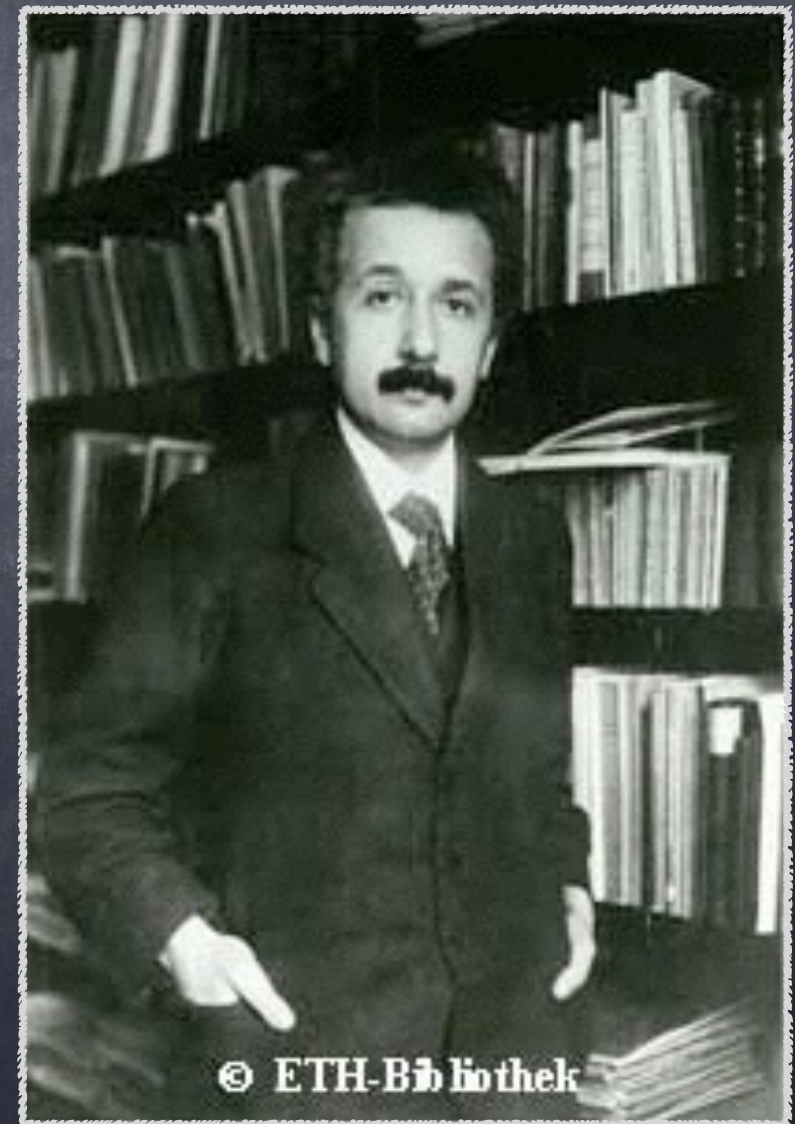
1915: Teoria da relatividade geral

Espaço-tempo curvo



Tudo o que existe “sente” a gravidade:
caio logo existo

$$E = mc^2$$



Relatividade Geral (1915): A teoria de Einstein da gravidade

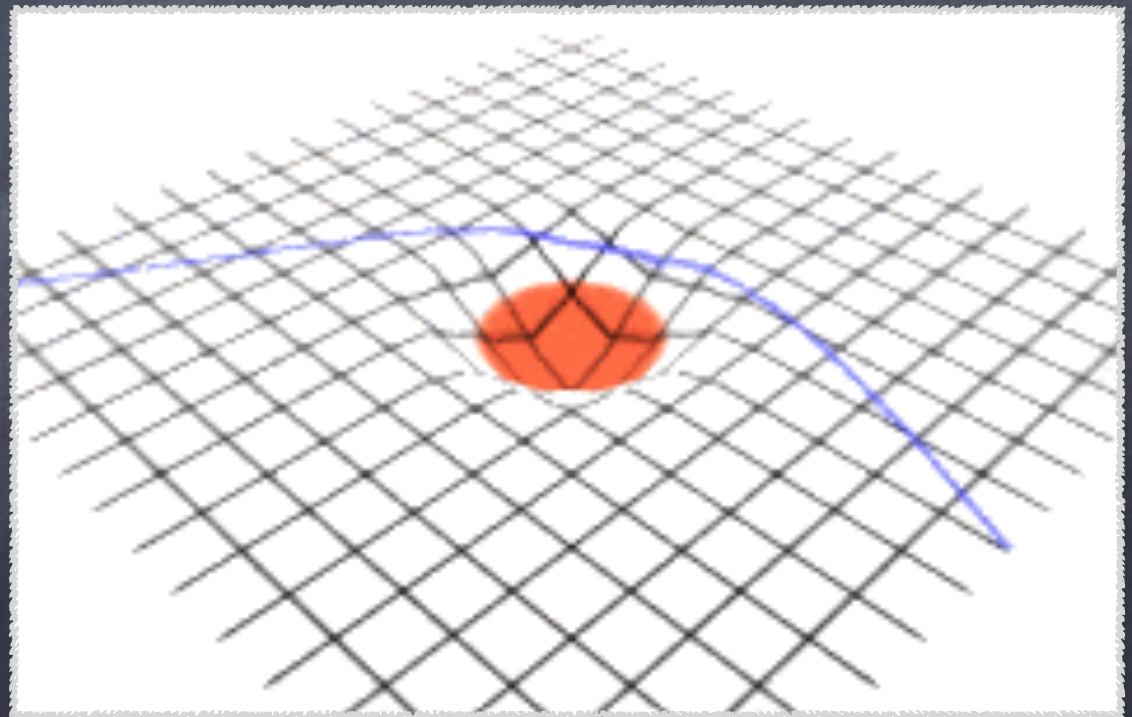
👁 10 anos depois da relatividade especial, uma nova revolução

O espaço-tempo é curvo! Seguindo a matemática do espaço-tempo curvo (Riemann) e a física física da gravidade

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

A matéria diz ao espaço como se curvar,
O espaço diz à matéria como se mover

“Tecido” do espaço-tempo, com
propriedades dinâmicas!



Relatividade Geral (1915): A teoria de Einstein da gravidade

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propriedades dinâmicas!

👁 Compatível com a lei
de Newton (para baixas
velocidades e baixas massas)

👁 Compatível com a
relatividade restrita

👁 Explica a precessão do
perihélio de mercúrio

👁 Prevê muitos
fenômenos novos

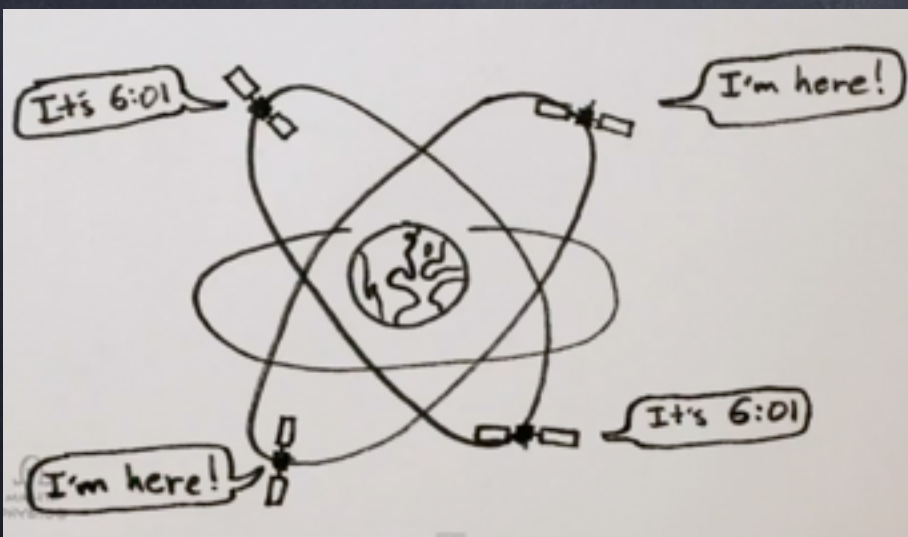
Relatividade Geral (1915): A teoria de Einstein da gravidade

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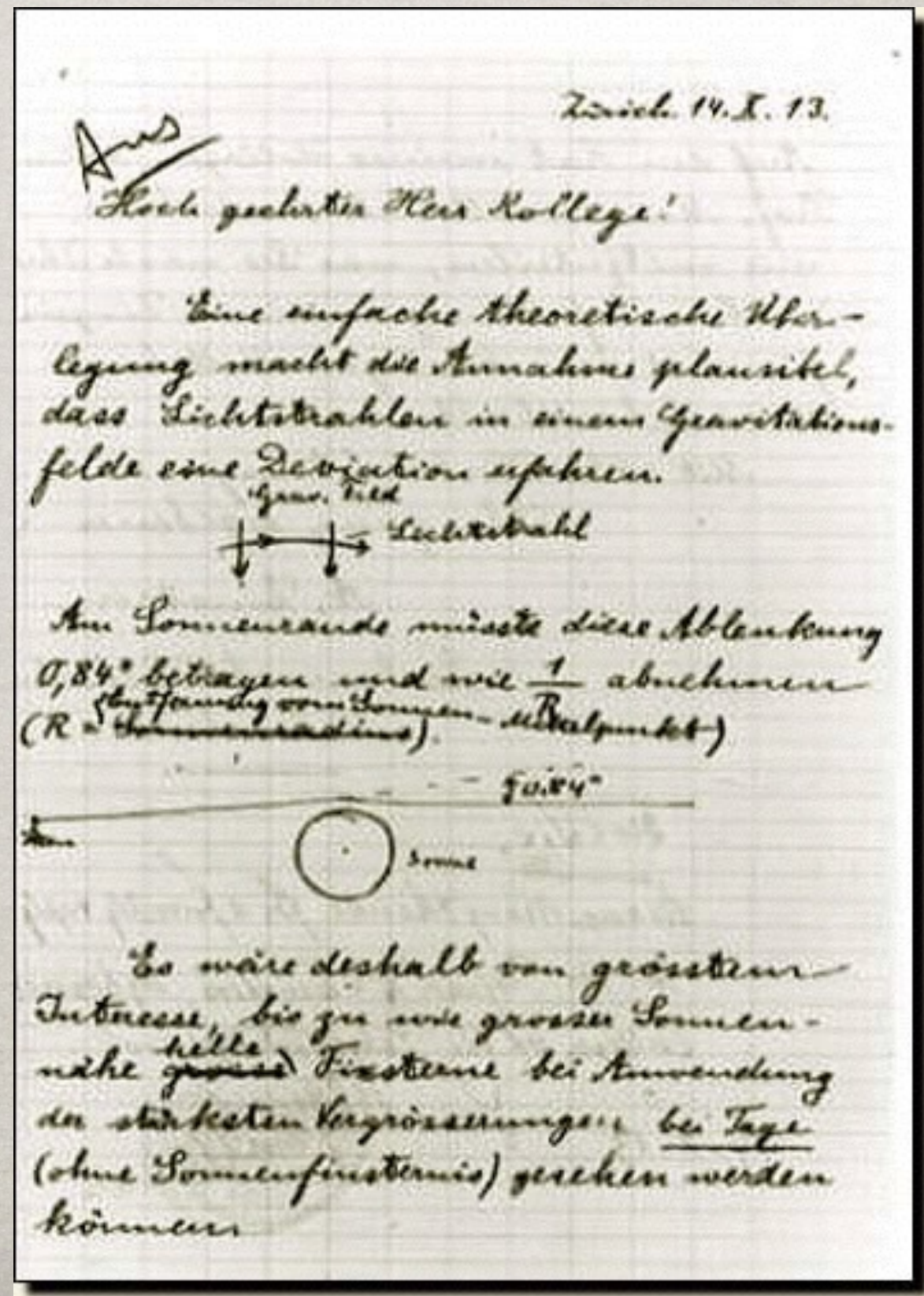
A matéria diz ao espaço como se curvar,
O espaço diz à matéria como se mover



- Desvio da luz (fator dois)
- Lentes gravitacionais
- Cosmologia moderna
- Buracos negros
- Ondas gravitacionais
- Referenciais acelerados (paradoxo dos gêmeos)
- GPS!..

DETECÇÃO DA DEFLEXÃO DA LUZ PELA GRAVIDADE

- Teste do desvio da luz:
Eclipse total do Sol



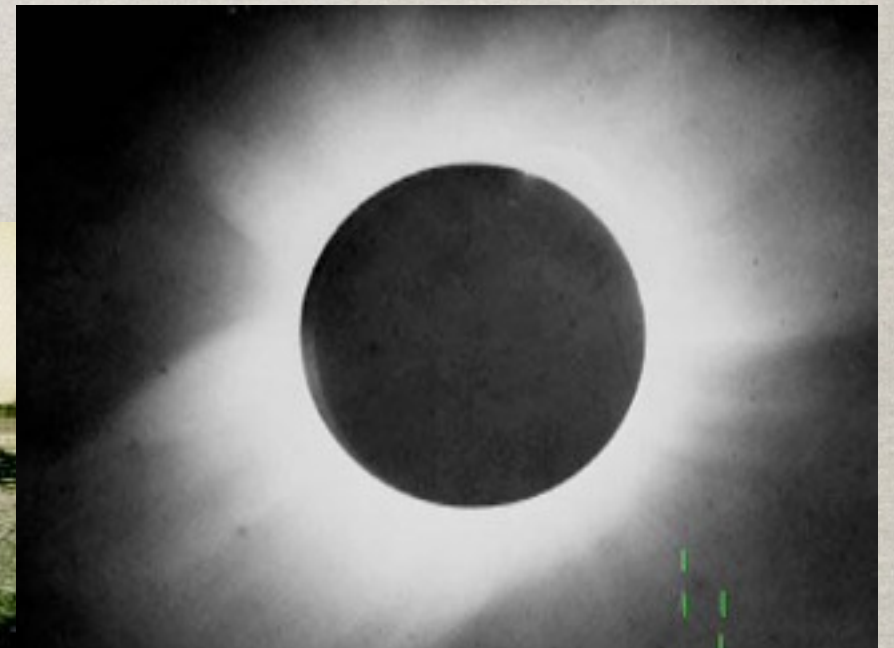
DETECÇÃO DA DEFLEXÃO DA LUZ PELA GRAVIDADE

- 1912: Expedição argentina para observar eclipse solar no Brasil
 - Choveu



DETECÇÃO DA DEFLEXÃO DA LUZ PELA GRAVIDADE

- 1912: Expedição argentina para observar eclipse solar no Brasil
 - Choveu
- 1914: Expedição de Erwin Freundlich's para observar um eclipse na Crimeia (Russia)
 - Presos por causa da primeira guerra mundial
- 1919: Sobral (Brasil)
 - Detecção do desvio da luz pela gravidade (tempo melhor do que na Ilha de Príncipe)



“A questão que minha mente formulou foi respondida pelo céu ensolarado do Brasil”

A. Einstein

BREVE HISTÓRICO DO LENTEAMENTO GRAVITACIONAL

- Einstein 1911-12: previsões de Einstein e primeira tentativa de observação
- Sobral 1919: deflexão da luz pela gravidade
- Chwolson 1924: primeiro artigo sobre lentes
- Einstein 1936: lenteamento por estrelas

“[...] there is no great chance of observing this phenomenon directly [...]”

Therefore, there is no great chance of observing this phenomenon, even if dazzling by the light of the much nearer star B is disregarded. This apparent amplification of q by the lens-like action of the star B is a most curious effect, not so much for its becoming infinite, with x vanishing, but since with increasing distance D of the observer not only does it not decrease, but even increases proportionally to $\sqrt{D}$.

ALBERT EINSTEIN

INSTITUTE FOR ADVANCED STUDY,
PRINCETON, N. J.

BREVE HISTÓRICO DO LENTEAMENTO GRAVITACIONAL

- **Einstein 1911-12:** previsões de Einstein e primeira tentativa de observação
- **Sobral 1919:** deflexão da luz pela gravidade
- **Chwolson 1924:** primeiro artigo sobre lentes
- **Einstein 1936:** lenteamento por estrelas

“Of course, there is not much hope of observing this phenomenon directly”

- **Zwicky 1937:** otimista

“The probability that nebulae which act as gravitational lenses will be found becomes practically a certainty”



- **Walsh, Carswell, Weymann 1979:** Descoberta do primeiro quasar lenteado (imagem dupla de QSO 0957+561)

DESCOBERTAS

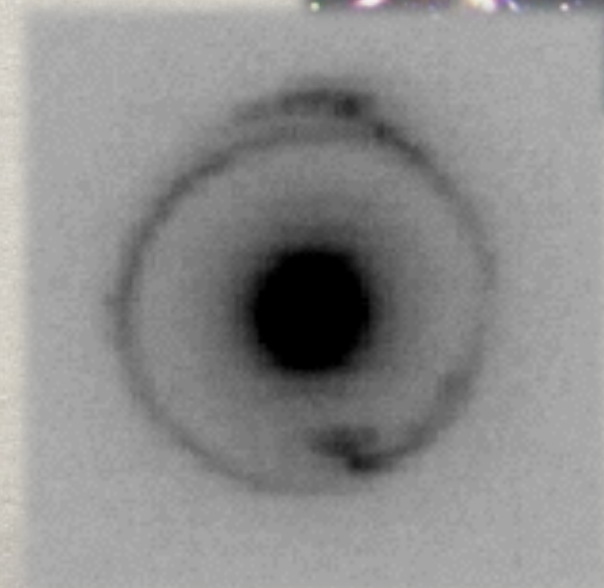
- Sobral 1919: deflexão da luz
- Walsh, Carswell, Weymann 1979: descoberta do primeiro quasar lenteado (imagem dupla de QSO 0957+561)
- Roger Lynds e Vahe Petrosian 1986, Soucail, Fort, & Picat 1987: Descoberta dos arcos gravitacionais:

arcos em A370, A2218, CL2244-02
ninguém previu...

- ✻ Hewitt, et al. 1988: anel de Einstein (no rádio)
- ✻ Irwin et al. 1989: microlenteamento de Quasares
- ✻ Colaborações EROS & MACHO, 1993:

Primeira detecção de microlenteamento

- ✻ Bond et al., 2003: Primeira detecção de planetas
 - ✻ ~2 planetas com massa de Júpiter em ~3 AU

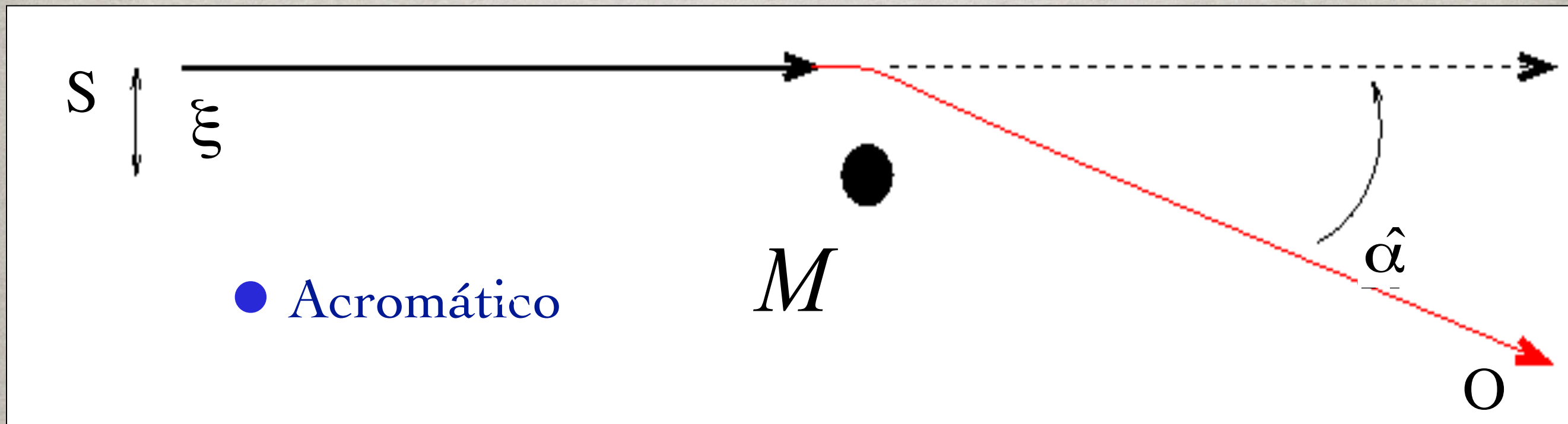


DESVIO DA LUZ PELA GRAVITAÇÃO

Geodésica nula,
princípio de Fermat

$$ds^2 = \left(1 + \frac{2\phi}{c^2}\right) c^2 dt^2 - \left(1 - \frac{2\phi}{c^2}\right) d\sigma^2$$

$$\frac{d\sigma}{dt} := c' = \sqrt{\frac{1 + 2\phi/c^2}{1 - 2\phi/c^2}} \simeq c(1 + 2\phi/c^2)$$



Desvio da Luz (fonte pontual)

$$\hat{\alpha} = \frac{4GM}{c^2} \frac{1}{\xi}$$

UMA MIRÍADE DE FENÔMENOS

Intensidade

- Lenteamento forte
 - Grandes magnificações
 - Imagens múltiplas
 - Distorções
 - Anéis
 - Arcos
- Lenteamento fraco
 - Pequena rotação
 - Pequena magnificação
 - Detectado estatisticamente

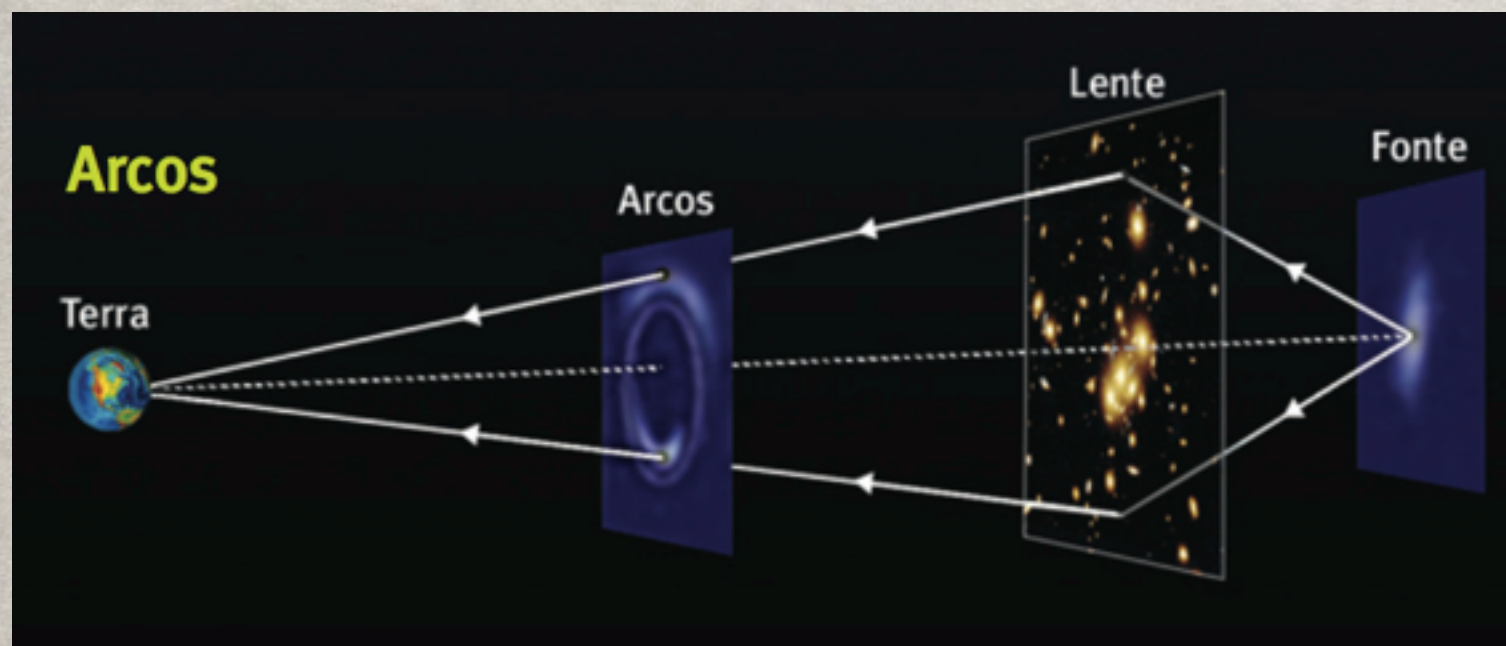
Escala angular

- Mili-lenteamento
 - MACHOS
 - Busca de planetas
- Micro-lenteamento
 - Quasares
- “Macro-lenteamento”
 - Galáxias
 - Aglomerados
 - Estrutura em grande escala

EFEITO FORTE DE LENTE

- Imagens múltiplas, grandes deformações, magnificação, atraso temporal
- Geodésicas nulas
 - brilho superficial conservado
 - acromático
- estrutura de galáxias e aglomerados → matéria escura e bárions
- informações complementares sobre a cosmologia e testes da gravidade

} → Telescópios gravitacionais



efeito forte de lentes, gravidade fraca

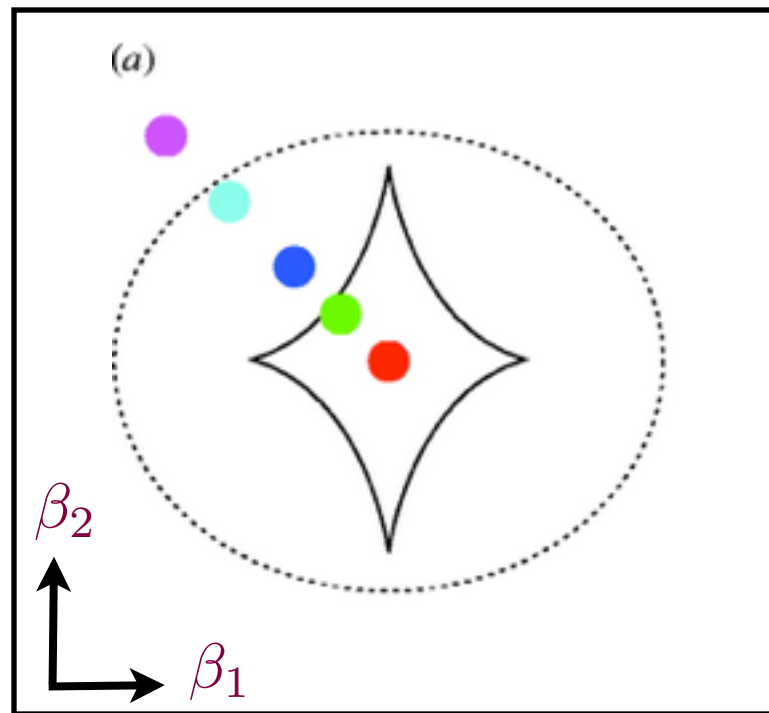


arcos gravitacionais

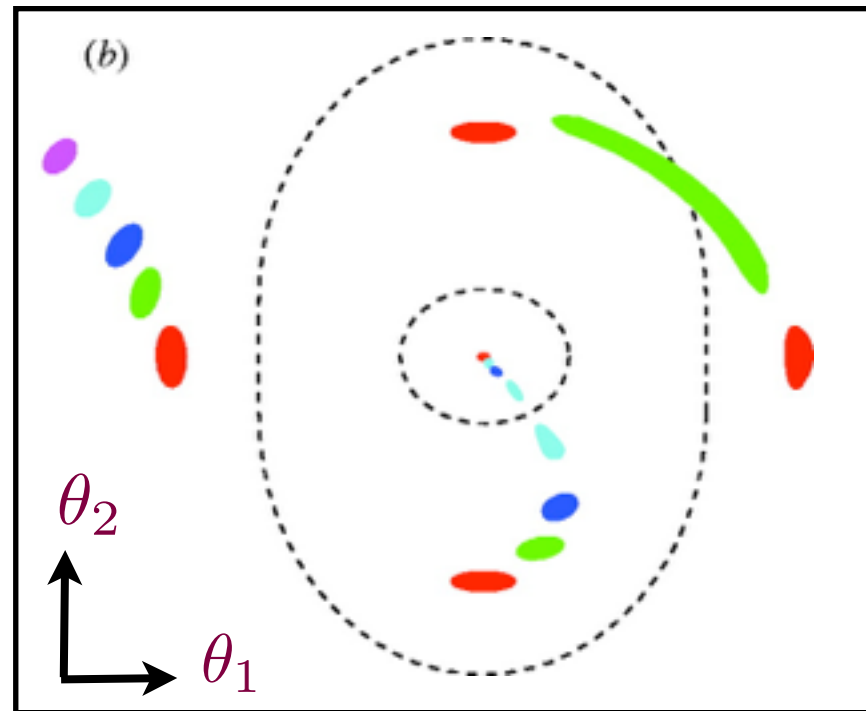
A wide-angle photograph of a beach. The foreground is a sandy beach with some small rocks and debris. The middle ground is the ocean, showing gentle waves with white foam washing onto the shore. The water is a vibrant turquoise color. The background is a clear, solid blue sky. The text 'EQUAÇÃO DA LENTE E LENTE PONTUAL' is overlaid in the center of the image in a white, serif font.

EQUAÇÃO DA LENTE E LENTE PONTUAL

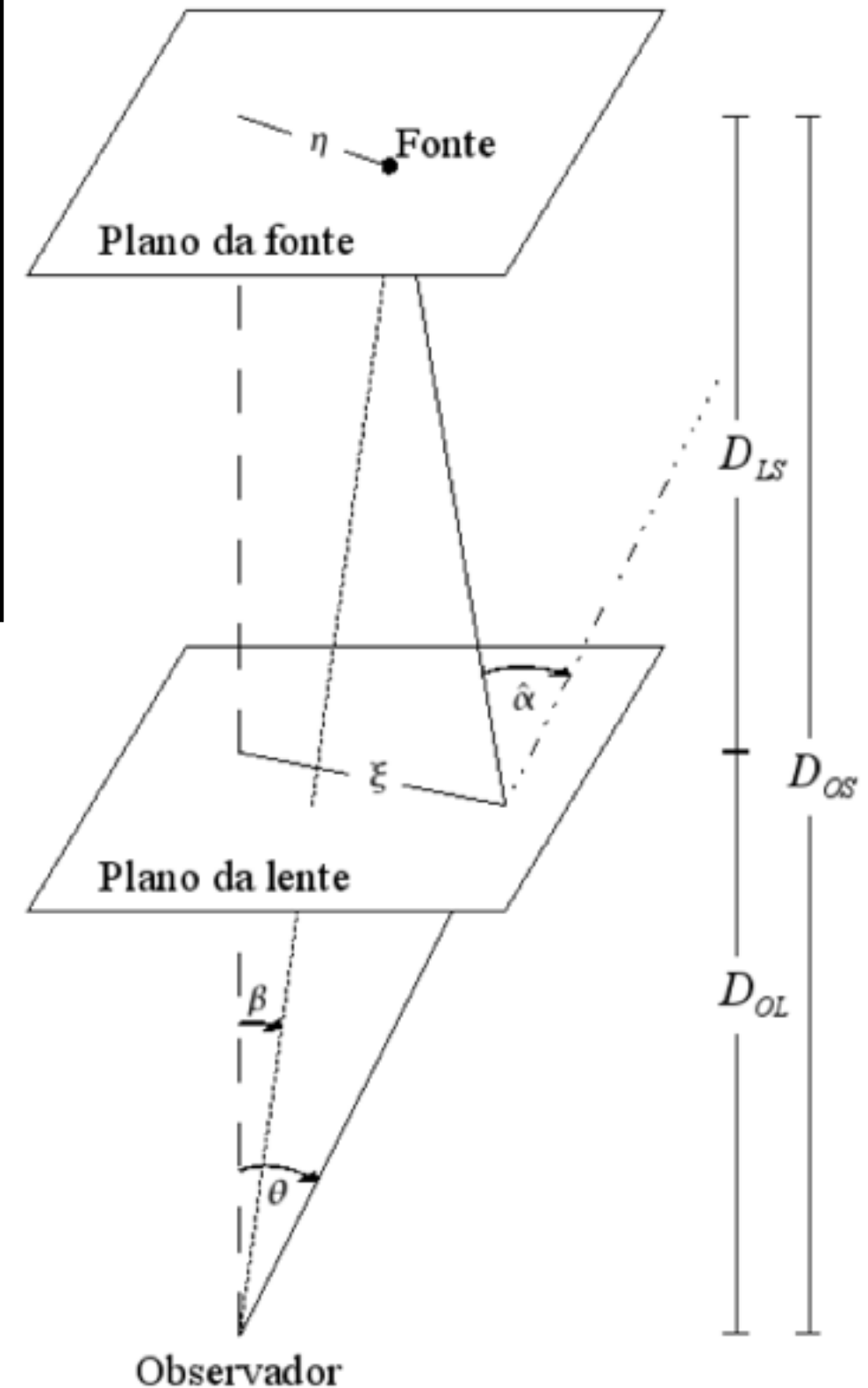
Geometria do lenteamento por um único plano



Plano das fontes



Plano da lente



Coordenadas
físicas (distância)

η

ξ

Coordenadas
angulares

β

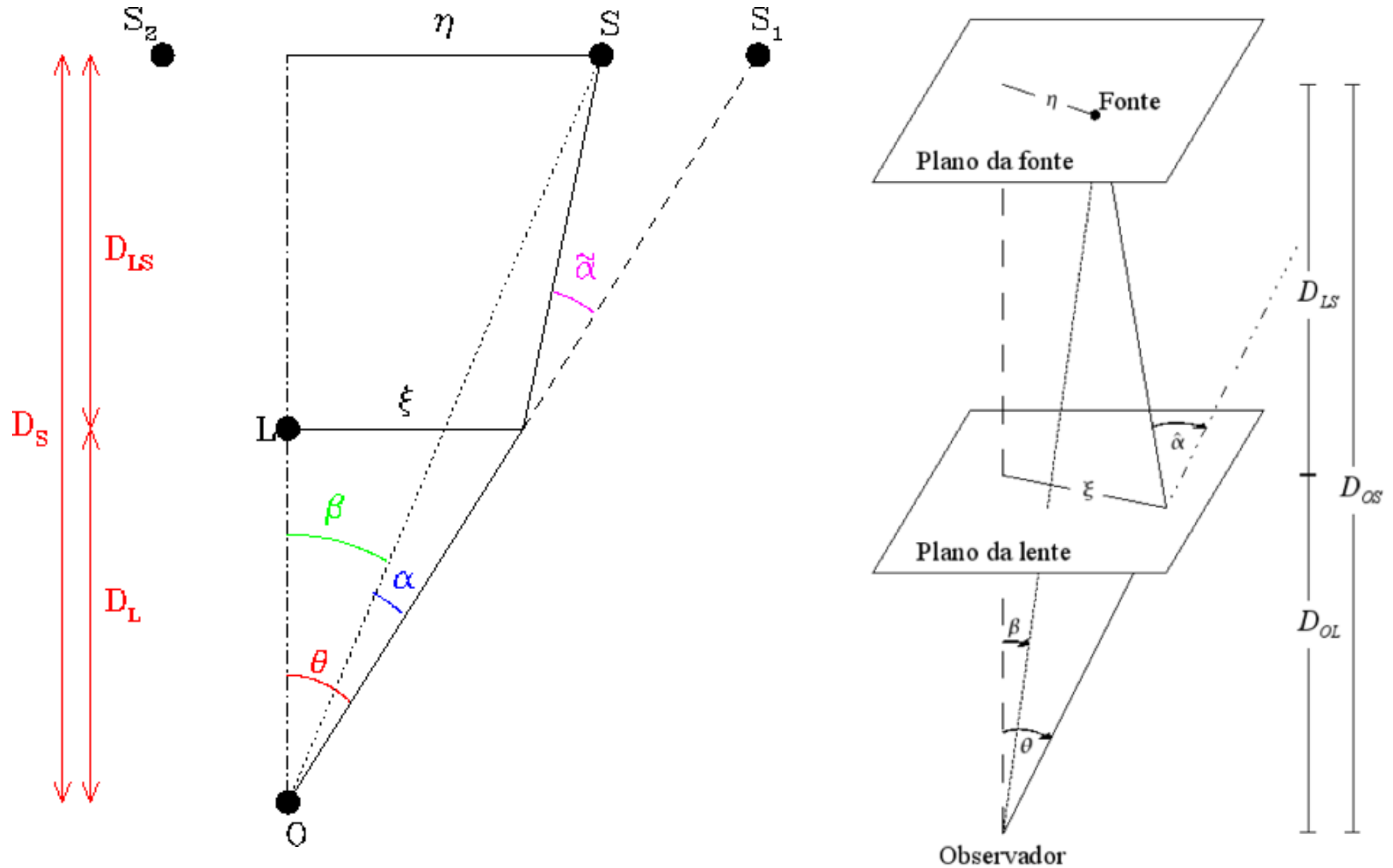
θ

Coordenadas
adimensionais

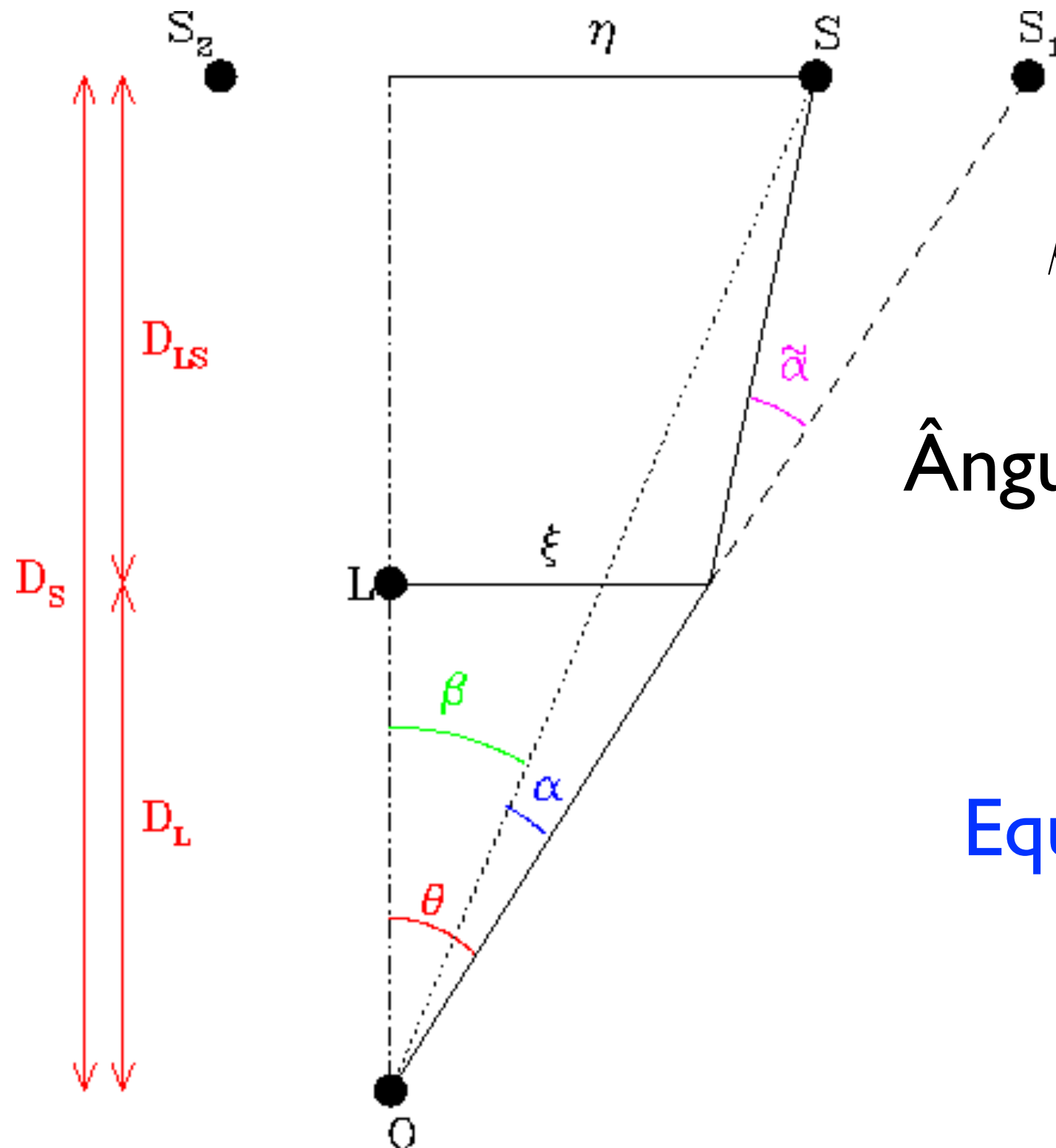
y

x

Geometria do lenteamento por um único plano



A Equação da Lente



$$\vec{\beta} D_{OS} = \vec{\theta} D_{OS} - \hat{\vec{\alpha}} D_{LS} \left(\vec{\theta} \right)$$

Ângulo de deflexão reduzido:

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}}$$

Equação da Lente

$$\vec{\beta} = \vec{\theta} - \vec{\alpha} \left(\vec{\theta} \right)$$

Lente pontual

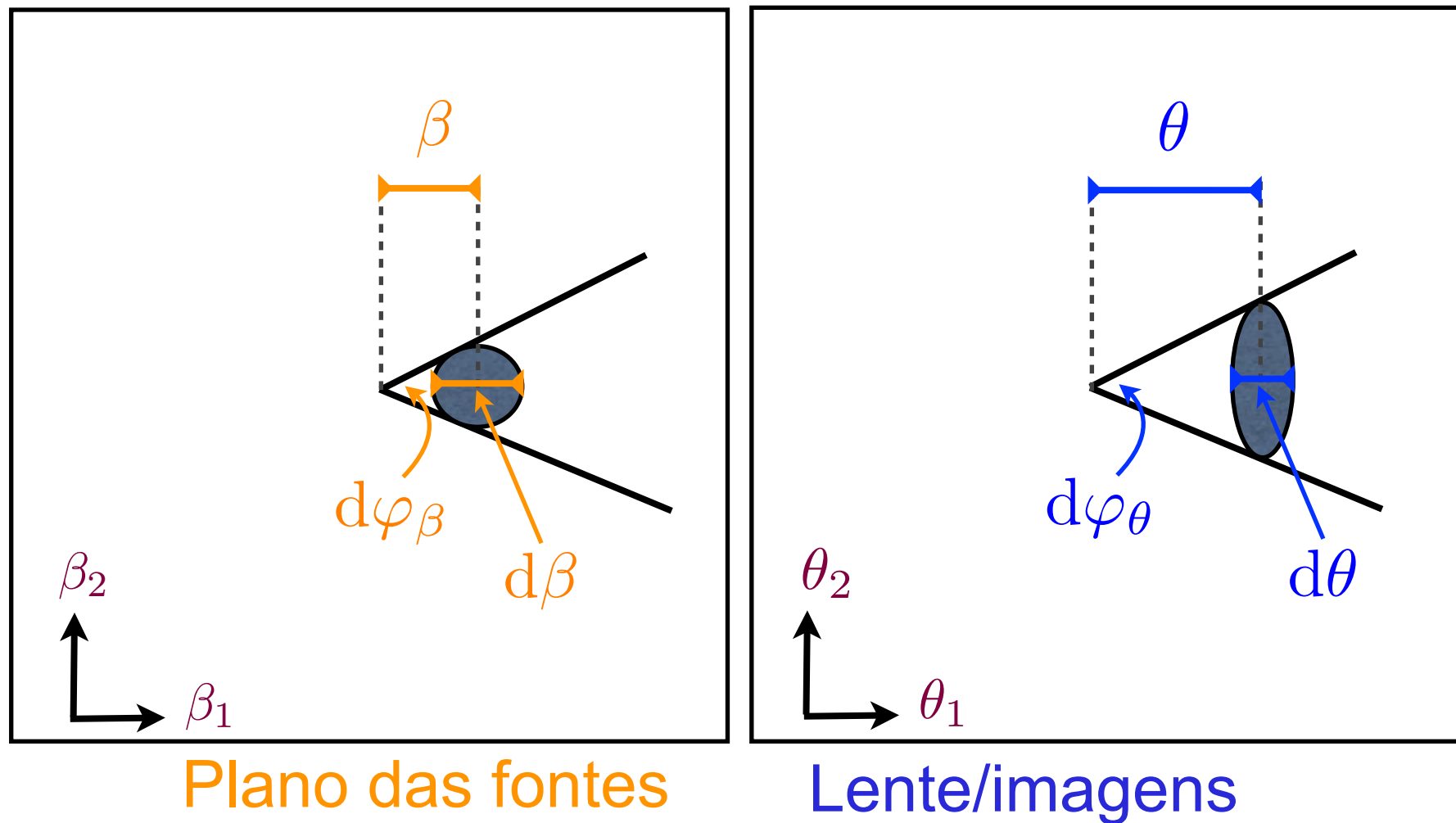
Lente pontual $\hat{\vec{\alpha}} = \frac{4GM}{c^2 \xi} \leftarrow D_{OL} \theta$

Ângulo de deflexão reduzido $\vec{\alpha} = \frac{D_{LS}}{D_{OS} D_{OL}} \frac{4GM}{c^2 \theta}$

Equação da Lente $\beta = \theta - \frac{\theta_E^2}{\theta}$

Ângulo de Einstein $\theta_E = \sqrt{\frac{D_{LS}}{D_{OS} D_{OL}} \frac{4GM}{c^2}}$

Magnificação



Comprimento
radial

$d\beta$

$d\theta$

Magnificação
radial

$$\frac{d\theta}{d\beta}$$

Comprimento
tangencial

$\beta d\varphi_\beta$

$\theta d\varphi_\theta$

Magnificação
tangencial

$$\frac{d\varphi_\theta}{d\varphi_\beta} \frac{\theta}{\beta}$$

$= 1$

Imagens e Magnificação

Equação da Lente $\beta = \theta - \frac{\theta_E^2}{\theta}$

Soluções $\theta_{1,2} = \frac{1}{2} \left(\beta \pm \sqrt{\beta^2 + 4\theta^2} \right)$

Magnificação $\mu = \frac{\theta}{\beta} \frac{d\theta}{d\beta}$

Imagens e Magnificação

Equação da Lente $\beta = \theta - \frac{\theta_E^2}{\theta}$

Soluções $\theta_{1,2} = \frac{1}{2} \left(\beta \pm \sqrt{\beta^2 + 4\theta^2} \right)$

Magnificação $\mu_{1,2} = \left(1 - \left[\frac{\theta_E}{\theta_{1,2}} \right]^4 \right)^{-1} = \frac{1}{2} \pm \frac{u^2 + 2}{2u\sqrt{u^2 + 4}}$

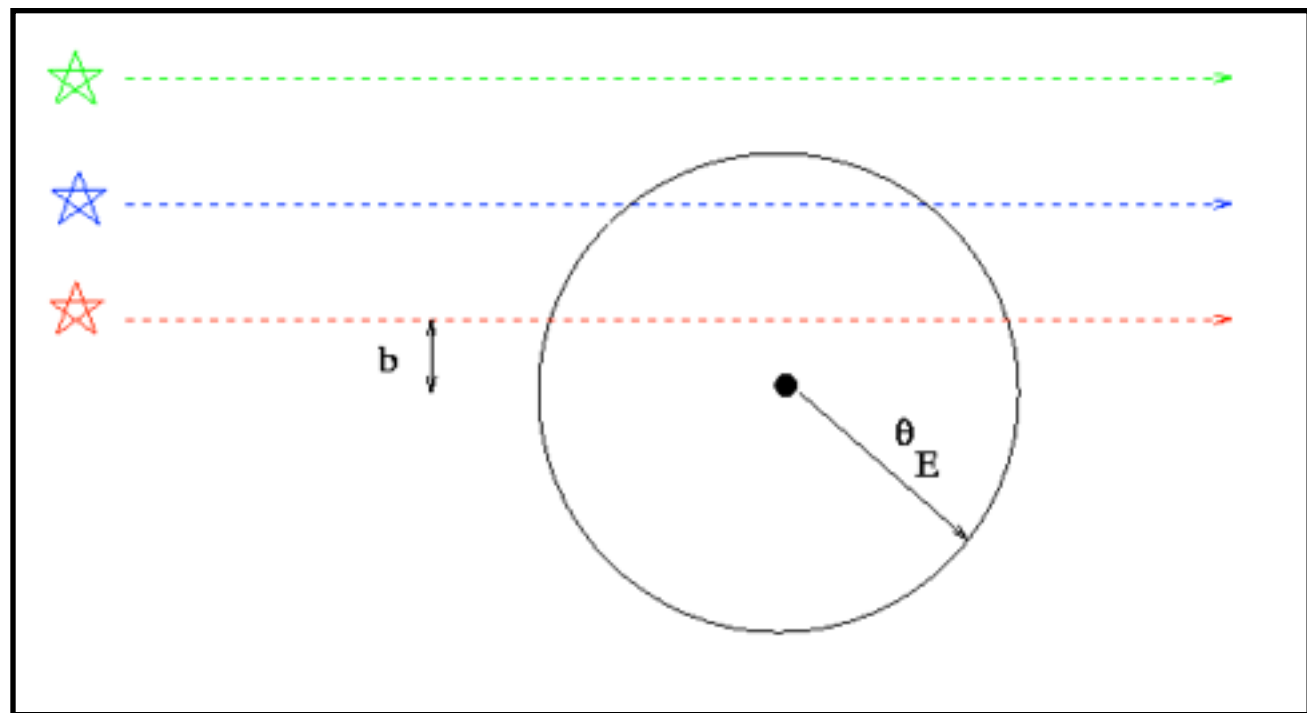
Distância em unidades do ângulo de Einstein $u = \beta/\theta_E$

Magnificação total $\mu = |\mu_1| + |\mu_2| = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}$

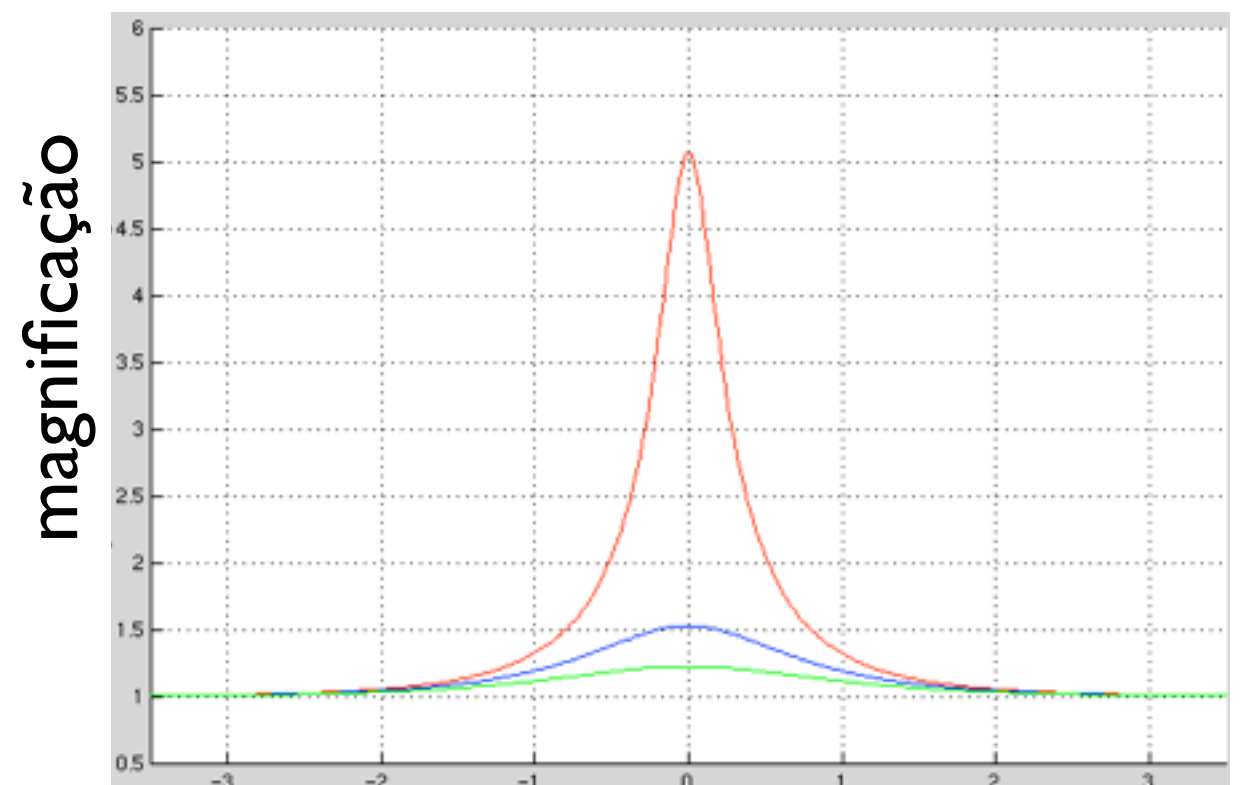
Microlenteamento I

Lente Pontual

$$\mu = \frac{u^2 + 2}{u\sqrt{u^2 + 4}} \quad u = \beta/\theta_E$$



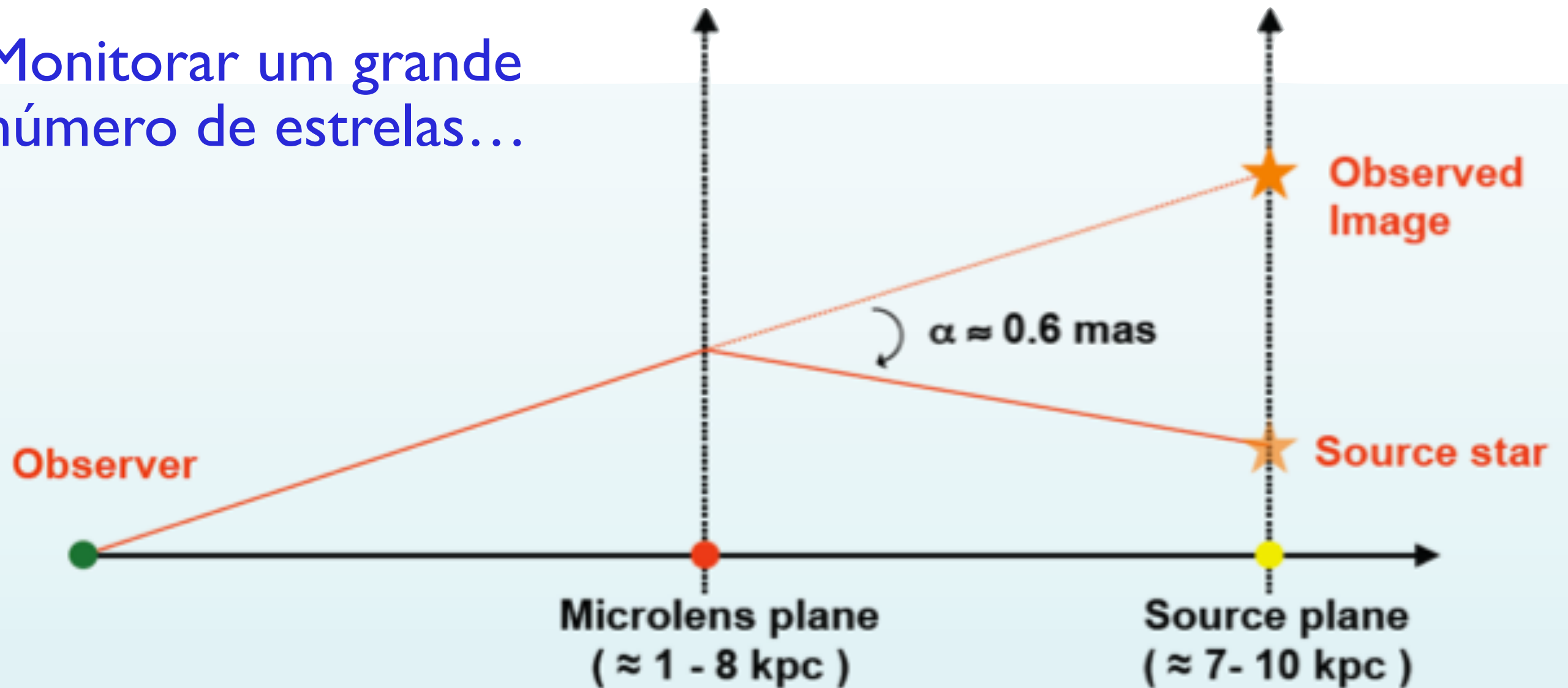
Plano das fontes



Distância à lente na direção x_l
em unidades do raio de Einstein

Micro(mili)lenteamento na galáxia

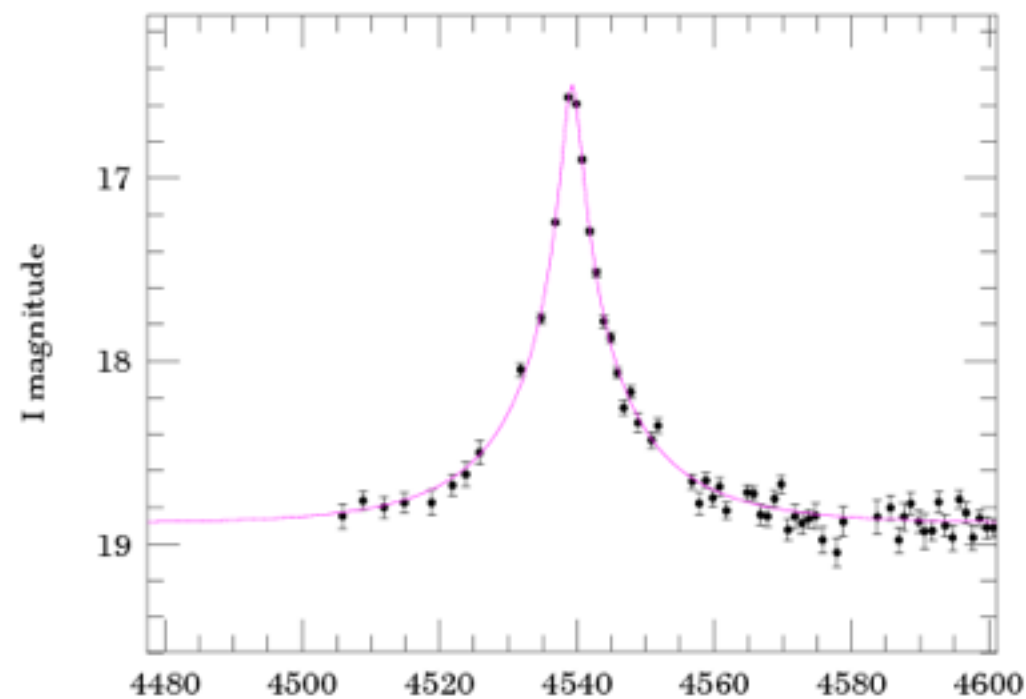
Monitorar um grande número de estrelas...



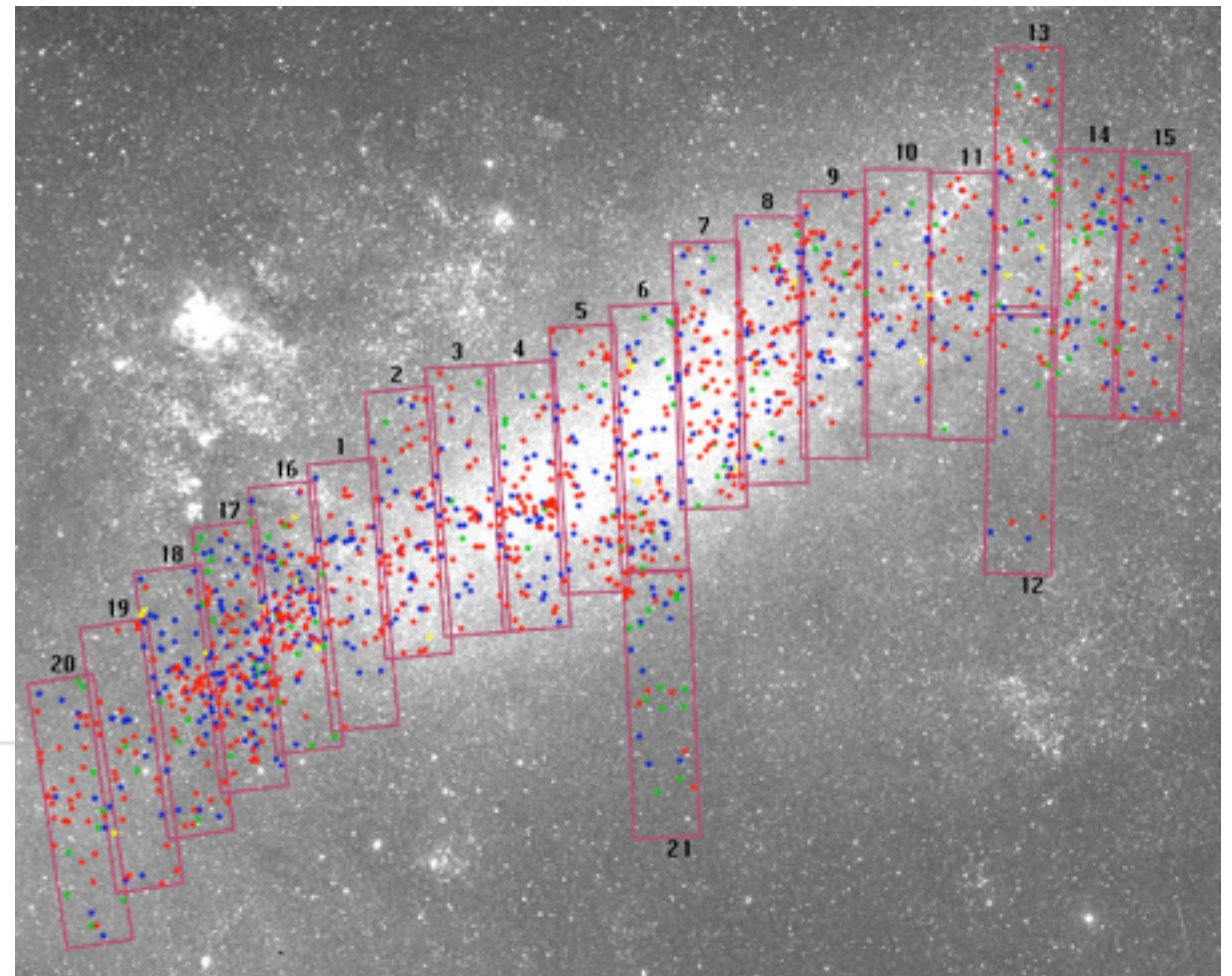
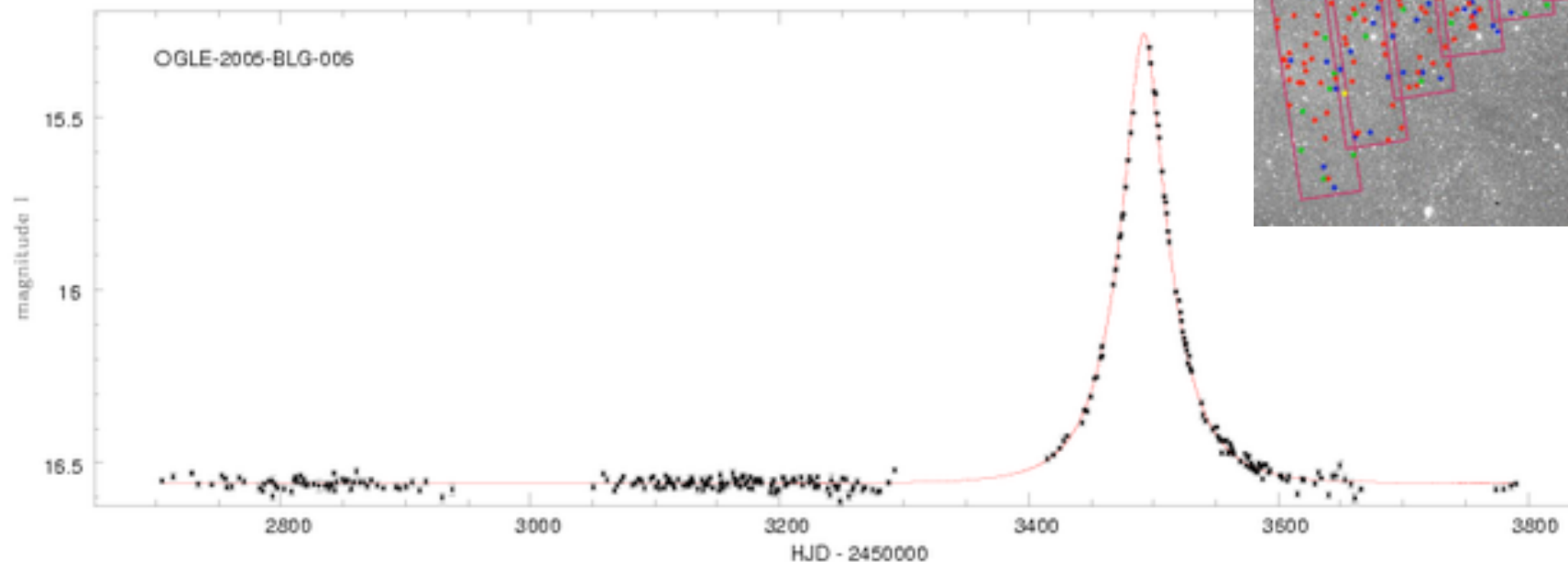
Observações de micro(mili)-lenteamento

Projetos MACHO, OGLE, EROS...

OGLE-2008-BLG-060

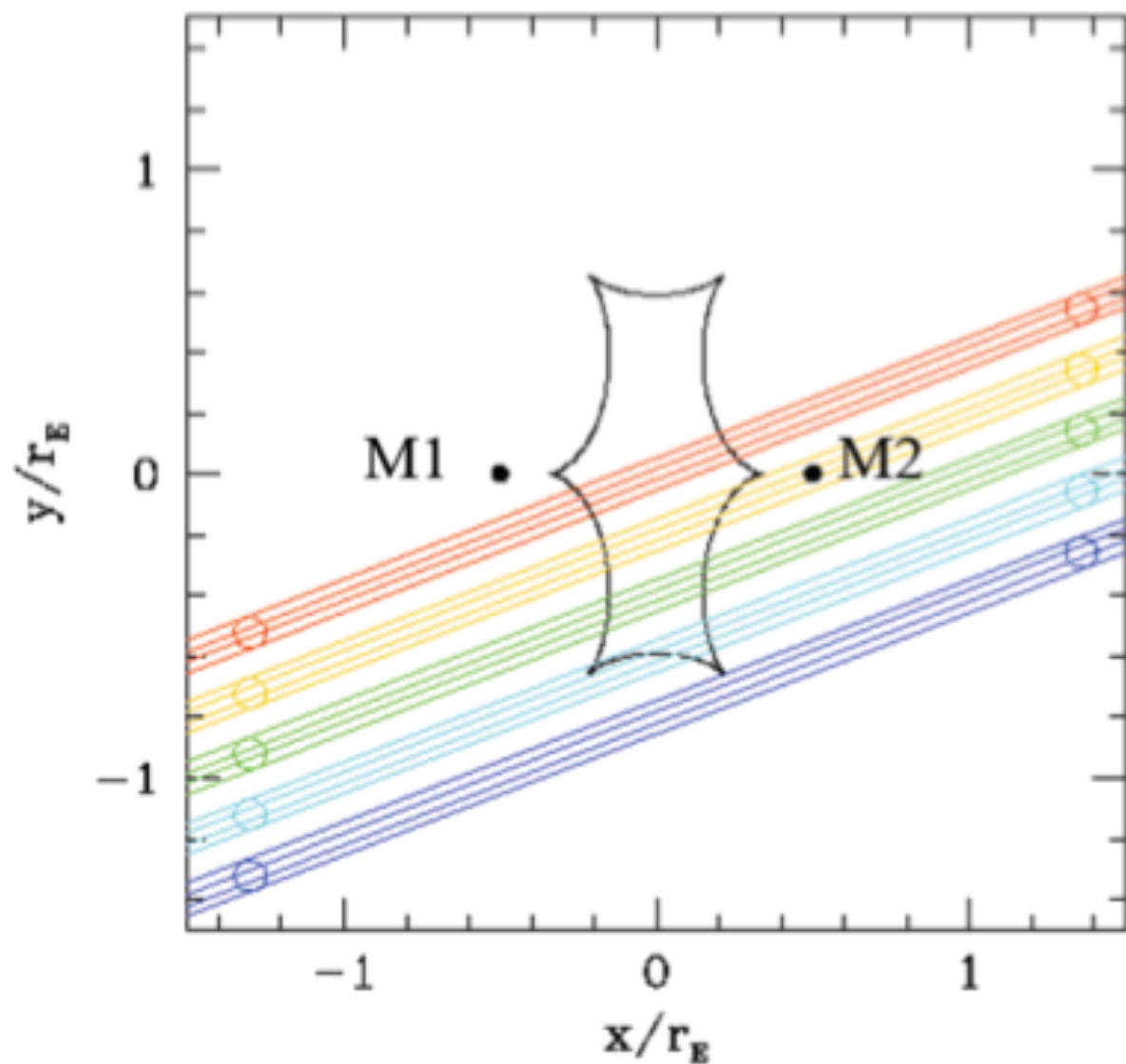


OGLE-2006-BLG-006

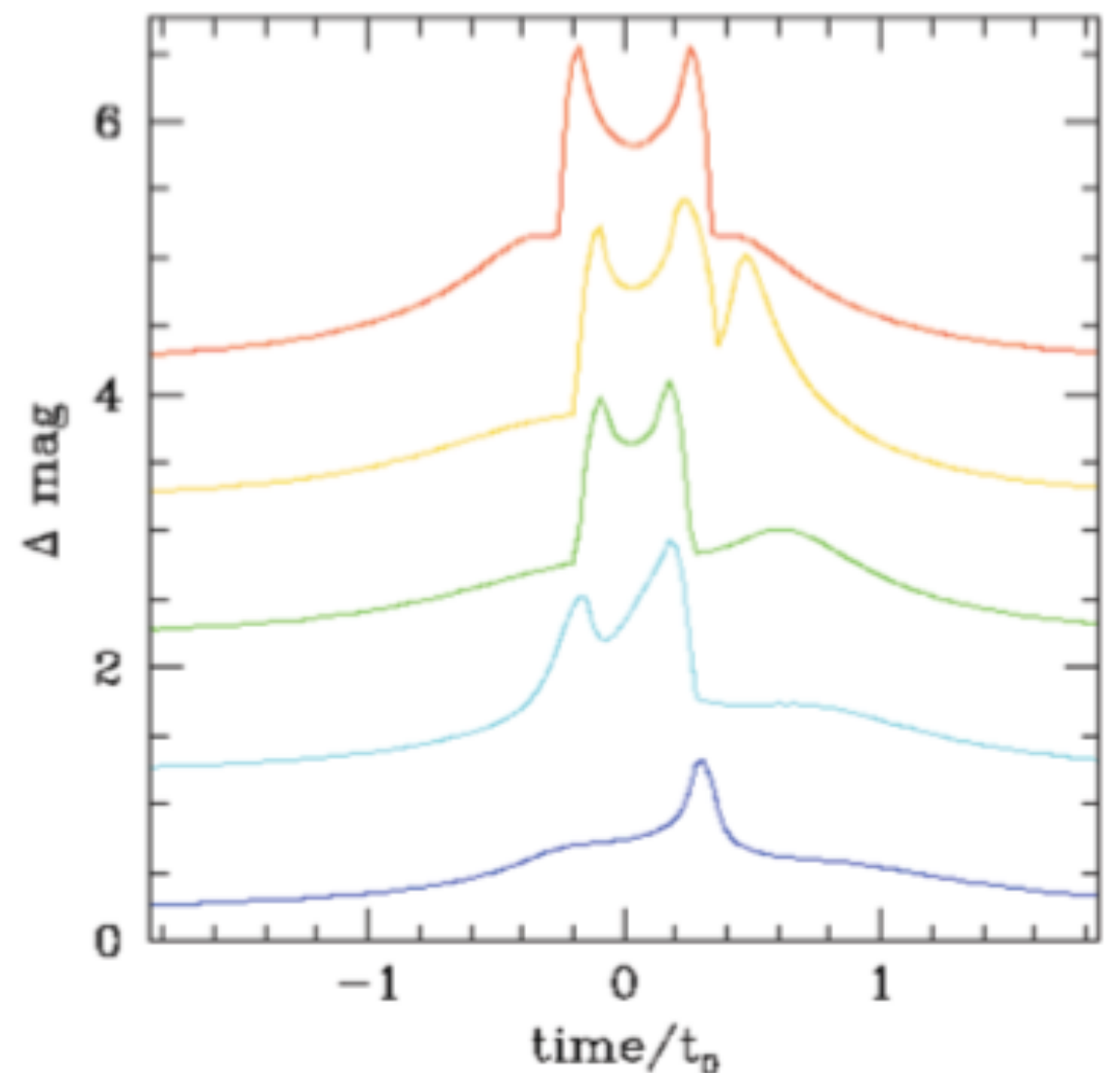


Microlenteamento por lente composta

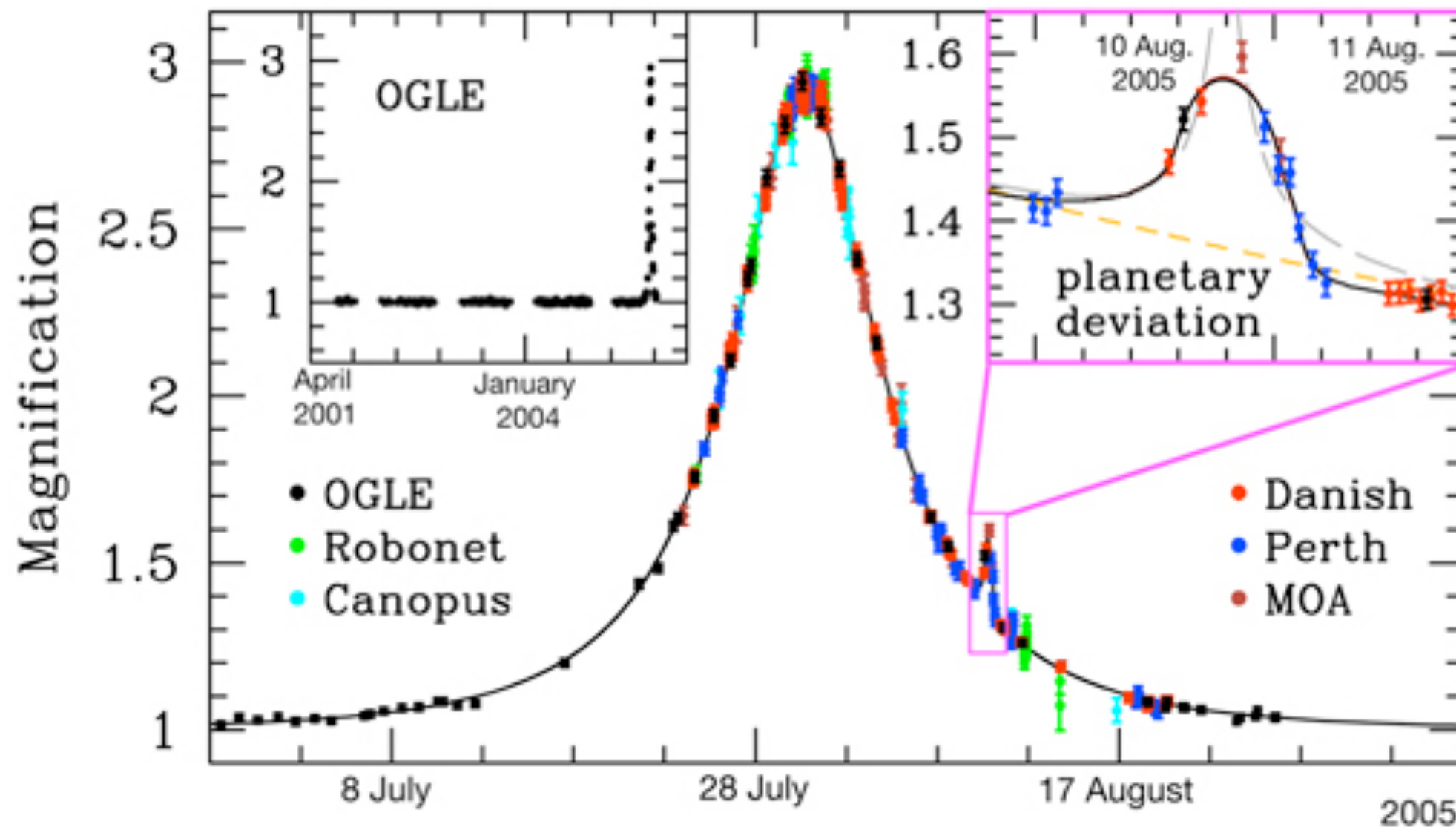
- Estrutura da cáustica



- Curvas de luz



Primeiro planeta: 2004



Fevereiro 2019

89 planetas
descobertos por
microlenteamento

Light Curve of OGLE-2005-BLG-390

ESO PR Photo 03b/06 (January 25, 2006)

© ESO



LEITURA SUGERIDA

- Curso de lentes com exemplos em python:
<http://pico.oabo.inaf.it/~massimo/teaching.html>
<https://github.com/maxmen/LensingLectures/>
- Peter Schneider, Christopher S. Kochanek, Joachim Wambsganss, 33rd Advanced Saas-Fee Course on Gravitational Lensing (2003):
 - Saas-Fee Advanced Course 33 - Springer
 - <https://astro.uni-bonn.de/~peter/SaasFee.html>
- Ramesh Narayan and Matthias Bartelmann, Lectures on Gravitational Lensing (Proceedings of the 1995 Jerusalem Winter School)
 - <http://arxiv.org/abs/astro-ph/9606001>,
 - <http://www.mpa-garching.mpg.de/Lenses/JeruLect.html>, (recomendo a opção: http://www.mpa-garching.mpg.de/Lenses/JL_co_art.ps.gz para baixar direto)
- Schneider P., Elhers J., Falco E. E., 1992, Gravitational lenses. Springer-Verlag, Berlin
- Mollerach, S, Roulet, E, 2002. Gravitational Lensing and Microlensing, World Scientific, Singapore.
- Petters A., O., Levine H., Wambsganss J., 2001, Singularity Theory and Gravitational Lensing. Birkhäuser, Boston

PROGRAMA

- Aula 1: *Overview* de lentes e formalismo básico
 - Introdução histórica
 - Equação da lente
 - Lente pontual
 - Microlensing
- Aula 2: Conceitos e formalismo
 - Lentes extensas, simetria axial, SIS
 - Cáusticas e curvas críticas
 - Arcos e anéis de Einstein
 - Reconstrução das massas e cosmologia
- Aula 3: Desvio temporal e efeito fraco de lentes
- Aula 4: Revisão de cosmologia
- Aula 5: Cosmologia e lentes (CMB, LSS, DM, MoG...)



LENTES EXTENSAS

Lentes Extensas

Fonte pontual $\hat{\vec{\alpha}} = \frac{4GM}{c^2 \xi}$

Densidade superficial de massa

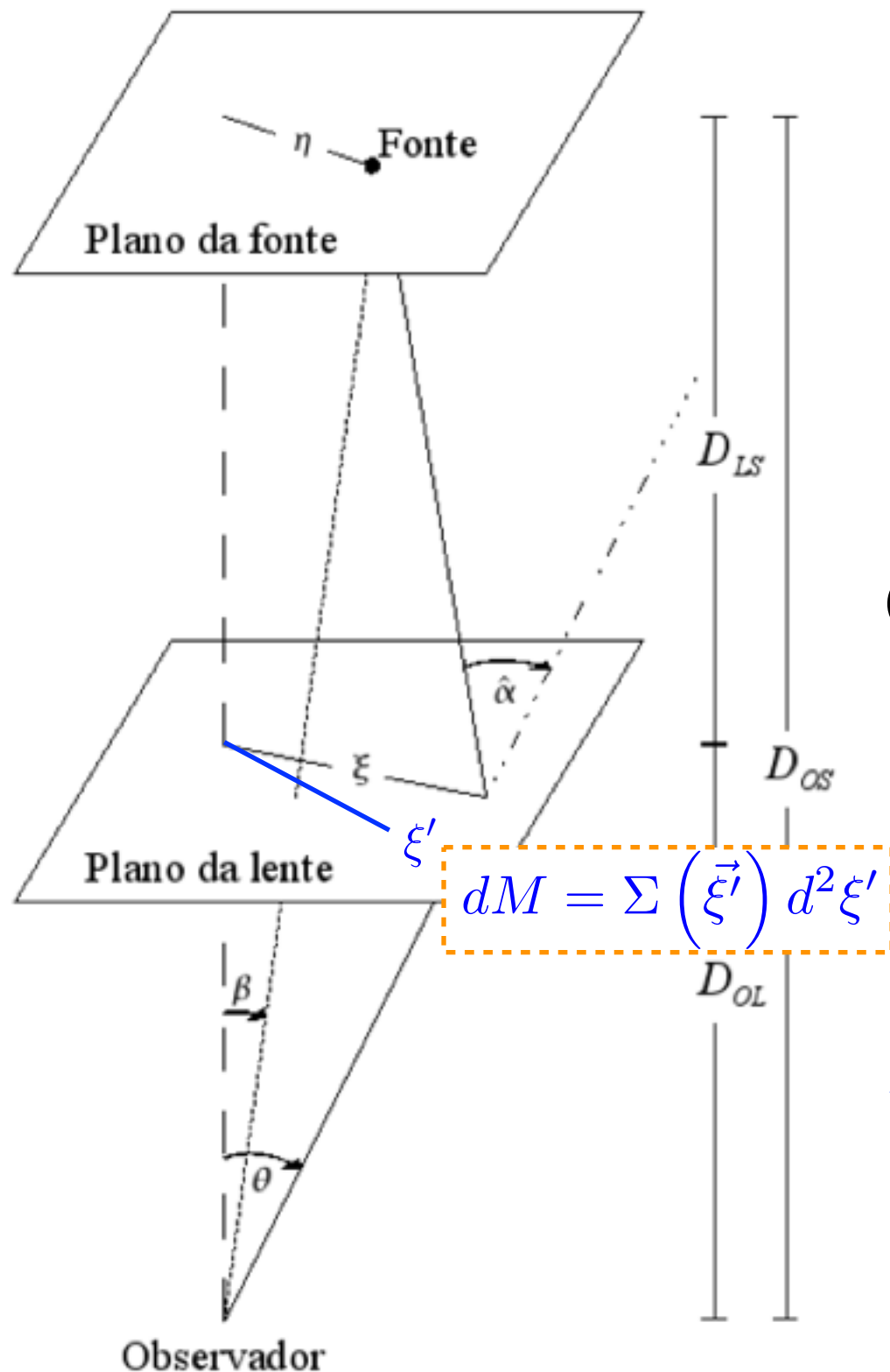
$$\Sigma(\vec{\xi}) = \int_0^\infty dz \rho(\vec{\xi}, z)$$

Contribuição do elemento de área

$$d\hat{\vec{\alpha}} = \frac{4G}{c^2} \Sigma(\vec{\xi}') d^2\xi' \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2}$$

Ângulo de deflexão

$$\hat{\vec{\alpha}} = \frac{4G}{c^2} \int d^2\xi' \Sigma(\vec{\xi}') \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2}$$



Lentes Extensas

Potencial projetado $\psi(\vec{\xi}) = \int dz \varphi(\vec{\xi}, z)$

Equação de Poisson $\nabla_{\xi}^2 \psi(\vec{\xi}) = 4\pi G \Sigma(\vec{\xi})$

Usando a função de Green 2D

$$\psi(\vec{\xi}) = 2G \int d^2\xi' \Sigma(\vec{\xi}') \ln |\xi - \xi'|$$

Comparando com o ângulo de deflexão

$$\hat{\vec{\alpha}} = \frac{4G}{c^2} \int d^2\xi' \Sigma(\vec{\xi}') \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2}$$

Obtemos $\hat{\vec{\alpha}} = \frac{2}{c^2} \vec{\nabla}_{\xi} \psi(\vec{\xi})$

Lentes Extensas

Ângulo de deflexão $\hat{\vec{\alpha}} = \frac{2}{c^2} \vec{\nabla}_{\xi} \psi \left(\vec{\xi} \right)$

convertendo variáveis $d\xi_i = D_{OL} d\theta_i$

Ângulo de deflexão reduzido

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}} = \frac{2}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \vec{\nabla}_{\theta} \psi \left(\vec{\theta} \right)$$

Potencial de Lente(amento) $\Psi(\vec{x}) \equiv \frac{2}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \psi$

Equação da lente $\vec{\beta} = \vec{\theta} - \vec{\alpha} \left(\vec{\theta} \right) = \vec{\theta} - \vec{\nabla}_{\theta} \Psi \left(\vec{\theta} \right)$

Mapeamento de Lentes

► mapeamento imagem $\rightarrow$ fonte

$$\frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial^2 \Psi(\vec{\theta})}{\partial \theta_i \partial \theta_j}$$

► plano único

$$\Psi = \frac{2}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \int \phi(\xi, z) dz$$

potencial gravitacional

distâncias cosmológicas

$$D_{LS} = D_A(z_L, z_S) \dots$$

► autovalores:

$$\mu_1 = \frac{1}{1 - \kappa + \gamma}, \mu_2 = \frac{1}{1 - \kappa - \gamma}$$

► magnificação local e razão axial:

$$\mu = \mu_1 \mu_2$$

$$r = \left| \frac{\mu_1}{\mu_2} \right|$$

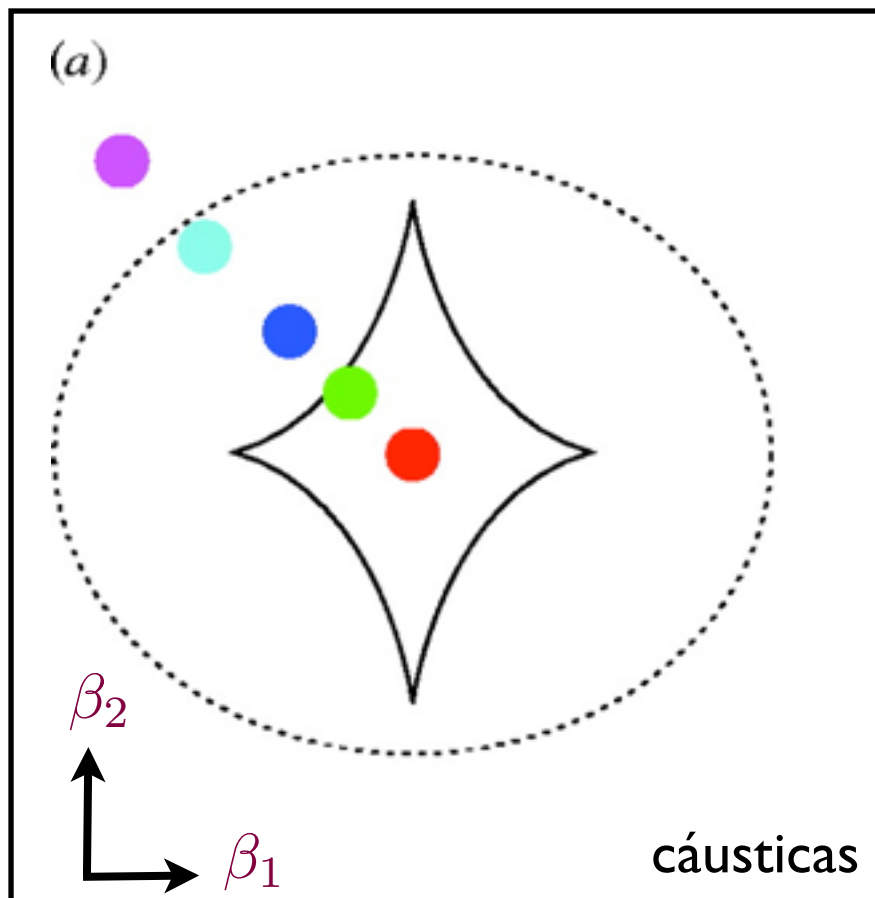
Mapeamento de Lentes

► mapeamento imagem $\rightarrow$ fonte

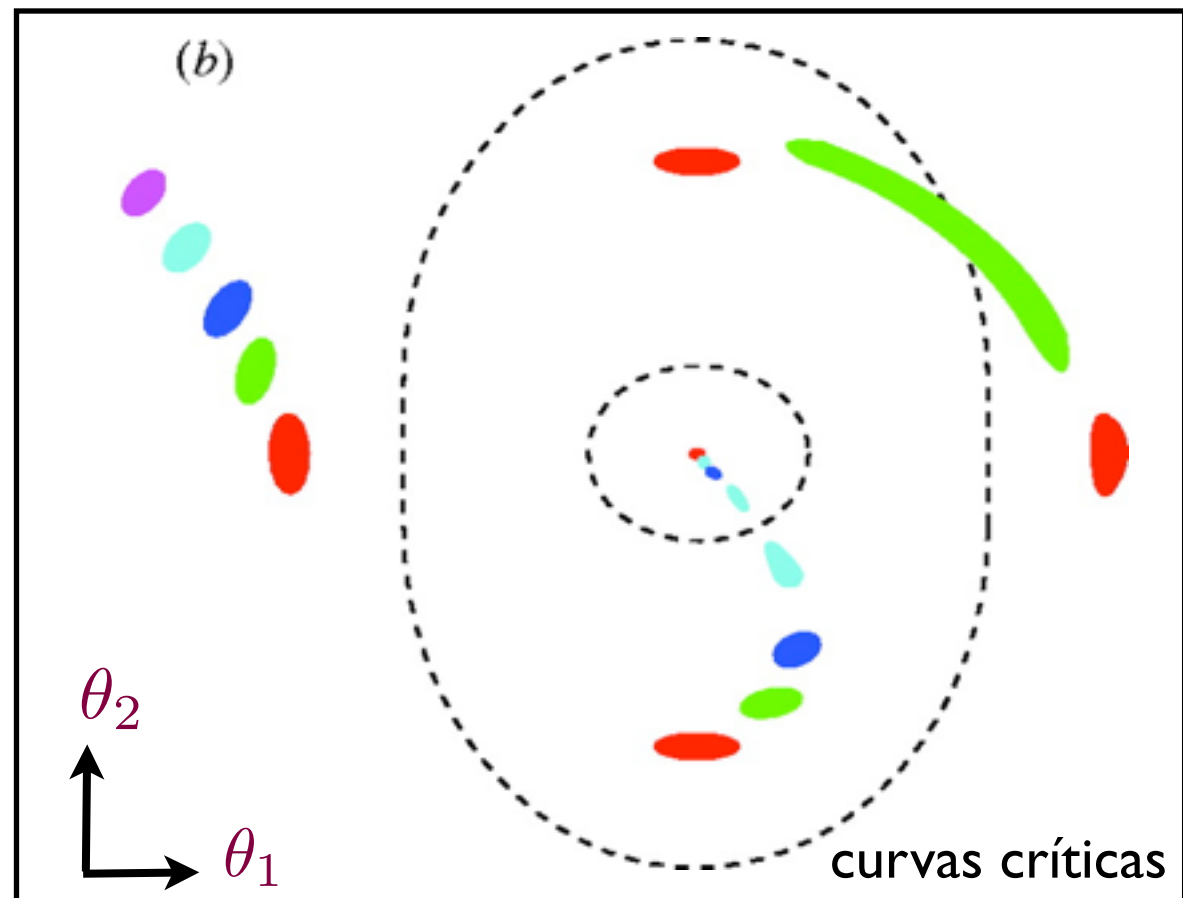
$$\frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial^2 \Psi(\vec{\theta})}{\partial \theta_i \partial \theta_j}$$

► autovalores:

$$\mu_1 = \frac{1}{1 - \kappa + \gamma}, \mu_2 = \frac{1}{1 - \kappa - \gamma}$$



Plano das fontes



Lente/imagens

ARCOS GRAVITACIONAIS

The background of the slide is a deep space photograph filled with numerous galaxies. Many of these galaxies are distorted into arcs and multiple images, which is a result of gravitational lensing by massive foreground objects. The galaxies vary in color, including yellow, orange, and blue, and are scattered across the dark cosmic background.

MACROLENTEAMENTO
EFEITO FORTE

“Anéis de Chwolson-Einstein”

Über eine mögliche Form fiktiver Doppelsterne. Von O. Chwolson.

Es ist gegenwärtig wohl als höchst wahrscheinlich anzunehmen, daß ein Lichtstrahl, der in der Nähe der Oberfläche eines Sternes vorbeigeht, eine Ablenkung erfährt. Ist γ diese Ablenkung und γ_0 der Maximumwert an der Oberfläche, so ist $\gamma_0 \geq \gamma \geq 0$. Die Größe des Winkels ist bei der Sonne $\gamma_0 = 1''.7$; es dürften aber wohl Sterne existieren, bei denen γ_0 gleich mehreren Bogensekunden ist; vielleicht auch noch mehr. Es sei A ein großer Stern (Gigant), T die Erde, B ein entfernter Stern; die Winkeldistanz zwischen A und B , von T aus gesehen, sei α , und der Winkel zwischen A und T , von B aus gesehen, sei β . Es ist dann

$$\gamma = \alpha + \beta.$$

Ist B sehr weit entfernt, so ist annähernd $\gamma = \alpha$. Es kann also α gleich mehreren Bogensekunden sein, und der Maximumwert von α wäre etwa gleich γ_0 . Man sieht den Stern B von der Erde aus an zwei Stellen: direkt in der Richtung TB und außerdem nahe der Oberfläche von A , analog einem Spiegelbild. Haben wir mehrere Sterne B, C, D , so würden die Spiegelbilder umgekehrt gelegen sein wie in

Petrograd, 1924 Jan. 28.

einem gewöhnlichen Spiegel, nämlich in der Reihenfolge D, C, B , wenn von A aus gerechnet wird (D wäre am nächsten zu A).



Der Stern A würde als fiktiver Doppelstern erscheinen. Teleskopisch wäre er selbstverständlich nicht zu trennen. Sein Spektrum bestände aus der Übereinanderlagerung zweier, vielleicht total verschiedenartiger Spektren. Nach der Interferenzmethode müßte er als Doppelstern erscheinen. Alle Sterne, die von der Erde aus gesehen rings um A in der Entfernung $\gamma_0 - \beta$ liegen, würden von dem Stern A gleichsam eingefangen werden. Sollte zufällig TAB eine gerade Linie sein, so würde, von der Erde aus gesehen, der Stern A von einem Ring umgeben erscheinen.

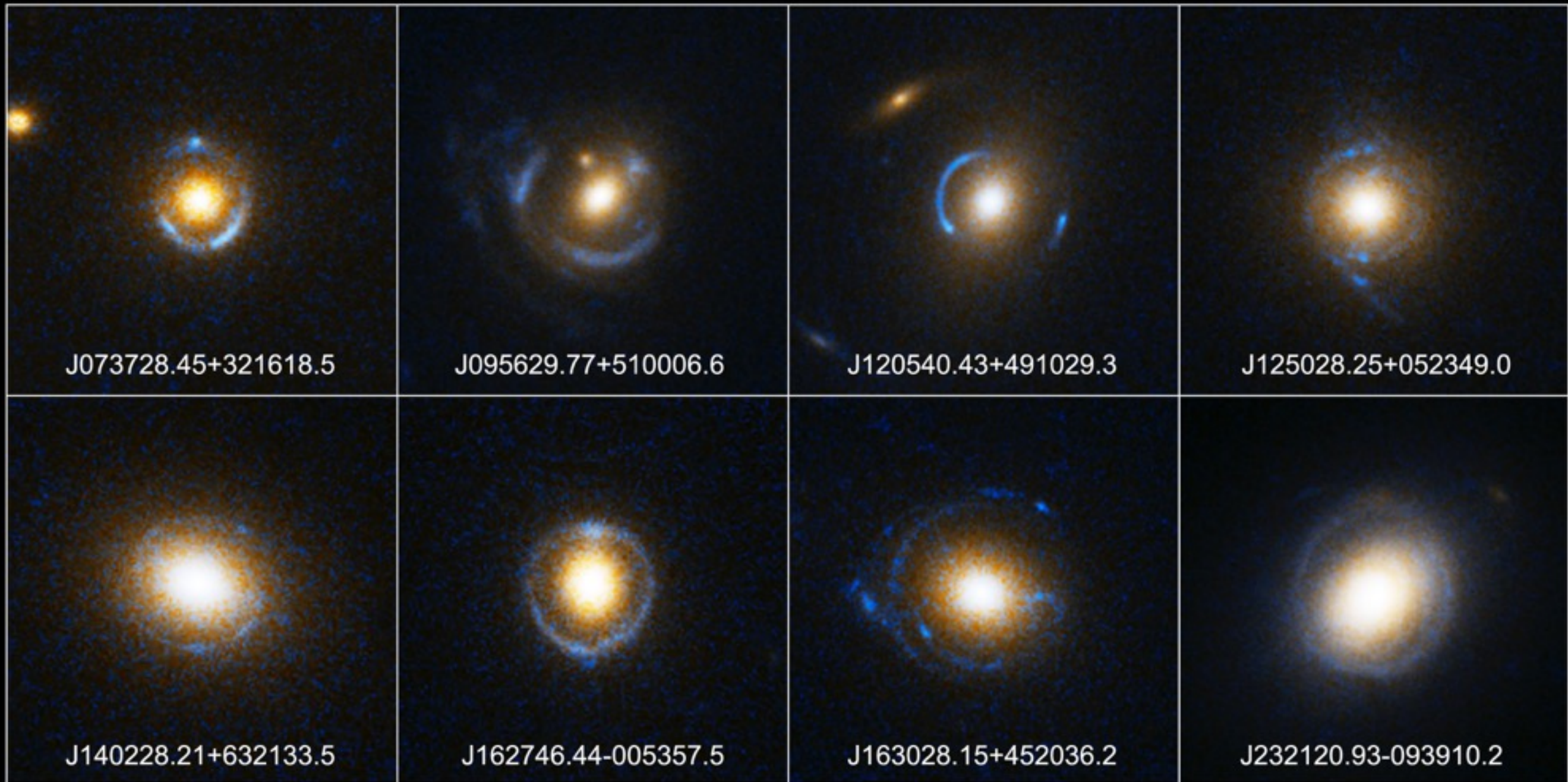
Ob der hier angegebene Fall eines fiktiven Doppelsternes auch wirklich vorkommt, kann ich nicht beurteilen.

O. Chwolson.

Einstein Ring Gravitational Lenses

Hubble Space Telescope • Advanced Camera for Surveys

“Anéis de Chwolson-Einstein”



Einstein Ring Gravitational Lenses
Hubble Space Telescope • Advanced Camera for Surveys

A RELATIVISTIC ECLIPSE

IT is a familiar aphorism that the theory of relativity, despite its enormous importance, both in physics and philosophy, may be forgotten in ordinary practical life. There is a good reason. In almost every known case its results agree so closely with those of the older "classical" theories that very accurate observations are required to distinguish between them. Thus, for example, the famous Michelson-Morley experiment involves four series of the

What Might be Seen from a Planet Conveniently Placed Near the Companion of Sirius . . . Perfect Tests of General Relativity that are Unavailable

By HENRY NORRIS RUSSELL, Ph. D.

Chairman of the Department of Astronomy and Director of the Observatory at Princeton University. Research Associate of the Mount Wilson Observatory of the Carnegie Institution of Washington. President of the American Astronomical Society.

FEBRUARY • 1937

SCIENTIFIC AMERICAN

77

cludes gravitation in its scope) are of this sort. The advance of the perihelion of Mercury as an instance is familiar, but definitely unaccountable until

stein himself. From a point exactly in line with the distant star would appear to be

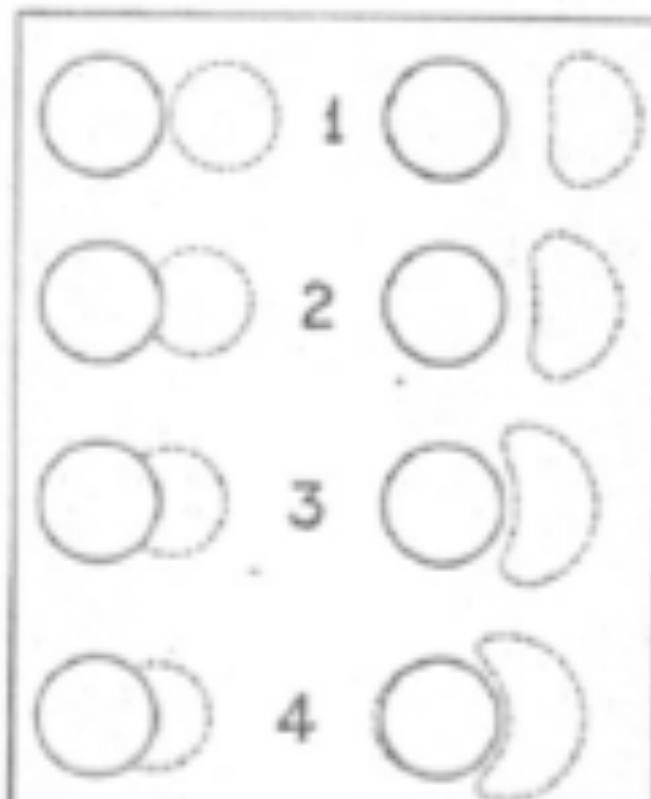
for it. The focusing effect, however, is not infinite. The light of the dis-

Ninguém previu os arcos gravitacionais?

For a star



be resolved by even the greatest tele-



point, with no angular diameter at all, the increase in brightness would be infinite. But any real star must have a finite angular size, however small, and for such a star the increase in brightness, though it might be large, would have a limit.

If a bright star passed in front of a fainter one, its own light would drown out the effect; but if one of the faint red stars, which are really more abundant than any other sort, should get directly

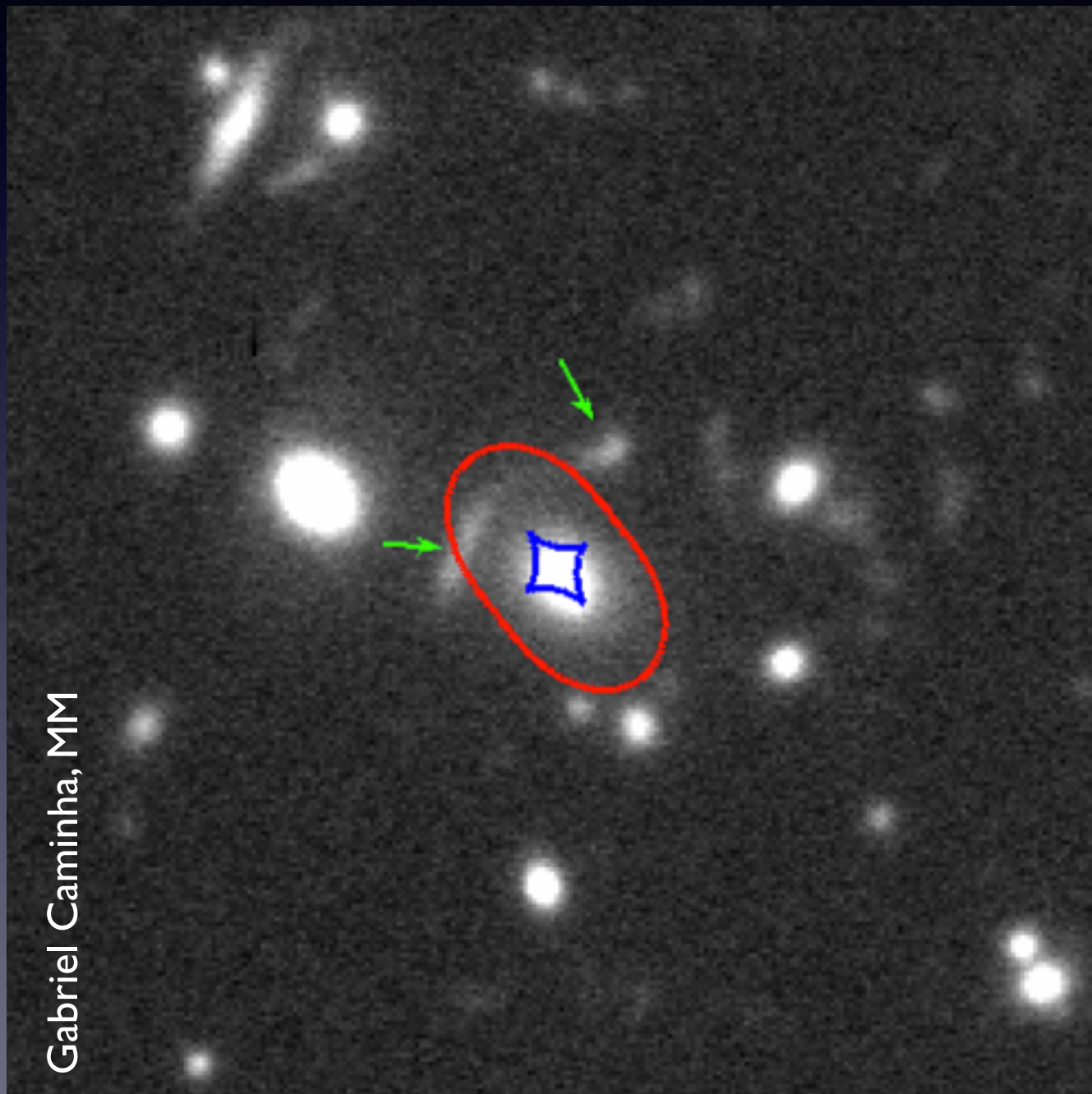
My hearty thanks are due to Professor Einstein, who permitted me to see the manuscript of his note before its publication.—*Princeton University Observatory, December 2, 1936.*

is, a million miles or so—from the line through the center of the two stars. The

A wide-angle photograph of a beach with turquoise water and a clear blue sky. The text is overlaid in the center.

RECONSTRUÇÃO DA MASSA E COSMOLOGIA

Modelagem inversa



Gabriel Caminha, MM

Arcos e imagens múltiplas

Parâmetros da distribuição de matéria

- ✓ Perfis de aglomerados em regiões centrais
- ✓ Subestrutura

Sensibilidade à cosmologia

Reconstrução das fontes

Desafios para usar arcos como sondas para a física fundamental:

- 🌀 estatística
- 🌀 sistemáticos

Modelagem Inversa

Através das posições das imagens múltiplas é possível determinar propriedades da distribuição de massa da lente

Definimos a função:

$$\chi_{\text{lente}}^2 := \sum_i \left(\frac{\vec{\theta}_i^{\text{obs}} - \vec{\theta}^{\text{mod}}(\vec{\beta}, \vec{\Pi})}{\sigma_i^{\text{obs}}} \right)^2$$

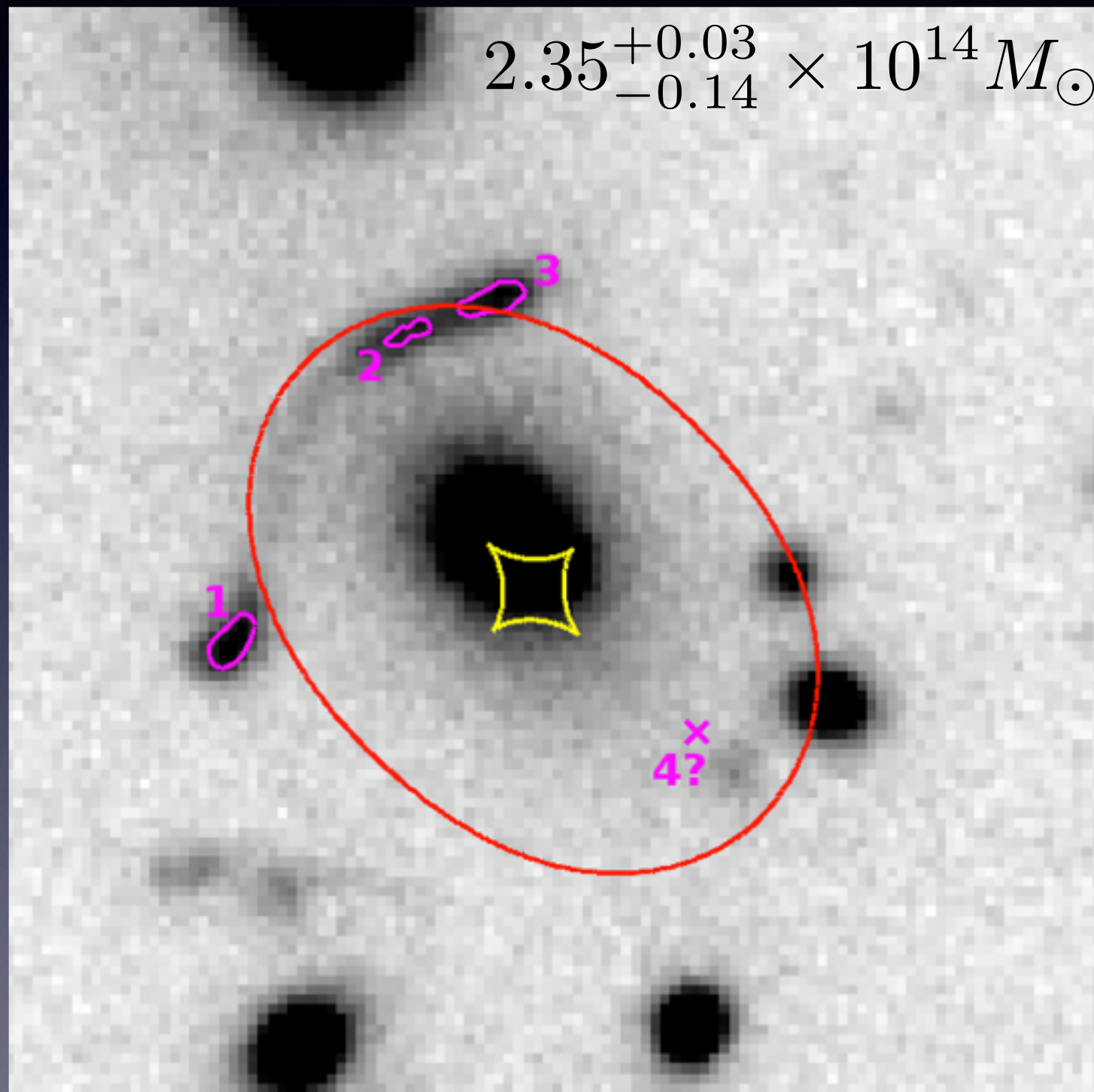
Posição das imagens múltiplas

Erro associado à
posição das imagens
múltiplas

Os parâmetros que minimizam esta função (ou maximizam a verossimilhança) são os parâmetros de melhor ajuste

Podemos citar os programas *lenstool*, *gravlens* e *glafic* que fazem a modelagem inversa da lente,

Modelagem inversa para o sistema SOGRAS0041-0043



Modelagem com lenstool (Jullo, Kneib)

Ajuste 1: 3 imagens

Ajuste 2: 4 imagens

	Ajuste 1	Ajuste 2
$\sigma_v [km/s]$	622^{+11}_{-13}	642^{+3}_{-3}
$\theta_{or} [^{\circ}]$	$135.2^{+0.7}_{-0.8}$	$135.2^{+1.5}_{-1.3}$
$x_{lente} ["]$	$0.50^{+0.2}_{-0.2}$	$0.50^{+0.05}_{-0.06}$
$y_{lente} ["]$	$-0.76^{+0.17}_{-0.15}$	$-0.86^{+0.06}_{-0.03}$
ϵ	—	$0.13^{+0.02}_{-0.03}$
χ^2/dof	0.25	0.22

~ 0.9'' de desvio entre galáxia central e centro da distribuição de massa
([Zitrin et al. 2012](#))

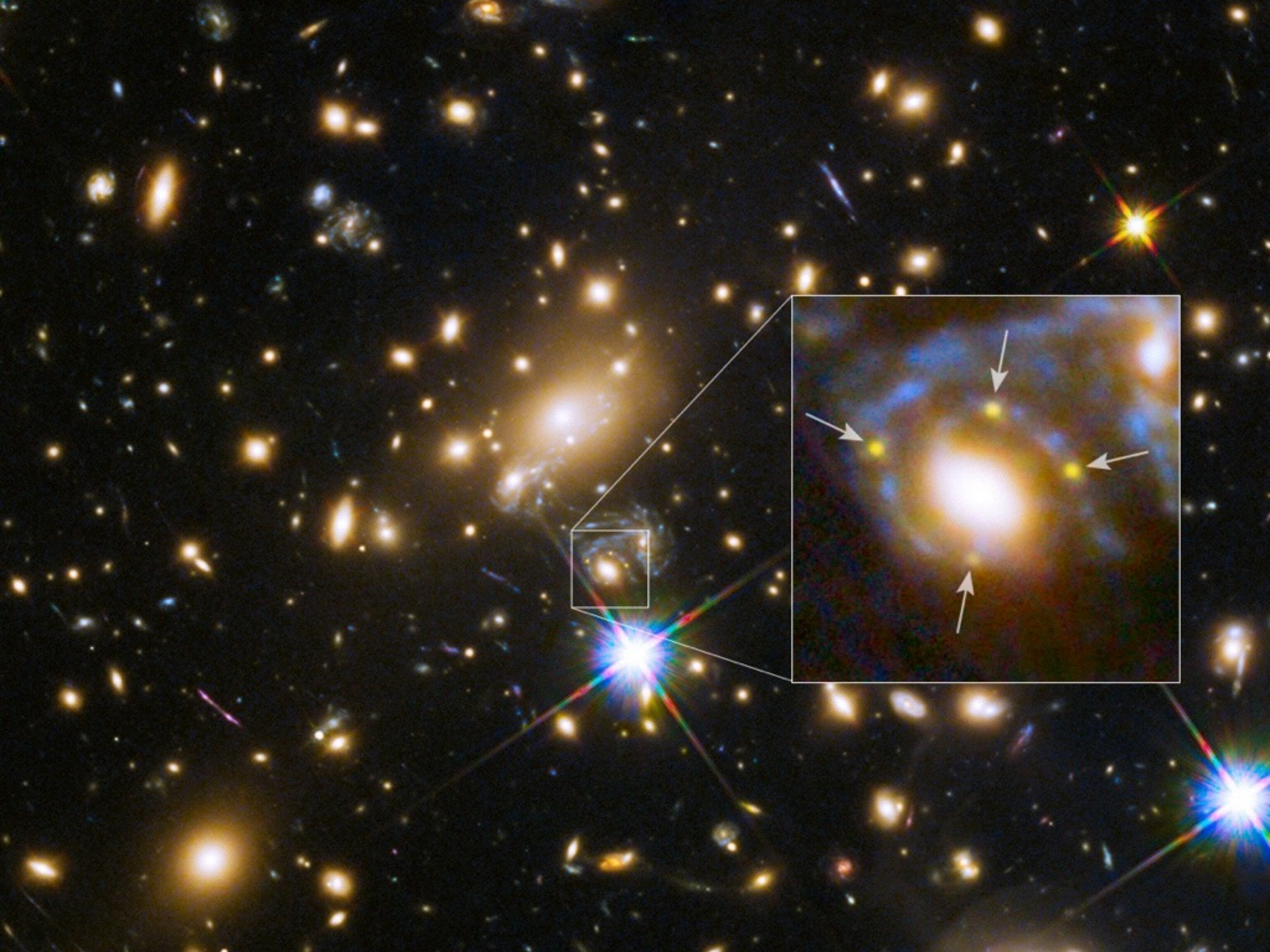
Estimativa de erro a partir das simulações (Caminha et al. in prep.):

~ 8% de viés na massa

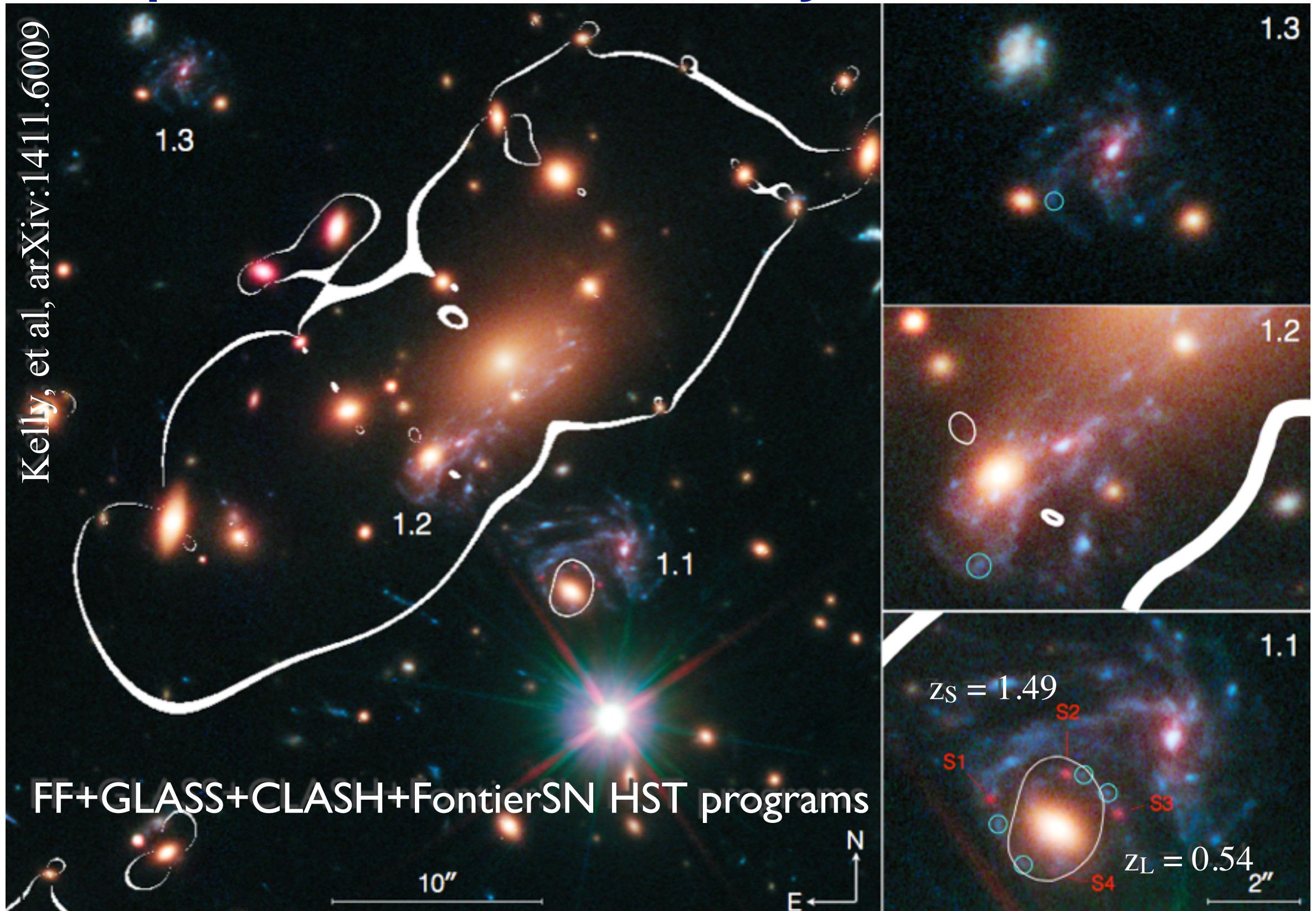
~ 5% incerteza estatística

A scenic view of a beach with turquoise water and a clear blue sky. The text is overlaid on the image.

MODELAGEM DE LENTES E MÉTODO CIENTÍFICO



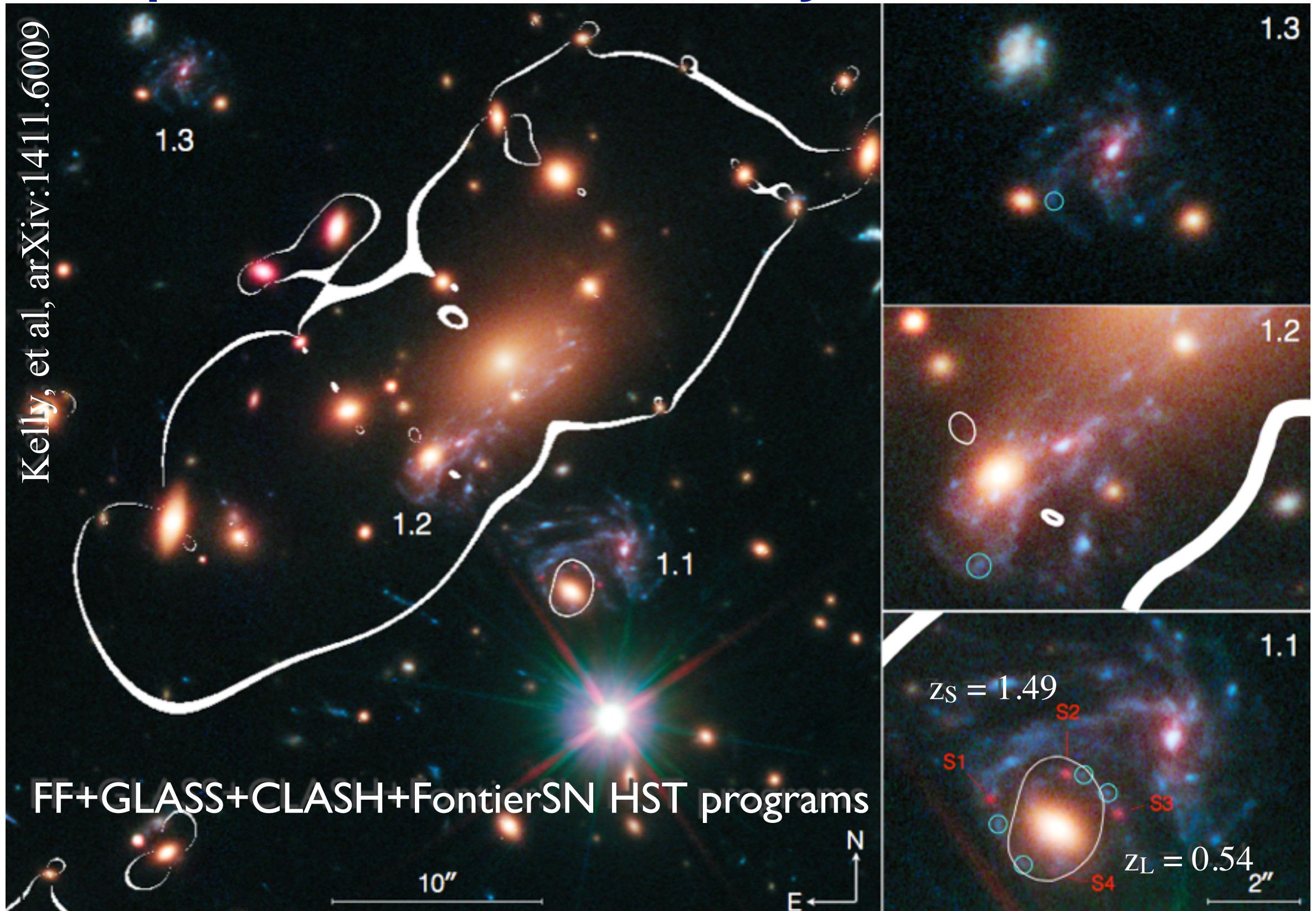
Supernova em MACS J1149.6+2223



Imagens múltiplas por galáxia do aglomerado
+ pelo potencial global (passado/futuro)

When Refsdal
meets Popper

Supernova em MACS J1149.6+2223



Usar a previsão do surgimento de imagens múltiplas da supernovas para testar os modelos

When Refsdal meets Popper!

THE STORY OF SUPERNOVA ‘REFSDAL’ TOLD BY MUSE*

C. GRILLO¹, W. KARMAN², S. H. SUYU³, P. ROSATI⁴, I. BALESTRA⁵, A. MERCURIO⁶, M. LOMBARDI⁷, T. TREU⁸,
G. B. CAMINHA⁴, A. HALKOLA, S. A. RODNEY^{9,10,11}, R. GAVAZZI¹², K. I. CAPUTI²

Draft version March 7, 2016

ABSTRACT

We present Multi Unit Spectroscopic Explorer (MUSE) observations in the core of the Hubble Frontier Fields (HFF) galaxy cluster MACS J1149.5+2223, where the first magnified and spatially-resolved multiple images of supernova (SN) ‘Refsdal’ at redshift 1.489 were detected. Thanks to a Director’s Discretionary Time program with the Very Large Telescope and the extraordinary efficiency of MUSE, we measure 117 secure redshifts with just 4.8 hours of total integration time on a single 1 arcmin² target pointing. We spectroscopically confirm 68 galaxy cluster members, with redshift values ranging from 0.5272 to 0.5660, and 18 multiple images belonging to 7 background, lensed sources distributed in redshifts between 1.240 and 3.703. Starting from the combination of our catalog with those obtained from extensive spectroscopic and photometric campaigns using the *Hubble Space Telescope*, we select a sample of 300 (164 spectroscopic and 136 photometric) cluster members, within approximately 500 kpc from the brightest cluster galaxy, and a set of 88 reliable multiple images associated to 10 different background source galaxies and 18 distinct knots in the spiral galaxy hosting SN ‘Refsdal’. We exploit this valuable information to build 6 detailed strong lensing models, the best of which reproduces the observed positions of the multiple images with a root-mean-square offset of only 0.26″. We use these models to quantify the statistical and systematic errors on the predicted values of magnification and time delay of the next emerging image of SN ‘Refsdal’. We find that its peak luminosity should occur between March and June 2016, and should be approximately 20% fainter than the dimmest (S4) of the previously detected images but above the detection limit of the planned *HST*/WFC3 follow-up. We present our two-dimensional reconstruction of the cluster mass density distribution and of the SN ‘Refsdal’ host galaxy surface brightness distribution. We outline the roadmap towards even better strong lensing models with a synergetic MUSE and *HST* effort.

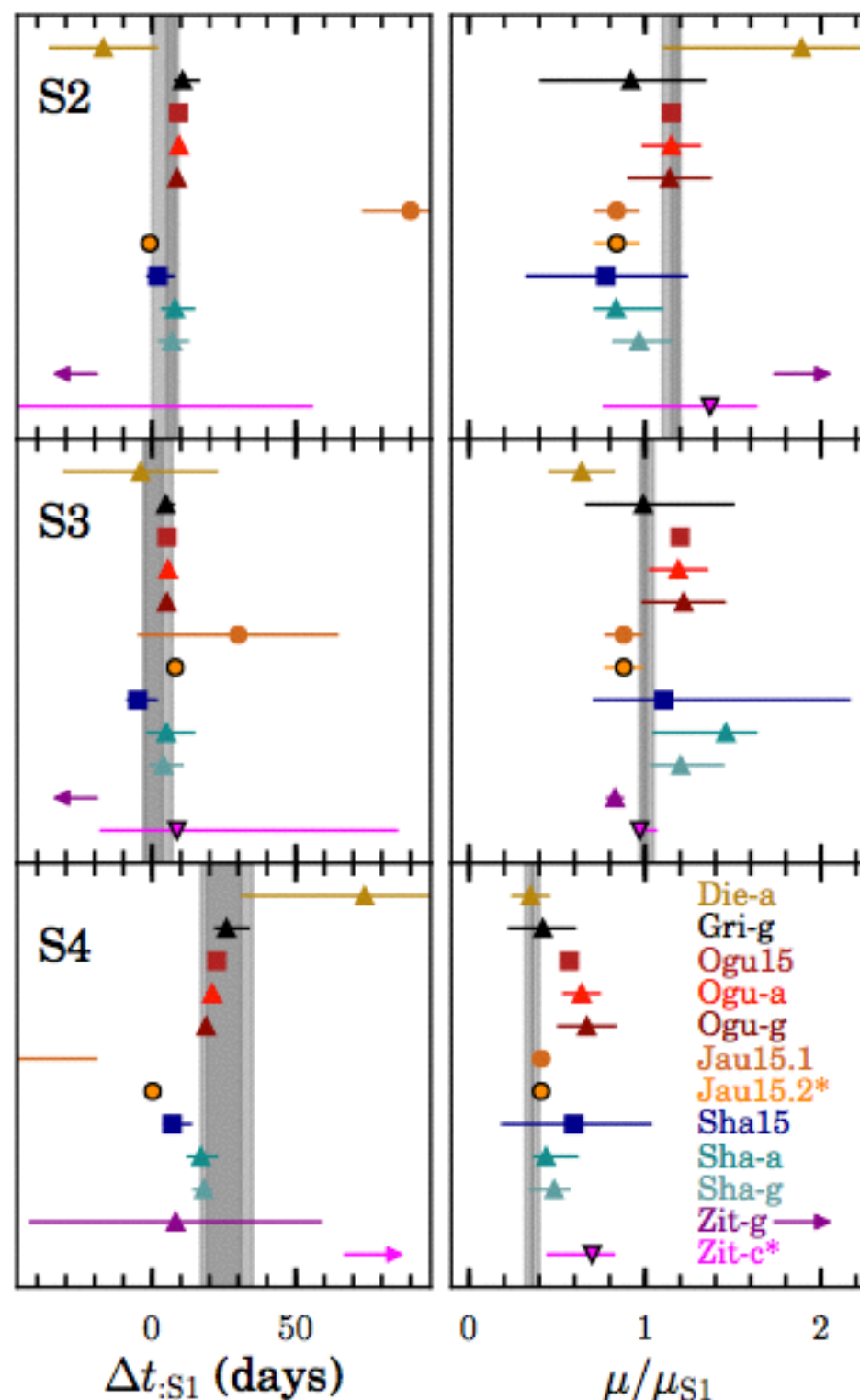
Subject headings: gravitational lensing — galaxies: clusters: general — galaxies: clusters: individuals:

MACS J1149.5+2223 — Dark matter

"REFSDAL" MEETS POPPER: COMPARING PREDICTIONS OF THE RE-APPEARANCE OF THE MULTIPLY IMAGED SUPERNOVA BEHIND MACSJ1149.5+2223

T. Treu^{1,28}, G. Brammer², J. M. Diego³, C. Grillo⁴, P. L. Kelly⁵, M. Oguri^{6,7,8}, K. Sharon¹², A. Zitrin^{13,29} [Show full author list](#)

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The Astrophysical Journal, Volume 817, Number 1



A free-form prediction for the reappearance of supernova Refsdal in the Hubble Frontier Fields cluster MACSJ1149.5+2223

Jose M. Diego,¹★ Tom Broadhurst,^{2,3} Cuncheng Chen,⁴ Jeremy Lim,⁴ Adi Zitrin,⁵† Brian Chan,⁴ Dan Coe,⁶ Holland C. Ford,⁶ Daniel Lam⁴ and Wei Zheng⁶

¹IFCA, Instituto de Física de Cantabria (UC-CSIC), Av. de Los Castros s/n, E-39005 Santander, Spain

²Fisika Teorikoa, Zientzia eta Teknologia Fakultatea, Euskal Herriko Unibertsitatea UPV/EHU, E-48080 Bilbao, Spain

³IKERBASQUE, Basque Foundation for Science, Alameda Urquijo 36-5, E-48008 Bilbao, Spain

⁴Department of Physics, The University of Hong Kong, 0000-0002-6536-5575, Pokfulam Road, Hong Kong

⁵Cahill Center for Astronomy and Astrophysics, California Institute of Technology, MS 249-17, Pasadena, CA 91125, USA

⁶Department of Physics and Astronomy, Johns Hopkins University, Baltimore, MD 21218, USA

THE STORY OF SUPERNOVA 'REFSDAL' TOLD BY MUSE*

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Draft version March 7, 2016

Monthly Notices

of the
ROYAL ASTRONOMICAL SOCIETY

MNRAS 457, 2029–2042 (2016)



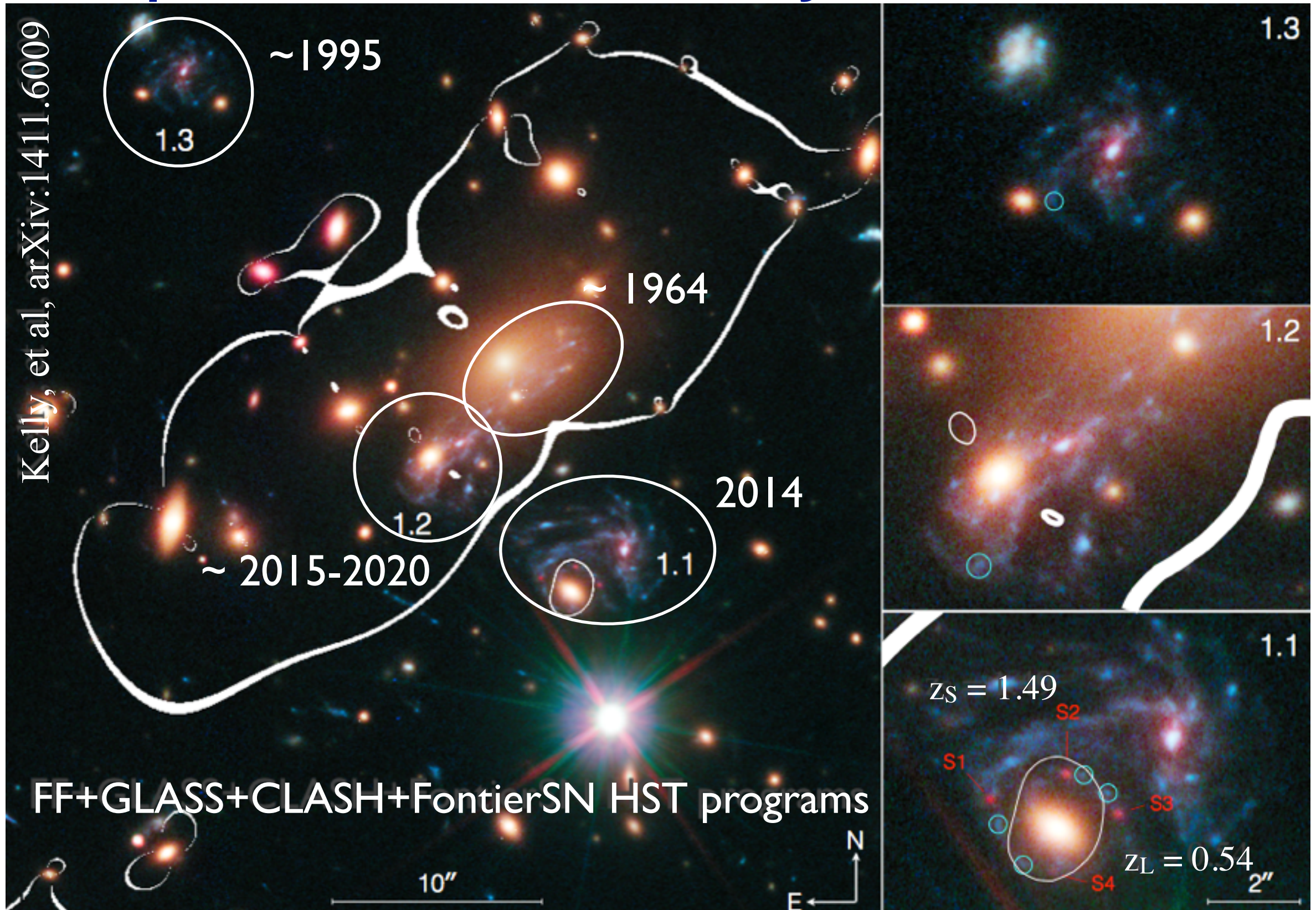
doi:10.1093/mnras/stw069

Hubble Frontier Fields: predictions for the return of SN Refsdal with the MUSE and GMOS spectrographs

M. Jauzac,^{1,2,3}★ J. Richard,⁴ M. Limousin,⁵ K. Knowles,³ G. Mahler,⁴ G. P. Smith,⁶ J.-P. Kneib,^{5,7} E. Jullo,⁵ P. Natarajan,⁸ H. Ebeling,⁹ H. Atek,⁸ B. Clément,⁴ D. Eckert,¹⁰ E. Egami,¹¹ R. Massey^{1,2} and M. Rexroth⁷

of MACS J0416.1–2403 and Abell 2744. In light of the discovery of the first resolved quadruply lensed supernova, SN Refsdal, in one of the multiply imaged galaxies identified in MACS J1149, we use our revised mass model to investigate the time delays and predict the rise of the next image between 2015 November and 2016 January.

Supernova em MACS J1149.6+2223

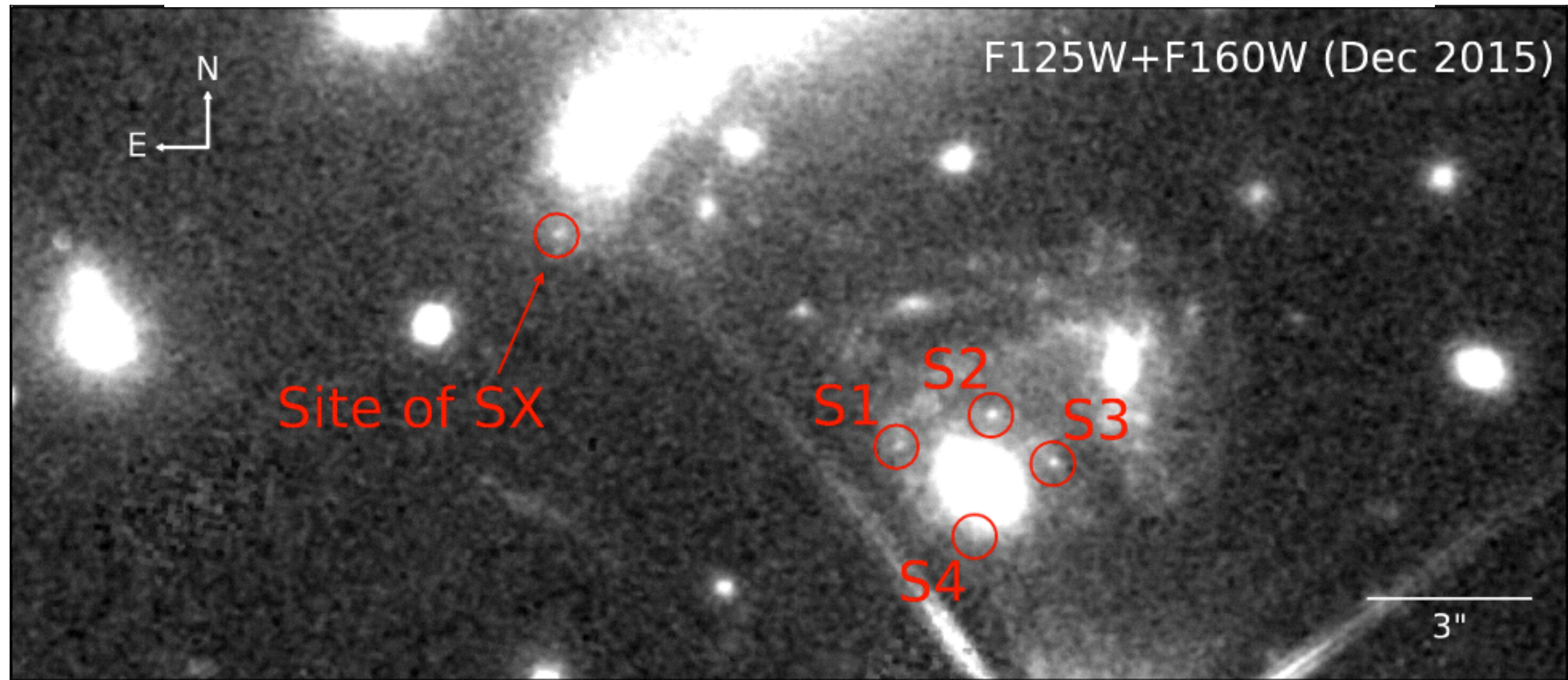


DEJA VU ALL OVER AGAIN: THE REAPPEARANCE OF SUPERNOVA REFSDAL

P. L. Kelly¹, S. A. Rodney², T. Treu^{3,3}, L.-G. Strolger⁴, R. J. Foley^{5,6}, S. W. Jha⁷, J. Selsing⁸, G. Brammer⁴, M. Bradač⁹, S. B. Cenko^{10,11} [Show full author list](#)

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[The Astrophysical Journal Letters](#), Volume 819, Number 1

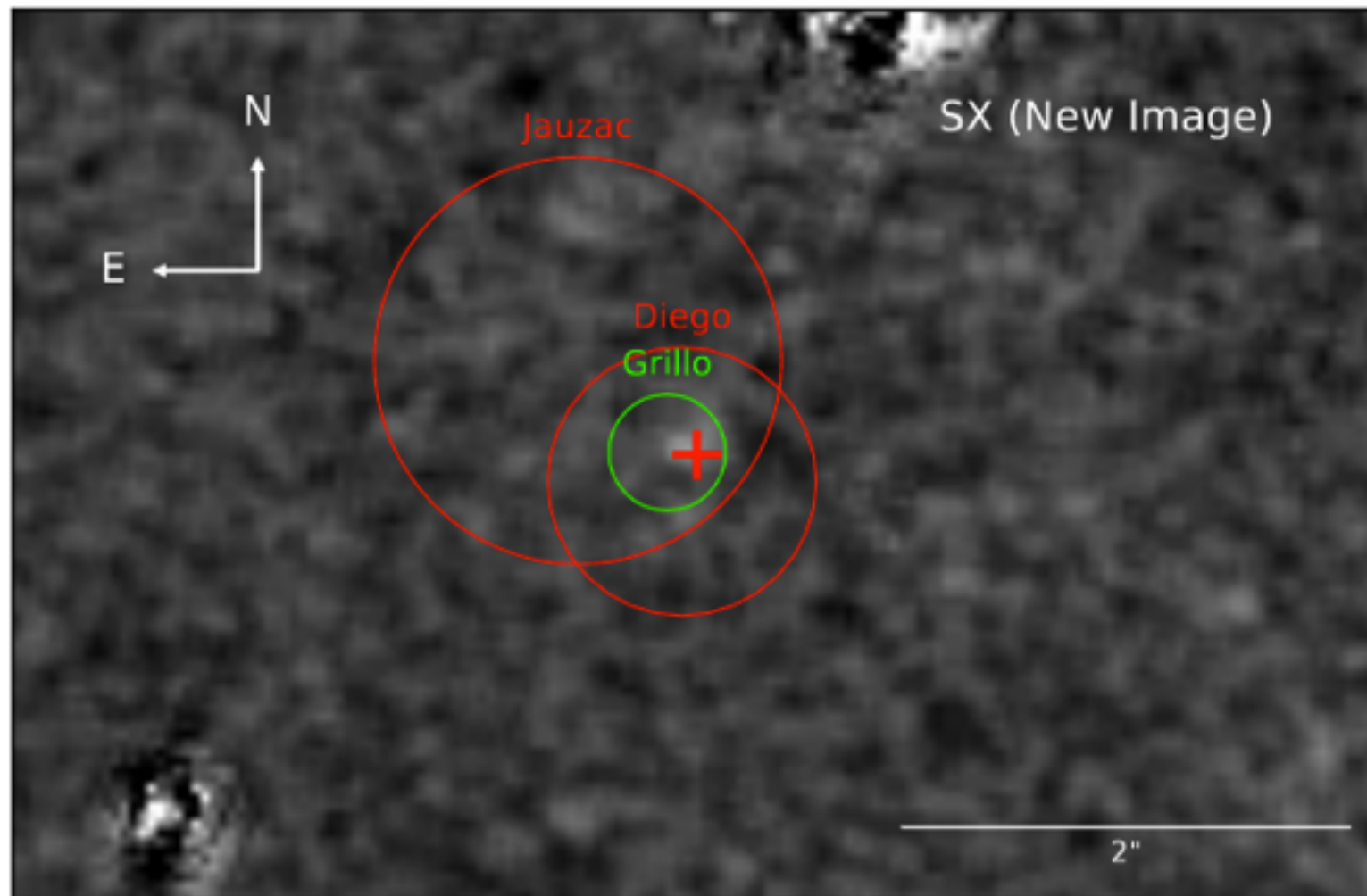


DEJA VU ALL OVER AGAIN: THE REAPPEARANCE OF SUPERNOVA REFSDAL

P. L. Kelly¹, S. A. Rodney², T. Treu^{3,3}, L.-G. Strolger⁴, R. J. Foley^{5,6}, S. W. Jha⁷, J. Selsing⁸, G. Brammer⁴, M. Bradač⁹, S. B. Cenko^{10,11} [Show full author list](#)

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[The Astrophysical Journal Letters](#), Volume 819, Number 1





ARCOS GRAVITACIONAIS E COSMOLOGIA: ENERGIA ESCURA

Aglomerados de Galáxias

$$\vec{\theta} = \vec{\beta} + \frac{2}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \nabla_{\theta} \psi \left(\vec{\theta} \right)$$

Famílias de imagens de fontes com distintos *redshifts*

☑️ **vínculos na cosmologia, além da distribuição de matéria**

A razão das distâncias diâmetro angular para 2 (ou mais) imagens com fontes em diferentes desvios para o vermelho define uma ‘razão de famílias’

$$\Xi(z_1, z_{s1}, z_{s2}; \Omega_M, \Omega_X, w_X) = \frac{D(z_1, z_{s1})}{D(0, z_{s1})} \frac{D(0, z_{s2})}{D(z_1, z_{s2})}$$

☑️ Jullo et al. 2010, Science: exemplo de limites competitivos na cosmologia com um sistema de lente forte (Abell 1689)

☑️ 8 famílias de fontes com desvios para o vermelho entre 1.15 e 4.86

☑️ Caminha et al. 2016: RXC J2248.7-4431 (Abell S1063), 16 fontes, 47 imagens

LEMBRETE: DISTÂNCIA DE DIÂMETRO ANGULAR

Caso plano:

$$D_A(z_1, z_2) = \frac{1}{1+z_2} \int_{z_1}^{z_2} \frac{dz}{H(z)}$$

No modelo wCDM

$$p = w\rho$$

$$H^2(a) = H_0^2 \left[\Omega_r a^{-4} + \Omega_M a^{-3} + \Omega_k a^{-2} + \Omega_{DE} a^{-3(1+w)} \right]$$

de modo que

$$D_A(z_1, z_2) = \frac{(1+z_2)^{-1}}{H_0} \int_{z_1}^{z_2} \frac{dz'}{\sqrt{\Omega_M (1+z')^3 + (1-\Omega_M) (1+z')^{3(1+w)}}}$$

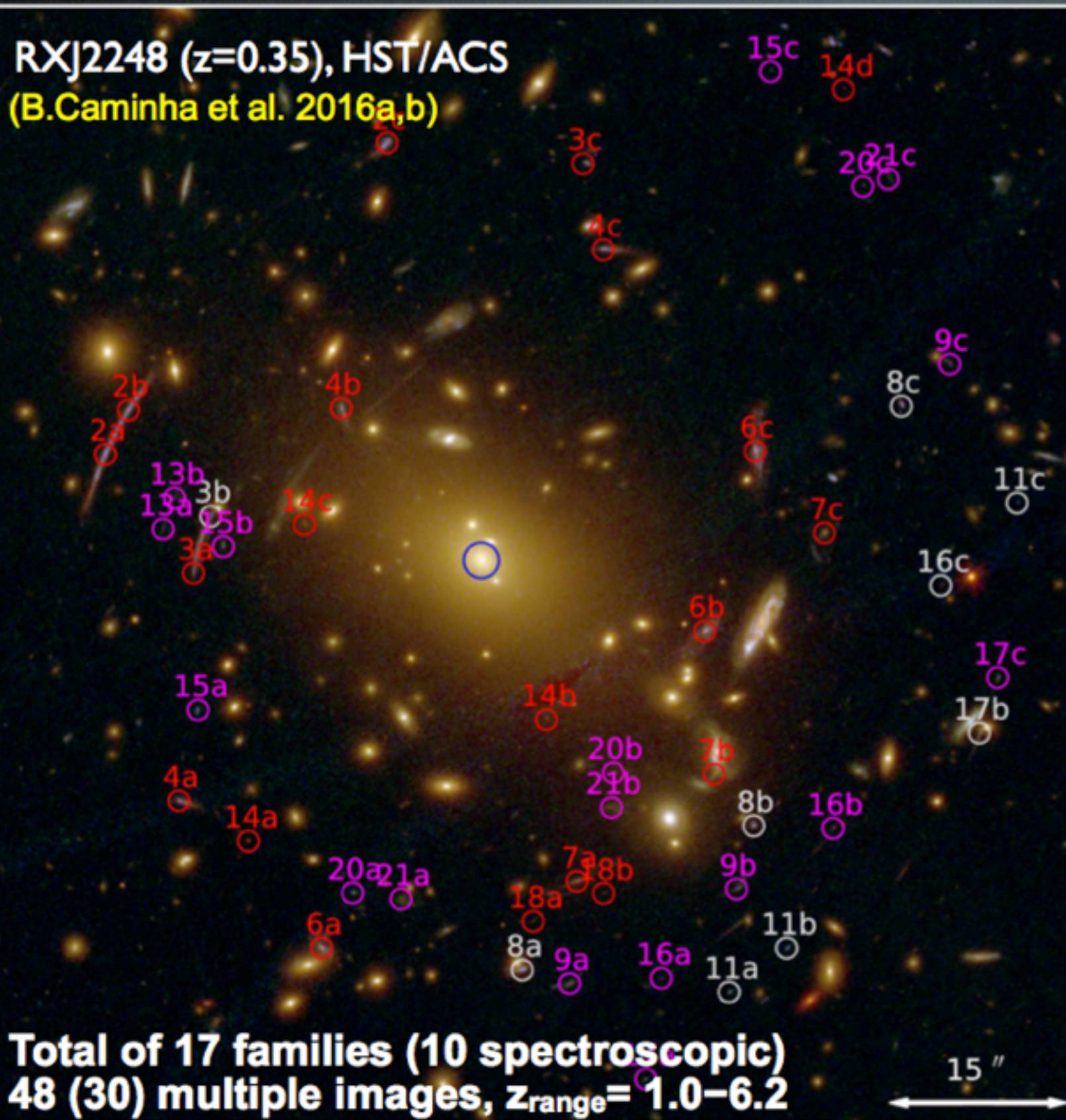
$$D_{LS} = D_A(z_L, z_S), \text{ etc.}$$

COSMOLOGICAL CONSTRAINTS

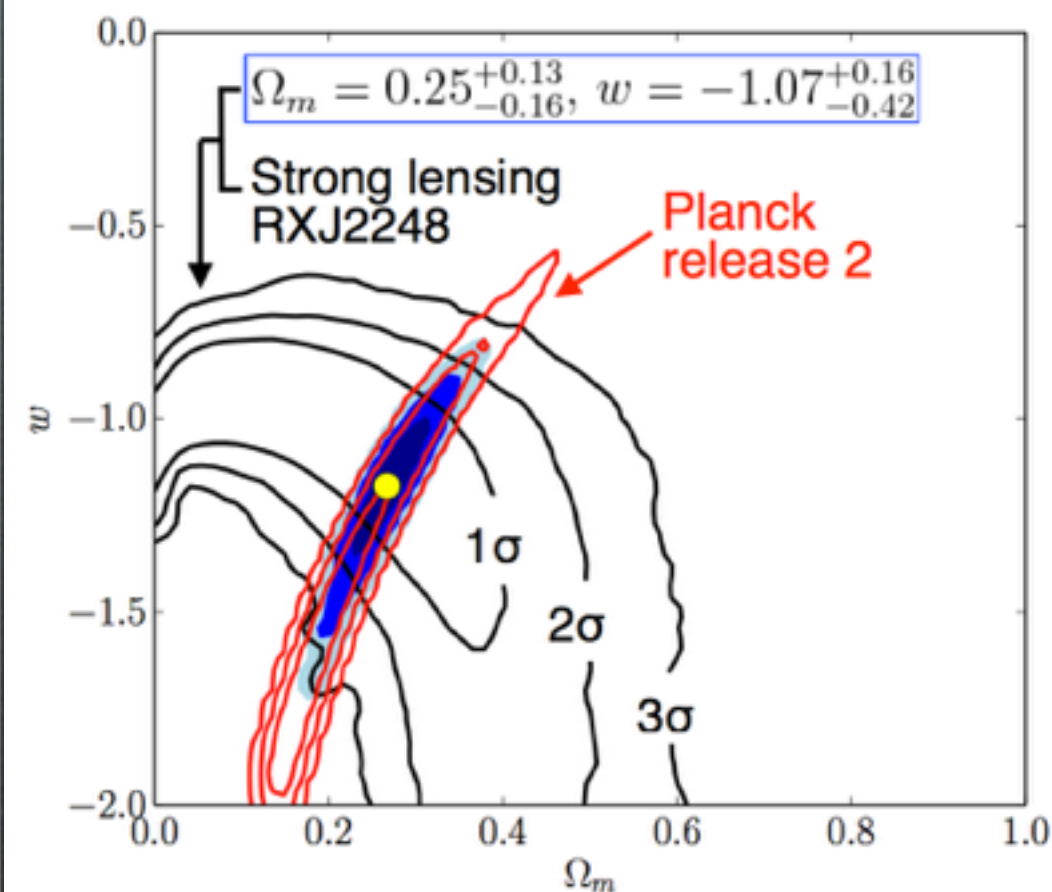
Frontier Field Cluster AS1063 (aka RXJ2248)

Caminha et al., 2016

RXJ2248 ($z=0.35$), HST/ACS
(B.Caminha et al. 2016a,b)



MUSE SV programme + GO (PI: K.Caputi)
(Karman et al. 2015)
(W.Karman et al. 2016, arXiv/160601471)

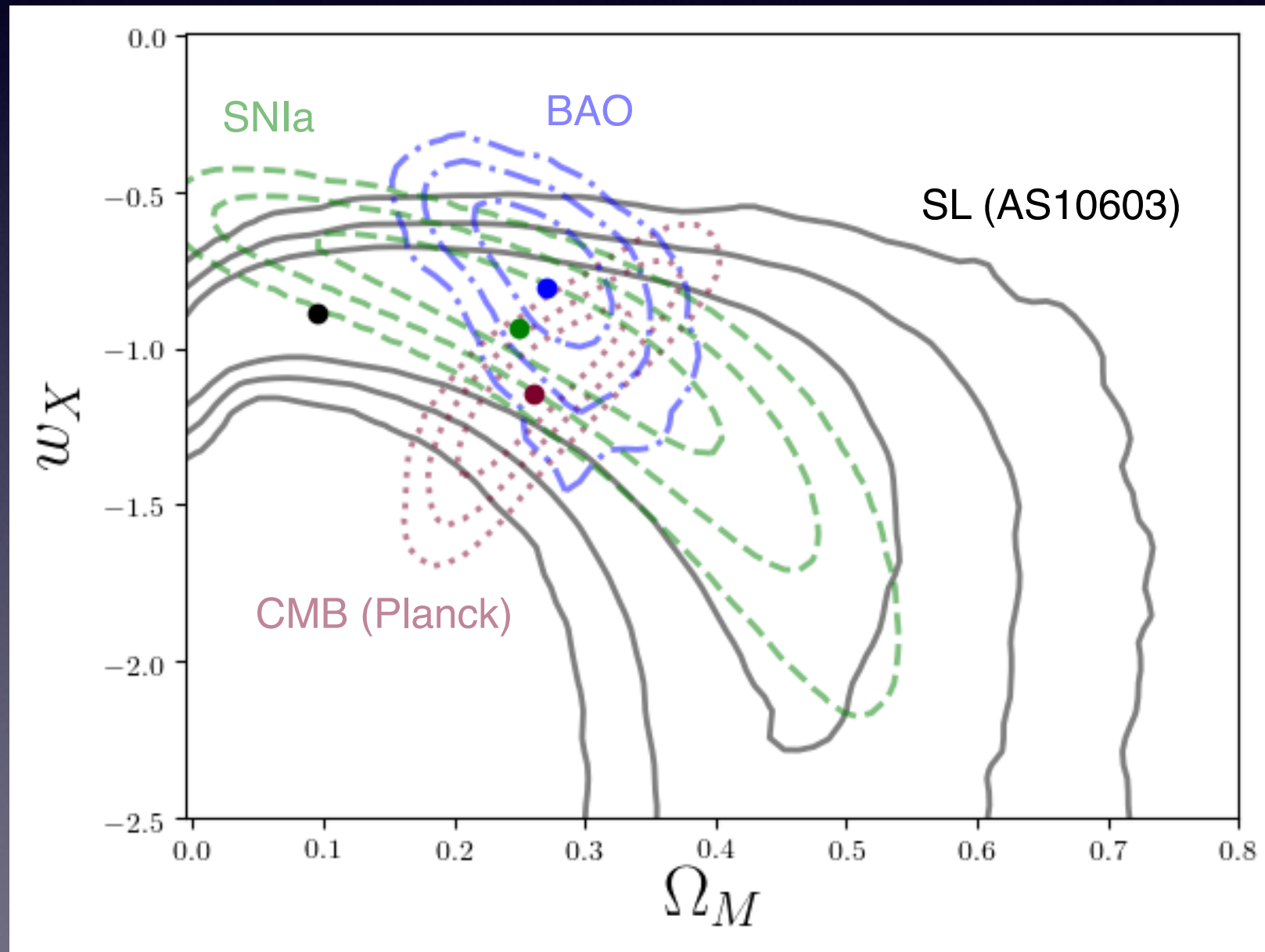


1 arcmin² FoV
2.6 Å resolution (4800-9300 Å)
0.2 arcsec/pxl
Exp. = 5 hrs

Novos limites com Abell S1063

- Dobro de famílias de imagens múltiplas
- Comparação e combinação com outros observáveis

Bom, Makler, Caminha, Vitenti, Penna-Lima (2019, in prep.)



Limites exclusivamente de Strong Lensing:

$$\Omega_M = 0.422^{+0.062}_{-0.274}$$

$$w = -0.911^{+0.030}_{-1.356}$$

Único sistema!

Teste de consistência

Refsdal

Futuro promissor,
especialmente para DM

THE *HST* FRONTIER FIELDS

THE DEEPEST DATA EVER OBTAINED FOR LENSING GALAXY CLUSTERS !!!

Abell 2744 - $z = 0.308$

Fully observed

Atek et al. 2014a, ApJ, 786, 60 ; Laporte et al. 2014, A&A, 562, 8; Zitrin et al. 2014, ApJ, 793, 12; Ishigaki et al. 2015, ApJ, 799, 12; Atek et al. 2015, ApJ, 800, 18; Jauzac et al. 2015, MNRAS, 452, 437 ; Wang et al., ApJ, 811, 29

MACS J0717 - $z = 0.545$

Fully observed

Diego et al. 2015, MNRAS, 451, 3920 ; Limousin et al. 2016, A&A, 588, 99; Kawamata et al. 2016, ApJ, 819, 14

MACS J1149 - $z = 0.543$

Fully observed

Kelly et al. 2015, Science, 347, 1123 ; Sharon & Johnson 2015, ApJ, 800, 26 ; Oguri 2015, MNRAS, 449, 86 ; Diego et al. 2016, MNRAS, 459, 344 ; Jauzac et al. 2016, MNRAS, 457, 2029 ; Treu et al. 2016, ApJ, 817, 60 ; Grillo et al. 2016, ApJ, 822, 78

Abell 370 - $z = 0.375$

ACS to go

MACS J0416 - $z = 0.396$

Fully observed

Jauzac et al. 2014, MNRAS, 443, 1549; Lam et al. 2014, ApJ, 797, 98; Jauzac et al. 2015, MNRAS, 446 4132; Grillo et al. 2015, ApJ, 800, 38 ; Harvey et al. 2016, MNRAS, 458, 660 ; Caminha et al., 2016, arXiv1607.0346

Abell S1063 - $z = 0.348$

Fully observed

Diego et al. 2016, MNRAS 459, 3447

**Limites mais fortes e
estudo de sistemáticos**

A wide-angle photograph of a beach. The foreground is a sandy beach with some small rocks and debris. The middle ground is the turquoise ocean with gentle waves breaking on the shore. The background is a clear blue sky. The text "LENTEs COM SIMETRIA AXIAL" is overlaid in the center of the image.

LENTEs COM SIMETRIA AXIAL

Teorema de Gauss

Equação de Poisson $\nabla_{\xi}^2 \psi \left(\vec{\xi} \right) = 4\pi G \Sigma \left(\vec{\xi} \right)$

Ângulo de deflexão $\hat{\vec{\alpha}} = \frac{2}{c^2} \vec{\nabla}_{\xi} \psi \left(\vec{\xi} \right)$

Assim: $\vec{\nabla} \cdot \hat{\vec{\alpha}} = \frac{8\pi G}{c^2} \Sigma \left(\vec{\xi} \right)$

Teorema de Gauss 2D

$$\int_{S_{\xi}} \vec{\nabla} \cdot \hat{\vec{\alpha}} dS = \oint_{C_{\xi}} \left(\hat{\vec{\alpha}} \cdot \hat{n} \right) dl$$

Como na lente pontual!
Raio de Einstein, etc.

$$\frac{8\pi G}{c^2} \underbrace{\int_{S_{\xi}} \Sigma(\xi) dS}_{M(\xi)} = \int_0^{2\pi} \hat{\alpha} \xi d\phi \Rightarrow \hat{\vec{\alpha}} = \frac{4GM(\xi)}{c^2 \xi} \hat{\xi}$$

Equação da Lente

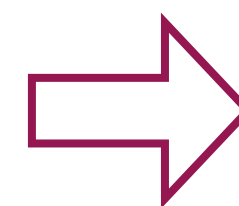
Ângulo de deflexão $\hat{\vec{\alpha}} = \frac{4GM(\xi)}{c^2\xi} \hat{\xi}$

Equação da lente $\vec{\beta} = \vec{\theta} - \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}} = \vec{\theta} - \frac{4GM(\theta)}{c^2\theta^2 D_{OL}} \vec{\theta} \frac{D_{LS}}{D_{OS}}$

$$\vec{\beta} = \left(1 - \frac{4GM(\theta) D_{LS}}{c^2 D_{OL} D_{OS}} \frac{1}{\theta^2} \right) \vec{\theta}$$

Ângulo de Einstein $(\vec{\beta} = 0)$

$$\theta_E = \sqrt{\frac{D_{LS}}{D_{OS} D_{OL}} \frac{4GM(\theta_E)}{c^2}}$$



Estimativa
da massa
em $\theta < \theta_E$

A wide-angle photograph of a beach. The foreground is a sandy beach with some small rocks and debris. The middle ground is the ocean, showing gentle waves and a clear turquoise color. The background is a clear blue sky. The text is overlaid in the center of the image.

ESFERA ISOTÉRMICA SINGULAR

Esfera Isotérmica Singular

$$\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}$$

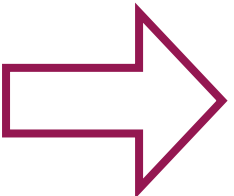
Densidade projetada (superficial)

$$\Sigma(\xi) = \frac{\sigma_v^2}{2\pi G} 2 \int_0^\infty \frac{dz}{\xi^2 + z^2} = \frac{\sigma_v^2}{2G\xi}$$

Massa contida em raio ξ

$$M(\xi) = \int_0^\xi \Sigma(\xi') 2\pi\xi' d\xi' = \sigma_v^2 \frac{\pi}{G} \xi$$

Ângulo de deflexão


$$\hat{\alpha} = \frac{4GM(\xi)}{c^2\xi} \hat{\xi} = 4\pi \left(\frac{\sigma_v}{c} \right)^2 \hat{\xi} \quad \text{constante!}$$

Magnificação

$$M = \sigma_v^2 \frac{\pi}{G} \xi = \sigma_v^2 \frac{\pi}{G} D_{OL} \theta$$

Magnificação tangencial

θ_E

$$\frac{\theta}{\beta} = \left(1 - \frac{M(\theta)}{\pi \Sigma_{\text{crit}} D_{OL}^2 \theta^2} \right)^{-1} = \left(1 - \boxed{4\pi \left(\frac{\sigma_v}{c} \right)^2 \frac{D_{LS}}{D_{OS}} \frac{1}{\theta}} \right)^{-1}$$

Magnificação radial

$$\frac{d\theta}{d\beta} = \left[1 - \frac{1}{\pi \Sigma_{\text{crit}} D_{OL}^2} \frac{d}{d\theta} \left(\frac{M(\theta)}{\theta} \right) \right]^{-1} = 1$$

Não há cóustica/
curva crítica
radial

Equação da Lente

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}} \qquad \hat{\vec{\alpha}} = 4\pi \left(\frac{\sigma_v}{c} \right)^2 \hat{\xi}$$

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}} = 4\pi \left(\frac{\sigma_v}{c} \right)^2 \frac{D_{LS}}{D_{OS}} \hat{\xi} = \theta_E \hat{\xi}$$

$$\vec{\beta} = \vec{\theta} - \theta_E \hat{\theta} = \left(1 - \frac{\theta_E}{\theta} \right) \vec{\theta}$$

$$\Rightarrow \beta = \left| 1 - \frac{\theta_E}{\theta} \right| \theta$$

Soluções: 1) se $1 - \frac{\theta_E}{\theta} > 0$

$$\text{então } \beta = \left(1 - \frac{\theta_E}{\theta} \right) \theta = \theta - \theta_E \Rightarrow \boxed{\theta = \beta + \theta_E}$$

Equação da Lente

$$\beta = \left| 1 - \frac{\theta_E}{\theta} \right| \theta$$

Soluções: I) se $1 - \frac{\theta_E}{\theta} > 0$

então $\beta = \left(1 - \frac{\theta_E}{\theta} \right) \theta = \theta - \theta_E \Rightarrow \theta = \beta + \theta_E$

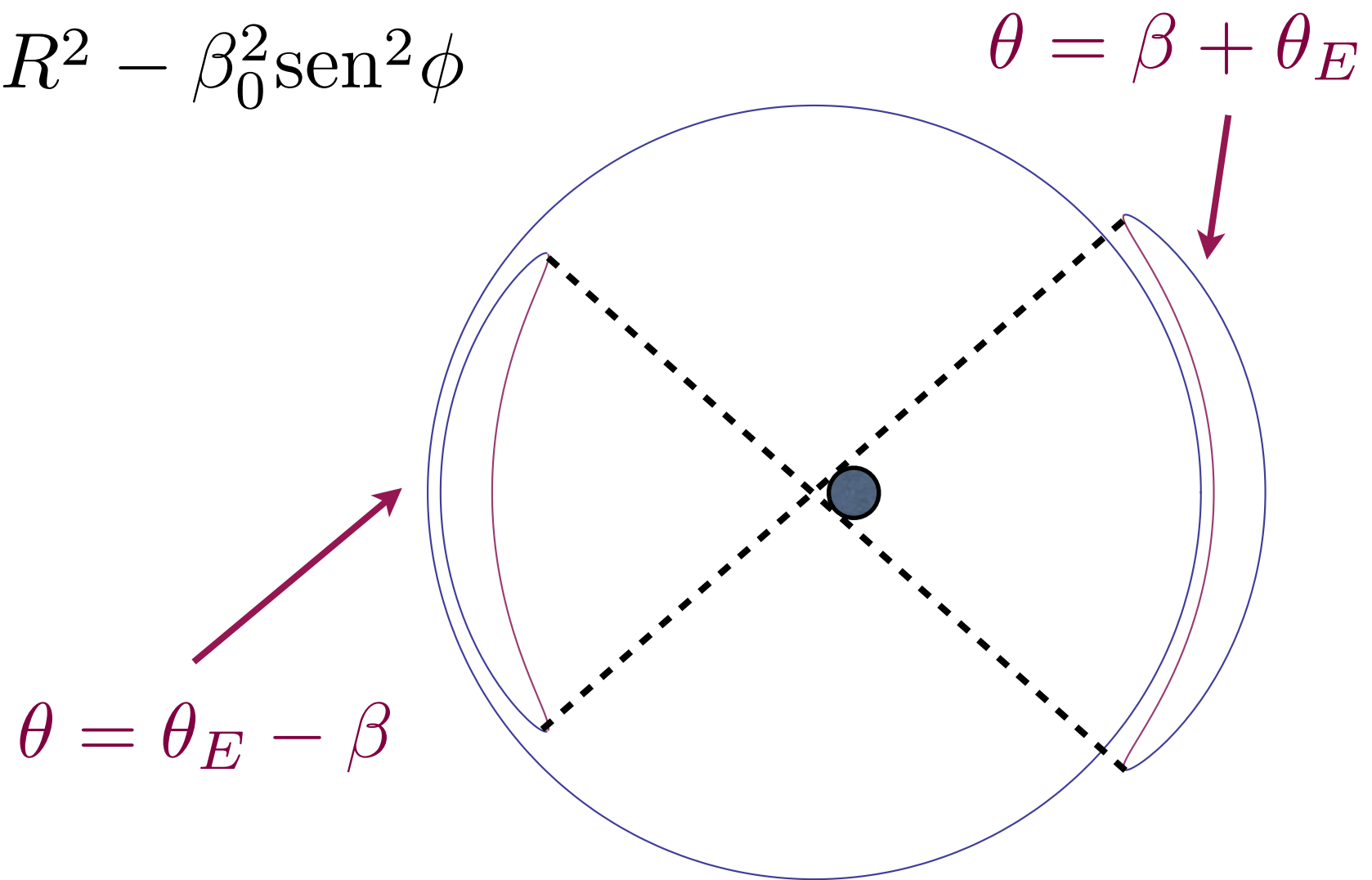
II) se $1 - \frac{\theta_E}{\theta} < 0$

então $\beta = - \left(1 - \frac{\theta_E}{\theta} \right) \theta = \theta_E - \theta \Rightarrow \theta = \theta_E - \beta$

Fonte circular

Círculo no plano das fontes: $|\vec{\beta} - \vec{\beta}_0| = R$

$$\beta = \beta_0 \cos \phi \pm \sqrt{R^2 - \beta_0^2 \sin^2 \phi}$$



Caso geral $\beta = \beta_0 \cos(\phi - \phi_0) \pm \sqrt{R^2 - \beta_0^2 \sin^2(\phi - \phi_0)}$

PROGRAMA

- Aula 1: *Overview* de lentes e formalismo básico
- Aula 2: Conceitos, formalismo, modelos e soluções, exemplos
- Aula 3: Desvio temporal e Efeito fraco de lentes
- Aulas 4 e 5: Revisão de cosmologia, Cosmologia e lentes
- Tópicos avançados:
 - Cosmologia (LSS + CMB)
 - Lentes e modelos de matéria escura
 - Testes de gravidade modificada





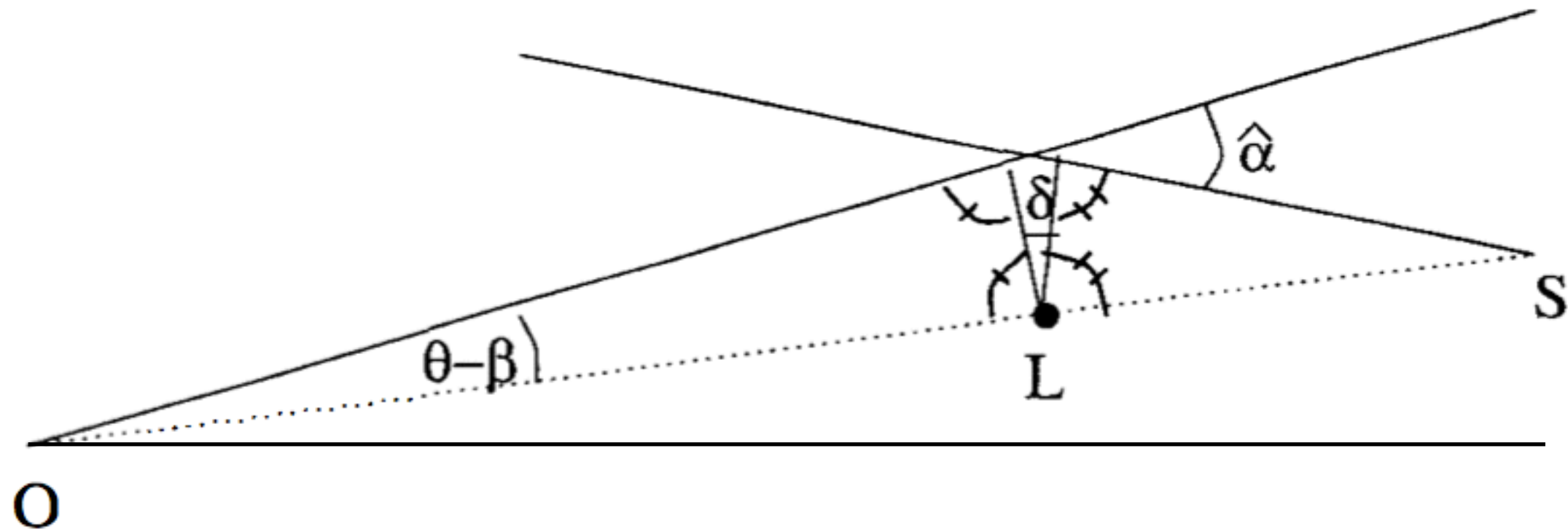
DESVIO TEMPORAL

Desvio temporal gravitacional

$$\delta t_{grav} = -\frac{2}{c^3} \int_{z_O}^{z_S} \varphi(z) dz := -\frac{2}{c^3} \psi$$

Obs.: Dedução rigorosa em Petters, Levine, Wambsganss

Desvio temporal geométrico



$$\delta L = \frac{D_{OS} D_{OL}}{2D_{LS}} (\vec{\theta} - \vec{\beta})^2$$

$$\delta t_{\text{geom}} = \delta L/c = \frac{D_{OS} D_{OL}}{2c D_{LS}} (\vec{\theta} - \vec{\beta})^2$$

Obs.: Dedução rigorosa em Petters, Levine, Wambsganss

Desvio temporal total

Desvio total no referencial da lente

$$\delta t_L = \delta t_{\text{geom}} + \delta t_{\text{grav}}$$

$$\delta t_{\text{geom}} = \delta L/c = \frac{D_{OS}D_{OL}}{2cD_{LS}}(\vec{\theta} - \vec{\beta})^2$$

$$\delta t_{\text{grav}} = -\frac{2}{c^3}\psi \quad \text{mas} \quad \Psi \equiv \frac{2}{c^2} \frac{D_{LS}}{D_{OS}D_{OL}}\psi$$

Assim

$$\delta t_L = \frac{D_{OS}D_{OL}}{cD_{LS}} \left(\frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \Psi \right)$$

Desvio temporal total

Efeito Doppler

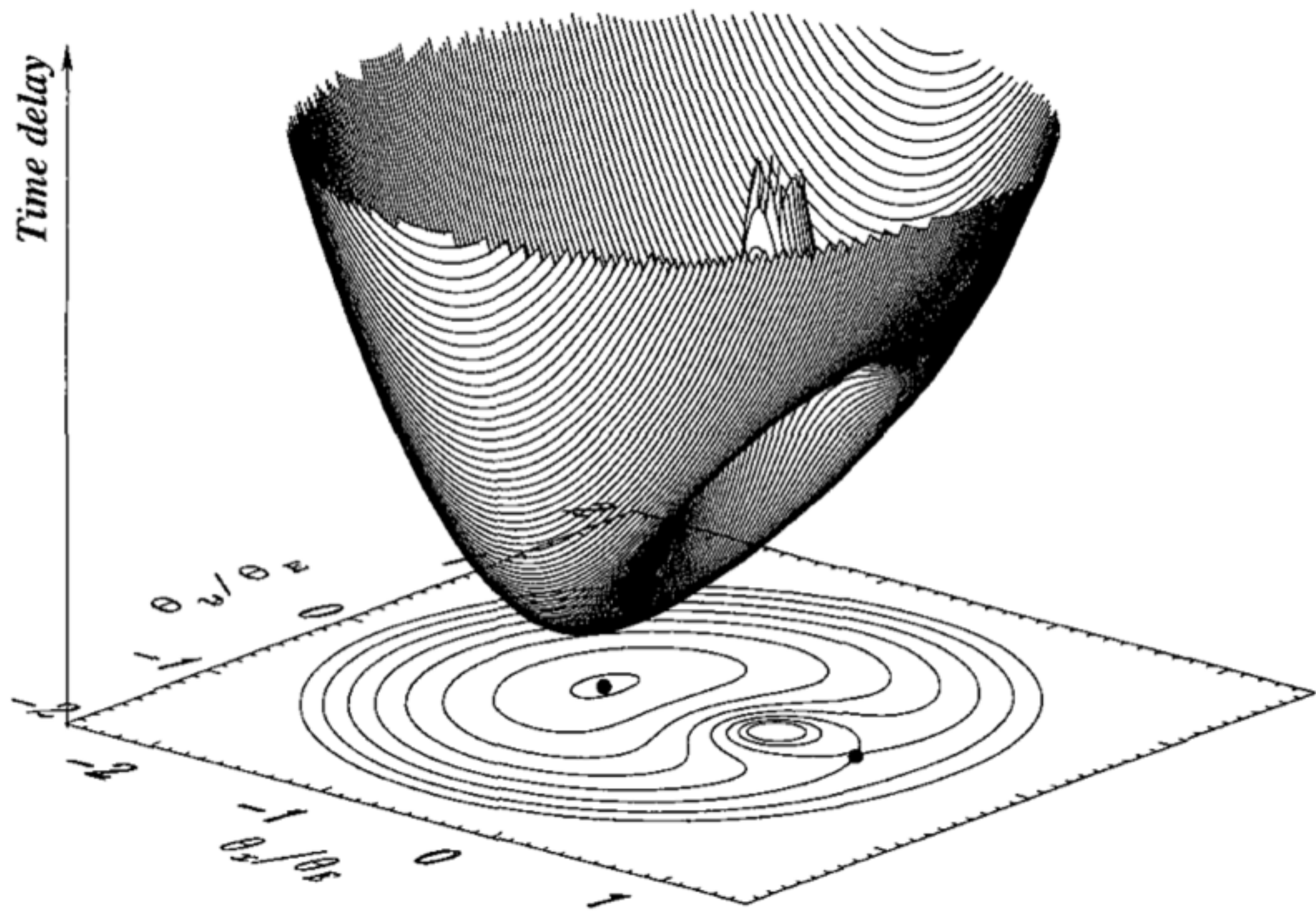
$$\delta t_O / \delta t_L = a_O / a_L = (1 + z_L)$$

$$\delta t_L = \frac{D_{OS} D_{OL}}{c D_{LS}} \left(\frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \Psi \right)$$

$$\delta t = (1 + z_L) \frac{D_{OS} D_{OL}}{c D_{LS}} \left(\frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \Psi \right)$$

$$\vec{\nabla}_{\theta}(\delta t) = 0 \quad \Rightarrow \quad \text{Equação da lente!}$$

Obs.: Dedução rigorosa em Petters, Levine, Wambsganss



Teorema de Burke

MULTIPLE GRAVITATIONAL IMAGING BY DISTRIBUTED MASSES¹

WILLIAM L. BURKE

Lick Observatory, Boards of Studies in Astronomy and Astrophysics and in Physics, University of California, Santa Cruz, CA 95064

Received 1980 September 26; accepted 1980 November 5

ABSTRACT

A transparent galaxy, not necessarily spherical, acting as a gravitational lens produces an odd number of images.

Subject headings: galaxies: general — gravitation — quasars

In a recent *Letter* (Dyer and Roeder 1980) it was shown that a transparent spherical galaxy of finite mass acting as a gravitational lens produces an odd number of images. The result is in fact true for arbitrary mass distributions as long as the bending (roughly m/r) remains bounded as r goes to infinity. This covers all realizable cases, since larger masses would be within their own event horizons.

A proof of this stronger result is easy. Parametrize the rays leaving the source by their impact parameters (u, v) , measured in some plane near the deflector and transverse to the rays. See Figure 1. You may think of this plane as the celestial sphere. The action of the gravitational lens bends a ray. Let us look at the intersection of this bent ray with another transverse plane, this one passing through the receiver. We can measure the bending by the direction cosines (s, w) between two lines: one from the source to the point (u, v) in the deflector plane, and one from that point to the point where the actual ray hits the receiver plane. By a transparent deflector we mean one for which all the rays hitting the first plane also hit the second. This excludes solid objects and black holes.

¹ *Lick Observatory Bulletin*, No. 848.

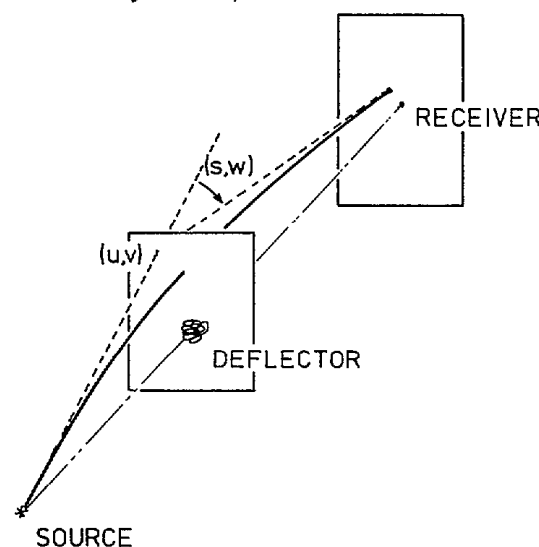


FIG. 1

The action of the gravitational lens is specified by giving the bendings (s, w) as functions of the impact parameters (u, v) . Such functions can be represented by a vector field on the (u, v) -plane with components (s, w) . A ray with impact parameters (u, v) will reach the receiver only for specific bendings (s, w) . This condition can also be represented as a vector field. The gravitational lens will form an image of the source at the receiver whenever these two vector fields are equal.

Subtract these two vector fields. Our problem is to study the zeros of this difference vector field. Since the bending is assumed bounded, for large impact parameter the difference vector field is dominated by the requirement that it takes a large bend for the ray to reach us. For large impact parameter the difference vector field is mainly radial. The zeros of a vector field on the plane are constrained by the Poincaré-Hopf index theorem (see Guillemin and Pollack 1974), which assigns an index to a region by counting the net rotation of the vector field around the boundary of the region. The index of a region is the sum of the indices around the zeros. Simple zeros of a vector field are either sources, sinks, or saddle points. Sources and sinks have index $+1$ and correspond to direct images, while saddle points have index -1 and correspond to inverted images (as in Dyer and Roeder's 1980 Fig. 2). The sum of the indices of the zeros for our difference vector field must be $+1$ because of the behavior at large impact parameter. Thus if there are only simple images, there must be an odd number of them, with one more direct image than inverted images.

Because this is a topological argument the transverse planes need not be flat, the bendings can be measured by angles, tangents, or whatever, and the straight lines need only be straight in some coordinate chart. The (u, v) -plane can be thought of as the celestial sphere of the source or, since rays are reversible, as the celestial sphere of the receiver.

This research was done at the Aspen Center for Physics, and their hospitality and the partial support of the National Science Foundation are gratefully acknowledged.

REFERENCES

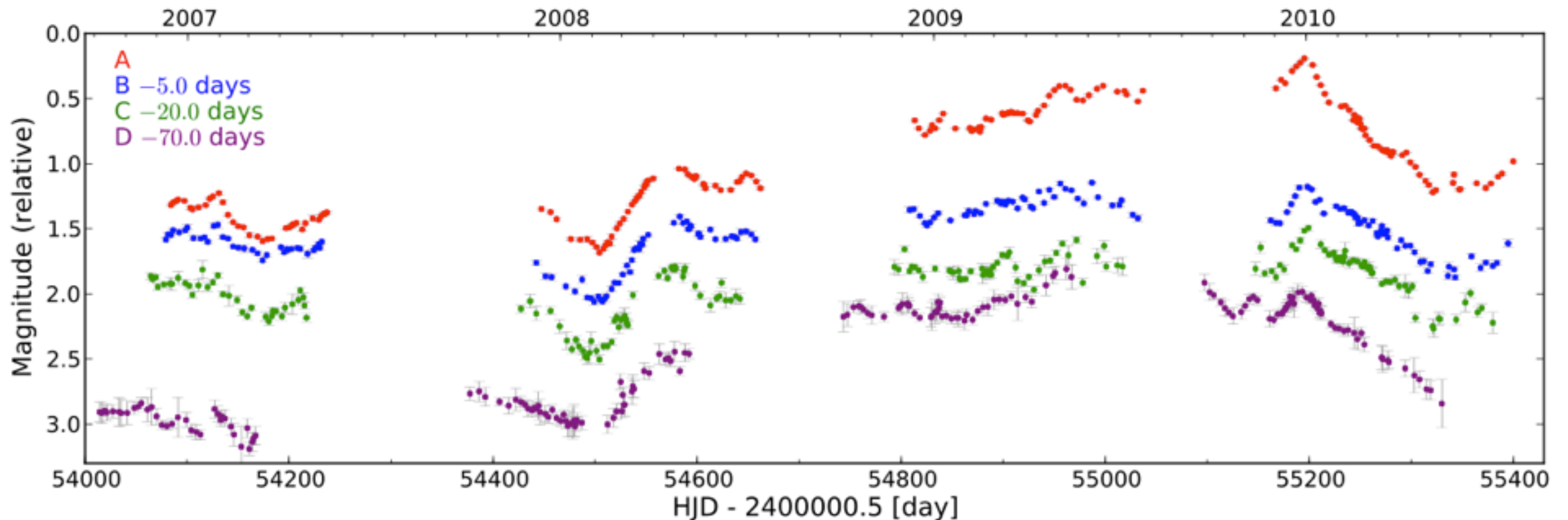
- Dyer, C. C., and Roeder, R. C. 1980, *Ap. J. (Letters)*, 238, L67.
 Guillemin, V., and Pollack, A. 1974, *Differential Topology* (Englewood Cliffs, N.J.: Prentice-Hall).

Curvas de luz de Quasares

Defasagem temporal entre imagens

$$\Delta t_{ij} = \delta t \left(\vec{\theta}_i, \vec{\beta} \right) - \delta t \left(\vec{\theta}_j, \vec{\beta} \right)$$

$$= (1 + z_L) \frac{D_{OS} D_{OL}}{c D_{LS}} \left(\frac{1}{2} (\vec{\theta}_i - \vec{\beta})^2 - \Psi(\theta_i) - \frac{1}{2} (\vec{\theta}_j - \vec{\beta})^2 + \Psi(\theta_j) \right)$$



Defasagem Temporal

$$\Delta t_{ij} = \frac{D_{\Delta}}{c} \left(\frac{1}{2} (\vec{\theta}_i - \vec{\beta})^2 - \Psi(\theta_i) - \frac{1}{2} (\vec{\theta}_j - \vec{\beta})^2 + \Psi(\theta_j) \right)$$

Onde $D_{\Delta} = (1 + z_L) \frac{D_{OS} D_{OL}}{D_{LS}}$

↑
“time delay distance”

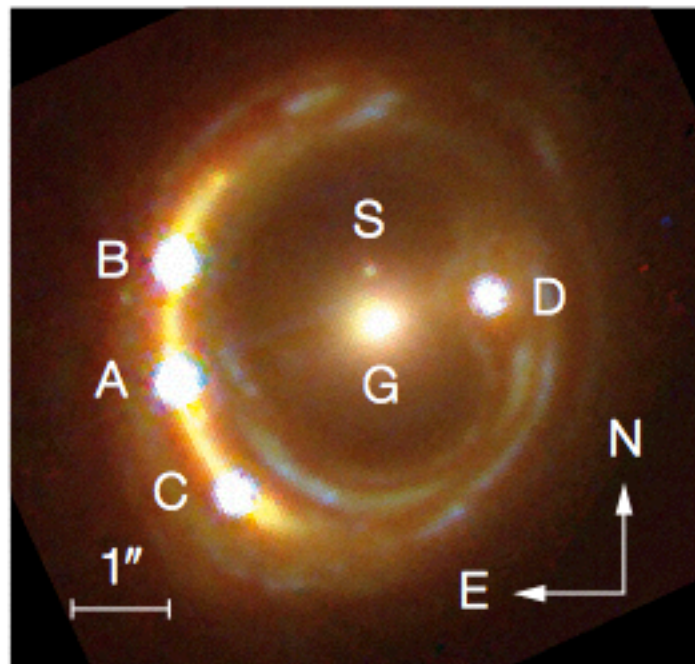
↑
modelagem inversa
(imagens múltiplas)

Principal dependência cosmológica: $\propto H_0^{-1}$

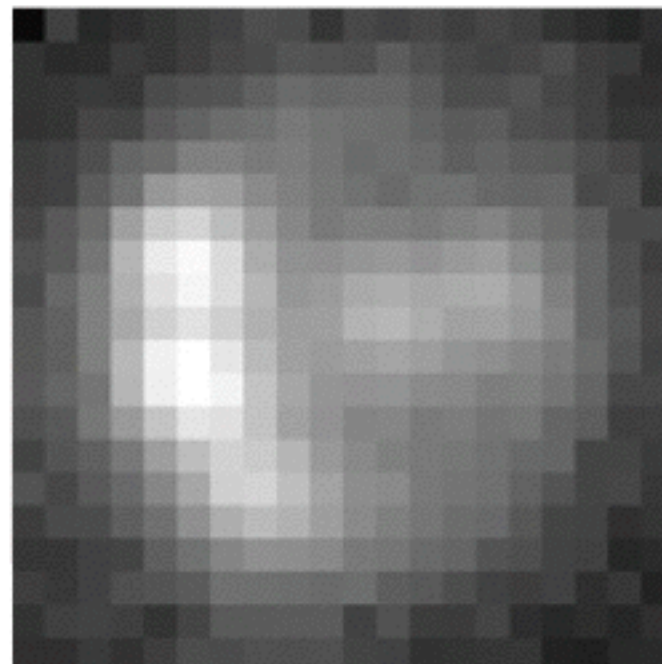
Exemplo: RX J1131-1231

- COSMOGRAIL: the COSmological MOnitoring of GRAvitational Lenses
- Curvas de luz + modelo da lente

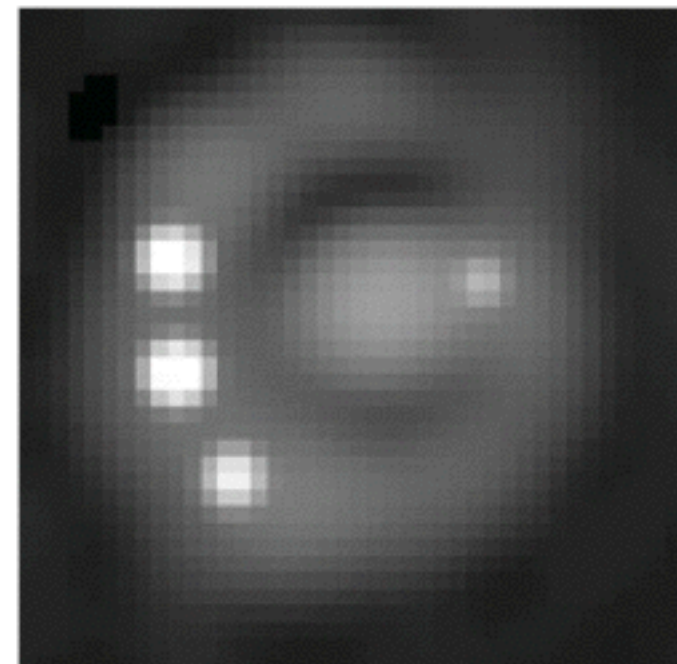
$$\kappa_{\text{pl}}(\theta_1, \theta_2) = \frac{3 - \gamma'}{2} \left(\frac{\theta_E}{\sqrt{q\theta_1^2 + \theta_2^2/q}} \right)^{\gamma' - 1}$$



Hubble Space Telescope

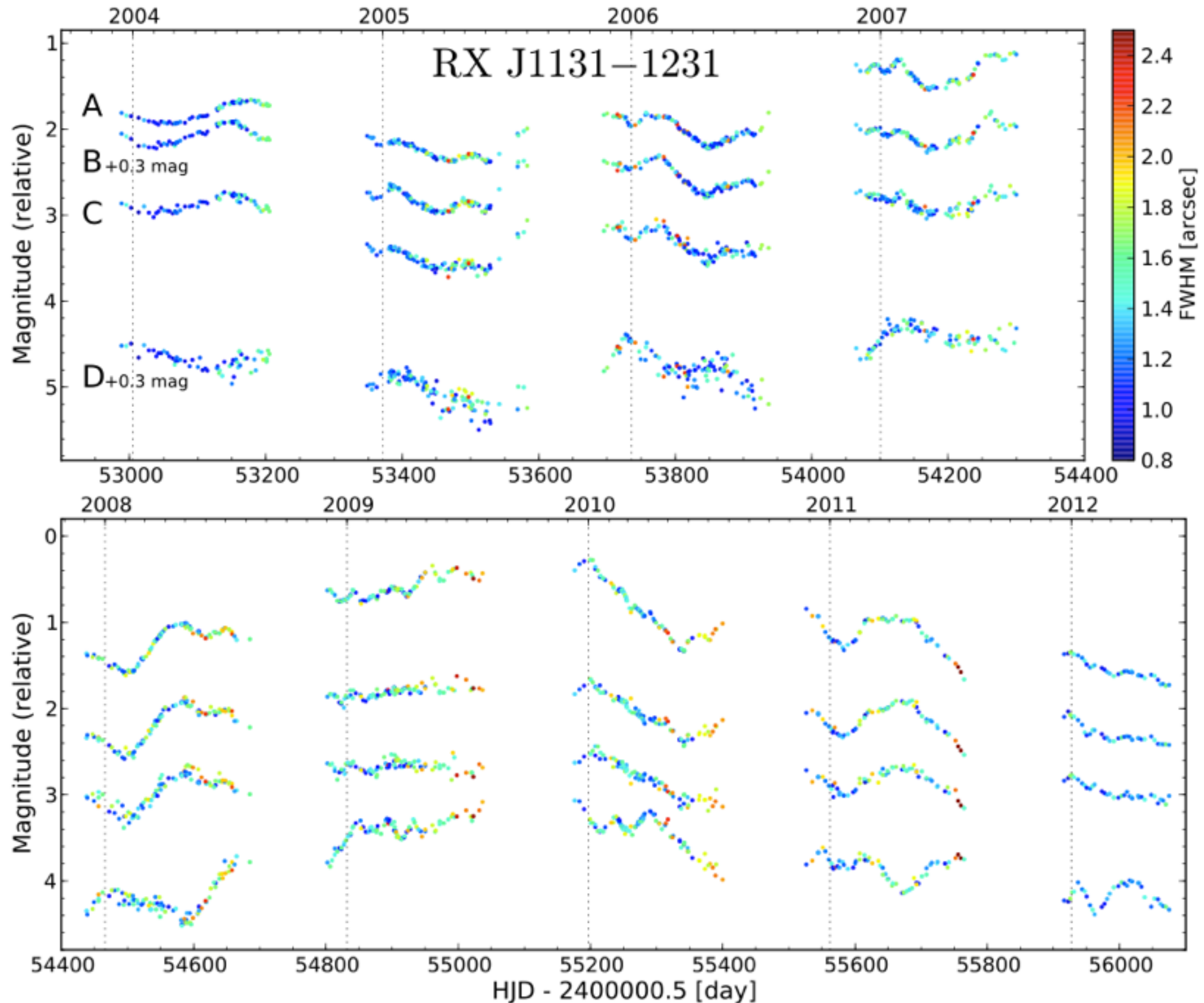


Swiss Leonhard Euler Telescope

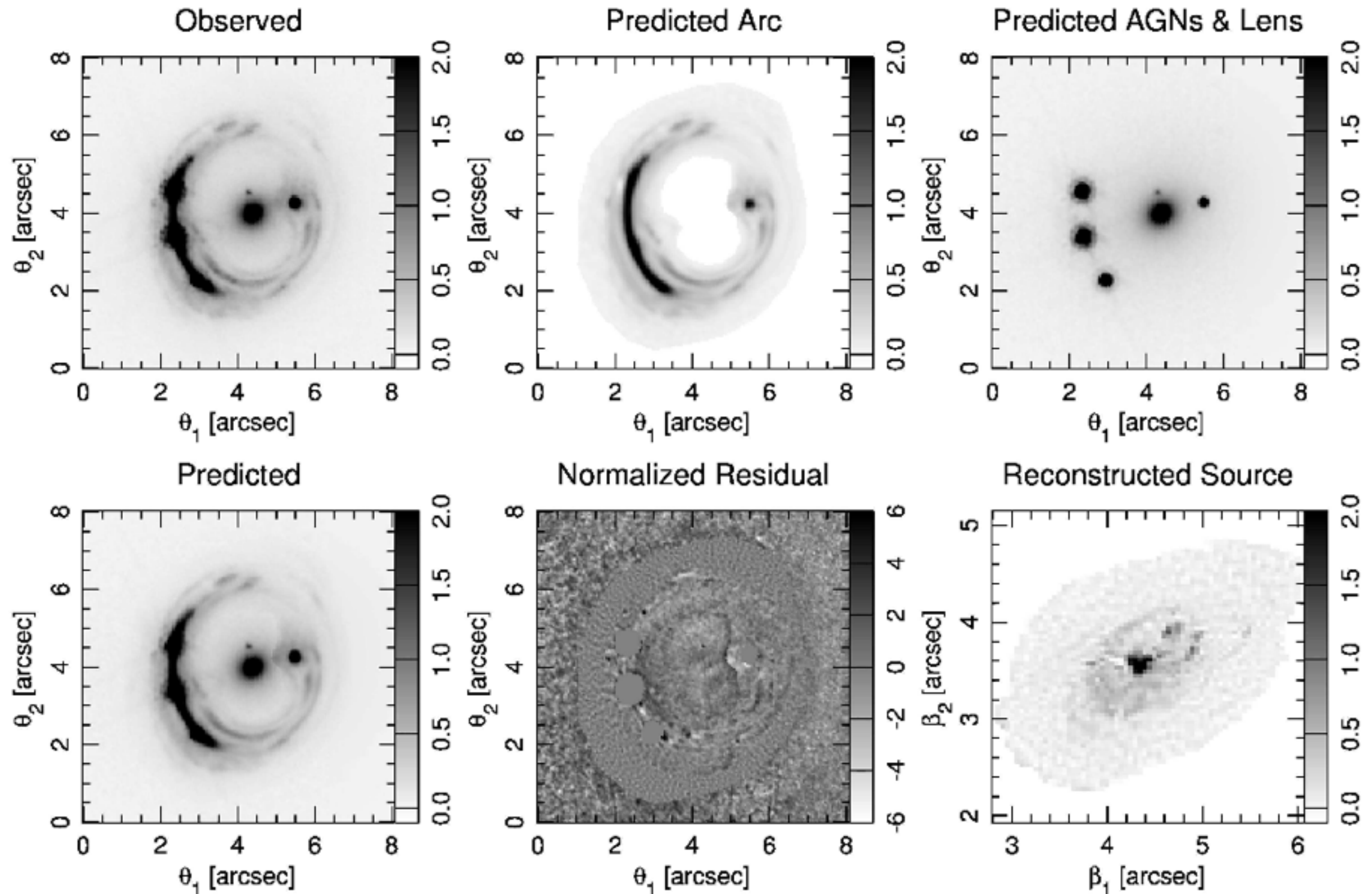


Euler deconvolved

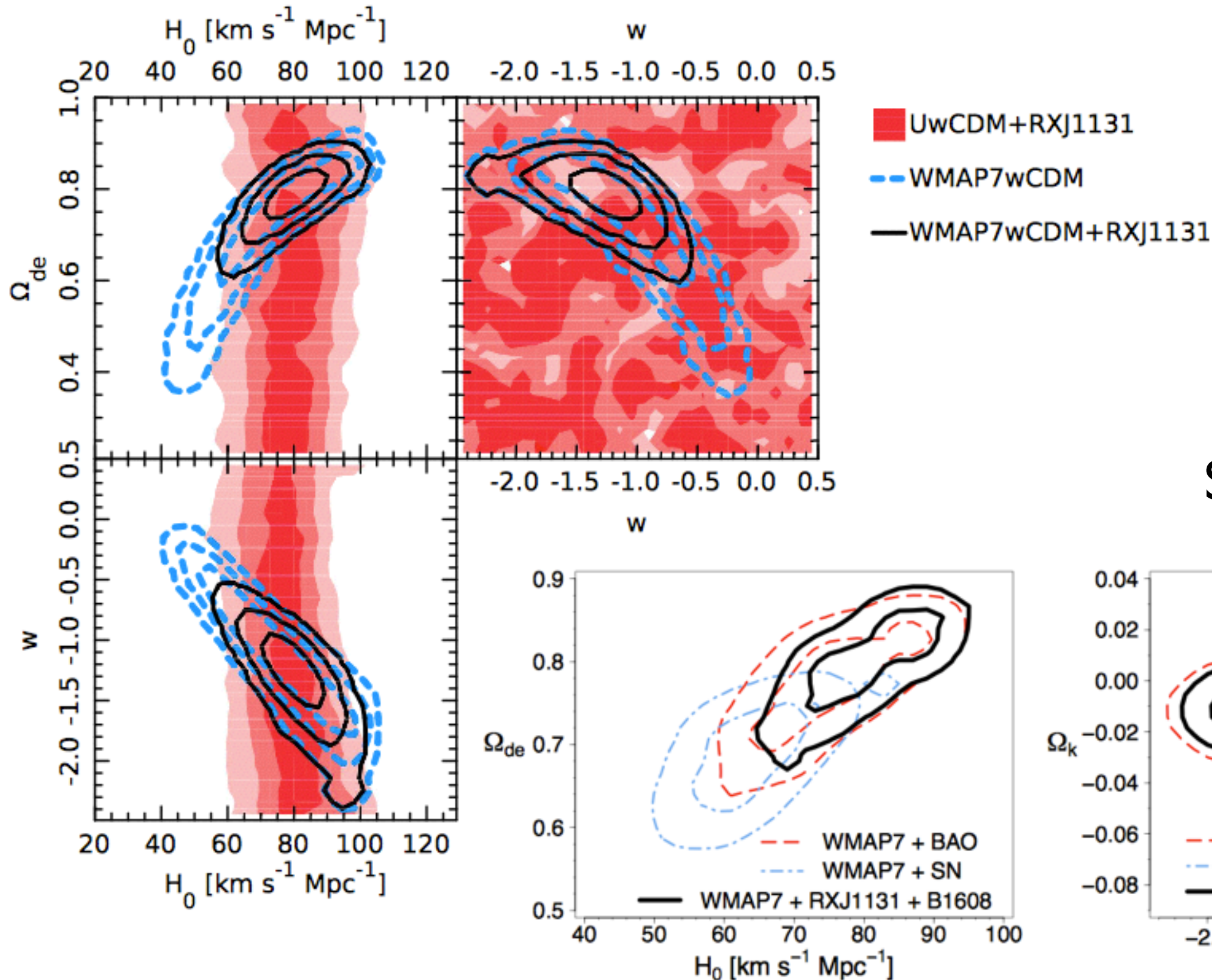
Curva de Luz de RX J1131-1231



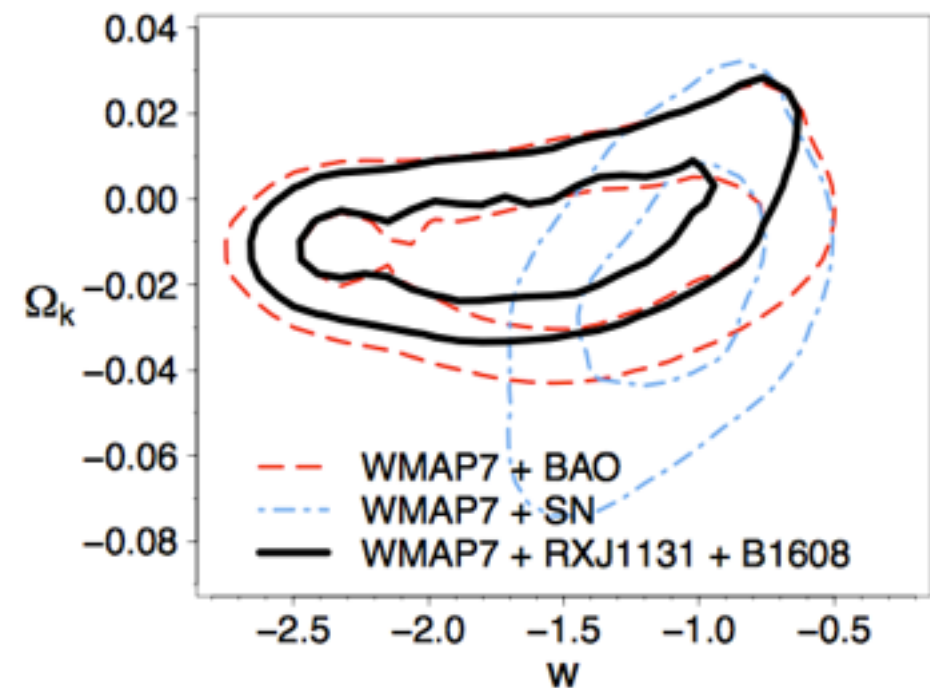
Modelagem de RX J1131-1231



Vínculos Cosmológicos



Suyu et al. 2012



A wide-angle photograph of a beach. The foreground is a sandy beach with some small rocks and debris. The middle ground is the turquoise ocean with gentle waves breaking on the shore. The background is a clear blue sky. The text is overlaid in the center of the image.

LENTE EXTENSA E JACOBIANA DA TRANSFORMAÇÃO

Lentes Extensas

Ângulo de deflexão $\hat{\vec{\alpha}} = \frac{4G}{c^2} \int d^2\xi' \Sigma(\vec{\xi}') \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2}$

Equação de Poisson $\nabla_{\xi}^2 \psi(\vec{\xi}) = 4\pi G \Sigma(\vec{\xi})$

Equação da lente $\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta}) = \vec{\theta} - \vec{\nabla}_{\theta} \Psi(\vec{\theta})$

Potencial de Lente(amento) $\Psi \equiv \frac{2}{c^2} \frac{D_{\text{LS}}}{D_{\text{OS}} D_{\text{OL}}} \psi$

convertendo variáveis $d\xi_i = D_{\text{OL}} d\theta_i$

$$\nabla_{\theta}^2 \Psi = \frac{2}{c^2} \frac{D_{\text{LS}}}{D_{\text{OS}} D_{\text{OL}}} D_{\text{OL}}^2 4\pi G \Sigma$$

Convergência

Equação de Poisson

$$\nabla_{\vec{\theta}}^2 \Psi = \frac{2}{c^2} \frac{D_{\text{LS}}}{D_{\text{OS}} D_{\text{OL}}} D_{\text{OL}}^2 4\pi G \Sigma(\vec{\theta}) = 2 \frac{\Sigma(\vec{\theta})}{\Sigma_{\text{crit}}}$$

Densidade superficial crítica

$$\Sigma_{\text{crit}} = \frac{c^2}{4\pi G} \frac{D_{\text{OS}}}{D_{\text{OL}} D_{\text{LS}}}$$

Convergência

$$\kappa(\vec{\theta}) = \frac{\Sigma(\vec{\theta})}{\Sigma_{\text{crit}}}$$

Equação de Poisson

$$\nabla_{\vec{\theta}}^2 \Psi = 2\kappa(\vec{\theta})$$

Jacobiana da transformação

$$\vec{\beta} = \vec{\theta} - \vec{\nabla}_{\theta} \Psi(\vec{\theta})$$

$$J_{ij} = \left(\frac{\partial \vec{\beta}}{\partial \vec{\theta}} \right)_{ij} = \delta_{ij} - \frac{\partial^2 \Psi}{\partial \theta_i \partial \theta_j}, \quad \text{definindo } \Psi_{ij} = \frac{\partial^2 \Psi}{\partial \theta_i \partial \theta_j}$$

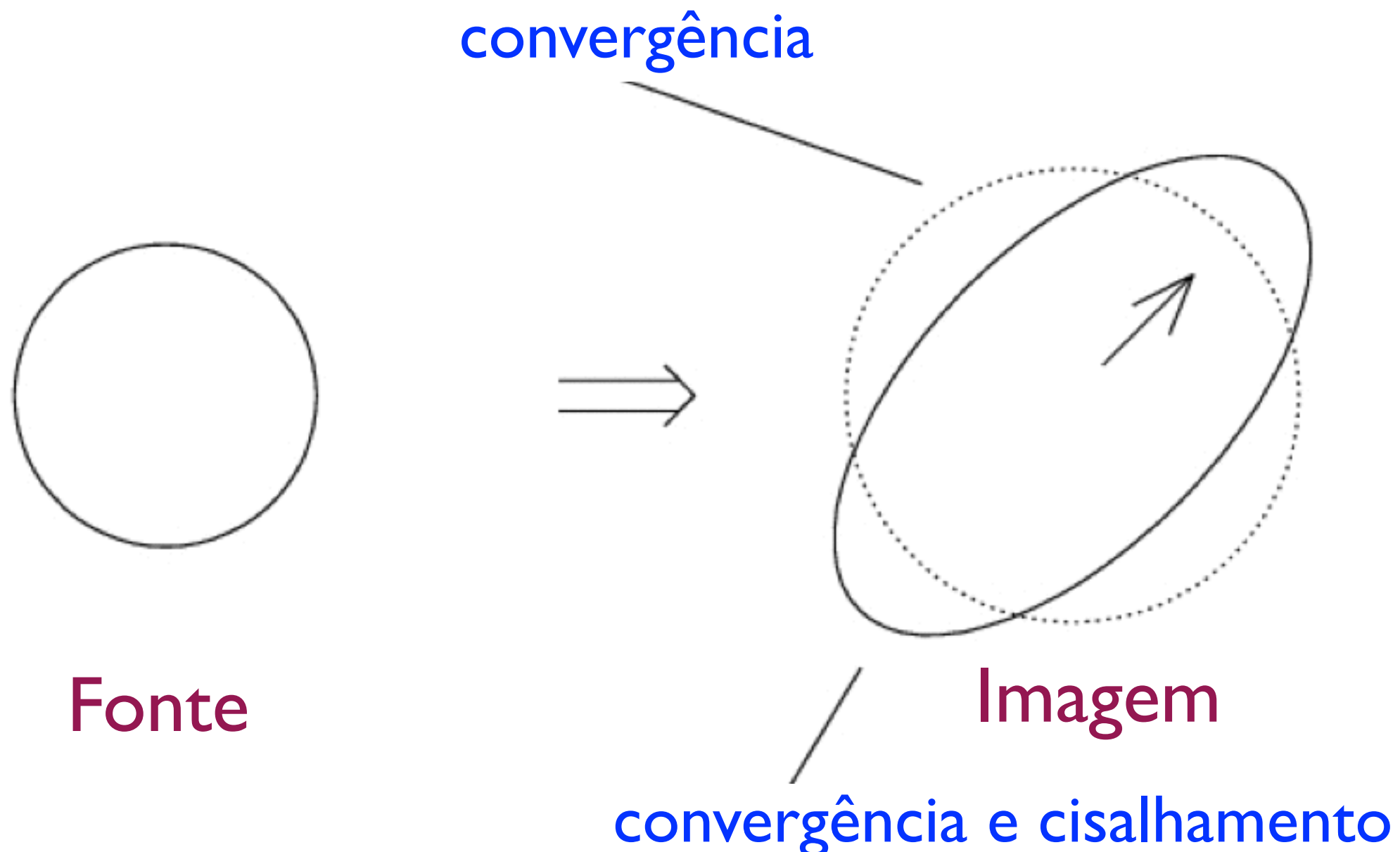
temos $\mathbf{J} = \begin{pmatrix} 1 - \Psi_{11} & -\Psi_{12} \\ -\Psi_{21} & 1 - \Psi_{22} \end{pmatrix}$

como $\nabla_{\theta}^2 \Psi = \Psi_{11} + \Psi_{22} = 2\kappa$

$$\mathbf{J} = (1 - \kappa) \mathbf{I} - \begin{pmatrix} \frac{1}{2} (\Psi_{11} - \Psi_{22}) & \Psi_{12} \\ \Psi_{21} & -\frac{1}{2} (\Psi_{11} - \Psi_{22}) \end{pmatrix}$$

Jacobiana da transformação

$$\mathbf{J} = (1 - \kappa)\mathbf{I} - \begin{pmatrix} \frac{1}{2}(\Psi_{11} - \Psi_{22}) & \Psi_{12} \\ \Psi_{21} & -\frac{1}{2}(\Psi_{11} - \Psi_{22}) \end{pmatrix}$$



Cisalhamento

$$\mathbf{J} = (1 - \kappa)\mathbf{I} - \begin{pmatrix} \frac{1}{2}(\Psi_{11} - \Psi_{22}) & \Psi_{12} \\ \Psi_{21} & -\frac{1}{2}(\Psi_{11} - \Psi_{22}) \end{pmatrix}$$

Cisalhamento:

$$\gamma_1(\vec{\theta}) = \frac{1}{2}(\Psi_{11} - \Psi_{22}) \quad \gamma = \sqrt{\gamma_1^2 + \gamma_2^2}$$

$$\gamma_2(\vec{\theta}) = \Psi_{12} = \Psi_{21}$$

Em termos do cisalhamento e convergência temos

$$\mathbf{J} = (1 - \kappa)\mathbf{I} + \begin{pmatrix} -\gamma_1 & -\gamma_2 \\ -\gamma_2 & \gamma_1 \end{pmatrix}$$

Autovalores de $\mathbf{J}^{-1}$

$$\mu_1 = \frac{1}{1 - \kappa - \gamma}, \quad \mu_2 = \frac{1}{1 - \kappa + \gamma}$$

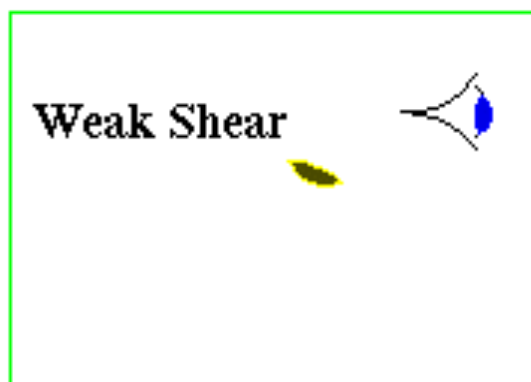
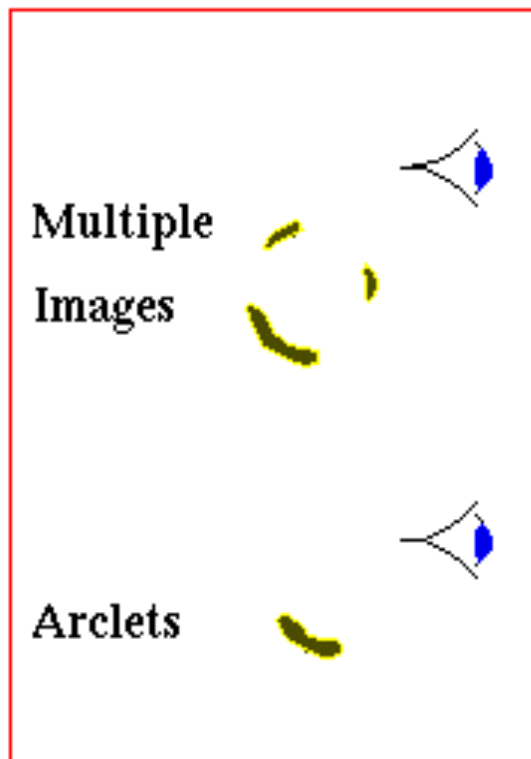
A scenic view of a beach with turquoise water and a clear blue sky. The text is overlaid on the image.

LENTEAMENTO GRAVITACIONAL FRACO

Efeito fraco e forte de lente

observador

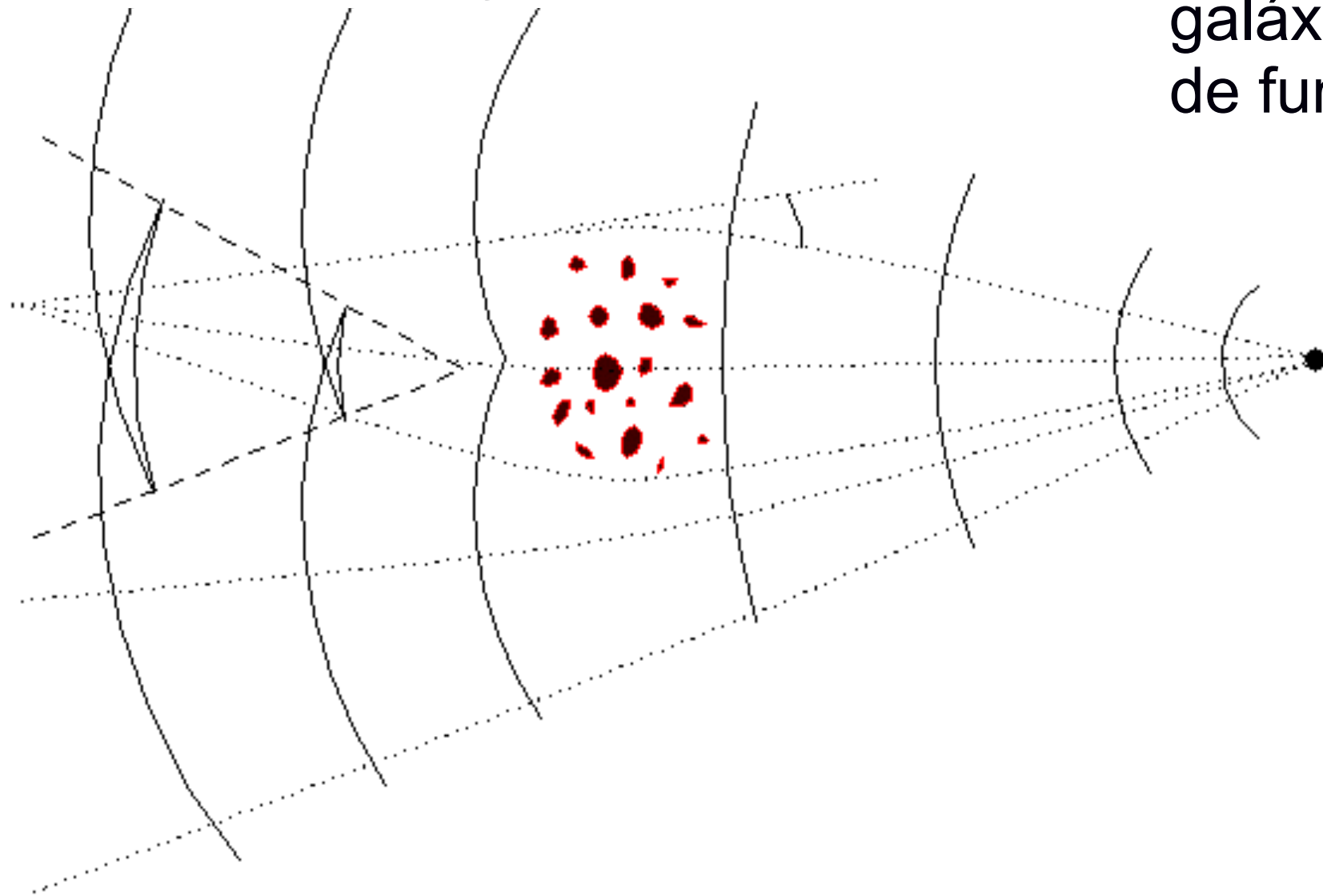
Non-Linear



Linear

aglomerado

galáxia
de fundo



MACRO LENTEAMENTO: DOIS REGIMES

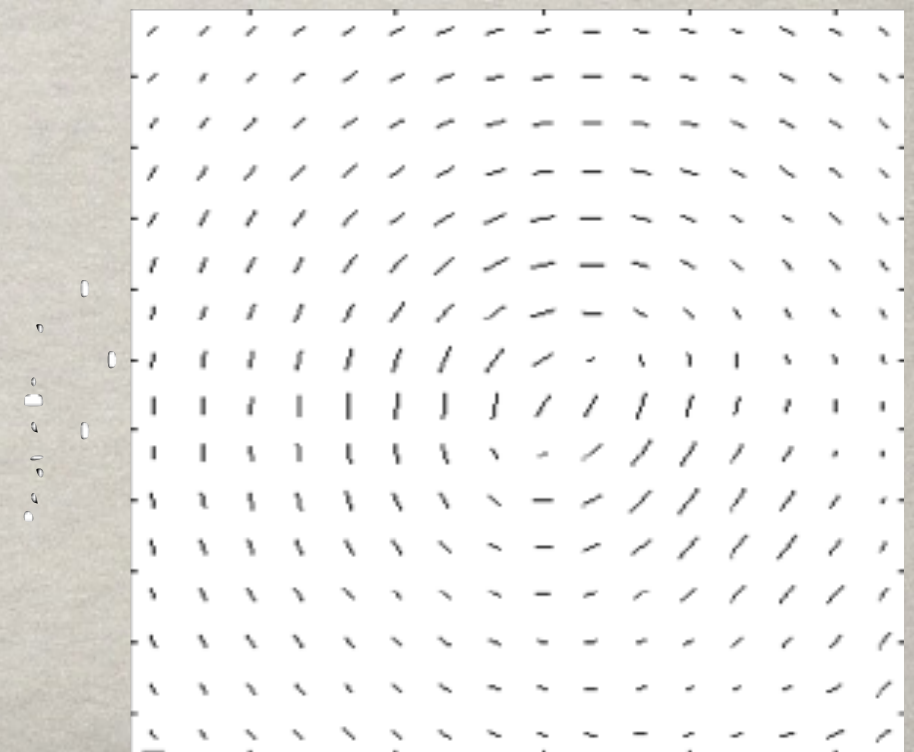
● Lenteamento forte:

- Grandes magnificações
- Imagens múltiplas
- Grandes distorções (arcos, anéis)
- Defasagem temporal
- $\Sigma > \Sigma_c$
- Regiões centrais de galáxias e aglomerados



● Weak Lensing:

- Mudança na “orientação” (+magnificação)
- Regiões não centrais/de baixa massa de galáxias e aglomerados
- Estrutura em grande escala



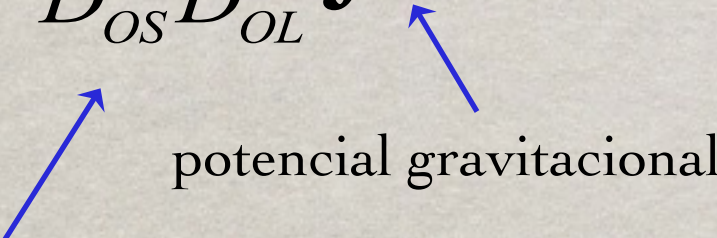
MAPEAMENTO DE LENTES

► mapeamento imagem $\rightarrow$ fonte

$$\frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial^2 \Psi(\vec{\theta})}{\partial \theta_i \partial \theta_j}$$

► plano único

$$\Psi = \frac{2}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \int \phi(\xi, z) dz$$



distâncias cosmológicas

$$D_{LS} = D_A(z_L, z_S) \dots$$

► autovalores:

$$\mu_1 = \frac{1}{1 - \kappa + \gamma}, \mu_2 = \frac{1}{1 - \kappa - \gamma}$$

► magnificação local e razão axial:

$$\mu = \mu_1 \mu_2 \quad r = \left| \frac{\mu_1}{\mu_2} \right|$$

► densidade superficial crítica

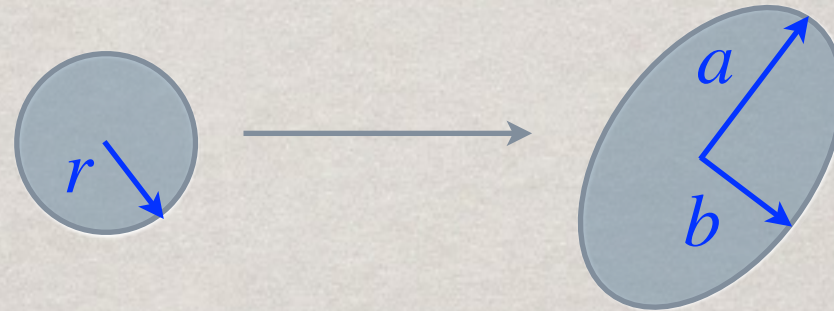
$$\Sigma_{\text{crit}} = \frac{c^2}{4\pi G} \frac{D_{OS}}{D_{OL} D_{LS}}$$

► convergência

$$\kappa = \frac{\Sigma}{\Sigma_{\text{crit}}}$$

MAPEAMENTO LINEAR

● Fonte circular



$$a = \left(\frac{1}{1 - \kappa - \gamma} \right) r$$

$$b = \left(\frac{1}{1 - \kappa + \gamma} \right) r$$

● Magnificação

$$\mu = \frac{A_{\text{imagem}}}{A_{\text{fonte}}} = \left[(1 - \kappa)^2 - \gamma^2 \right]^{-1}$$

● Elipticidade

$$\epsilon := \frac{a - b}{a + b} = \frac{\gamma}{1 - \kappa} =: g$$

MAPEAMENTO LINEAR

- Efeitos:
 - Distorção: variação na orientação (mais usual)
 - Magnificação: mudança na área e na densidade de objetos
 - Analogamente a γ a elipticidade possui 2 componentes: matriz, pseudo-vetor ou número complexo

$$\epsilon = \epsilon_1 + i\epsilon_2 = |\epsilon|e^{2i\varphi}$$

$$\epsilon := \frac{a - b}{a + b} = \frac{\gamma}{1 - \kappa}$$

$$\kappa \ll 1 \text{ e } \gamma \ll 1$$

FONTES REAIS

- Centro do objeto (imagem)

$$\bar{\theta}_i = \frac{\int d^2\theta q_I [I(\theta)] \theta_i}{\int d^2\theta q_I [I(\theta)]}$$

- Momenta de segunda ordem

$$Q_{ij} = \frac{\int d^2\theta q_I [I(\theta)] (\theta_i - \bar{\theta}_i) (\theta_j - \bar{\theta}_j)}{\int d^2\theta q_I [I(\theta)]}$$

- Área

$$\Omega = (Q_{11}Q_{22} - Q_{12}^2)^{1/2}$$

- Elipticidade

$$\epsilon := \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22} + 2(Q_{11}Q_{22} - Q_{12}^2)^{1/2}}$$

FONTES REAIS

● Momenta de segunda ordem

$$Q_{ij} = \frac{\int d^2\theta q_I[I(\theta)](\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j)}{\int d^2\theta q_I[I(\theta)]}$$

● Elipticidade

$$\epsilon := \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22} + 2(Q_{11}Q_{22} - Q_{12}^2)^{1/2}}$$

● Distorção linear

$$\epsilon_S = \frac{\epsilon_I - g}{1 - g^* \epsilon_I} \quad g_i := \frac{\gamma_i(\theta)}{1 - \kappa(\theta)} \quad |g| \leq 1$$

LENTEAMENTO FRACO

- Regime de lenteamento fraco

$$\epsilon = \epsilon_I = \epsilon_S + g$$

- “teorema fundamental do lenteamento fraco”:

$$\langle \epsilon \rangle = g$$

- mas

$$g \ll \epsilon$$

- desafios observacionais, instrumentais, computacionais e teóricos!

Medindo o lenteamento fraco

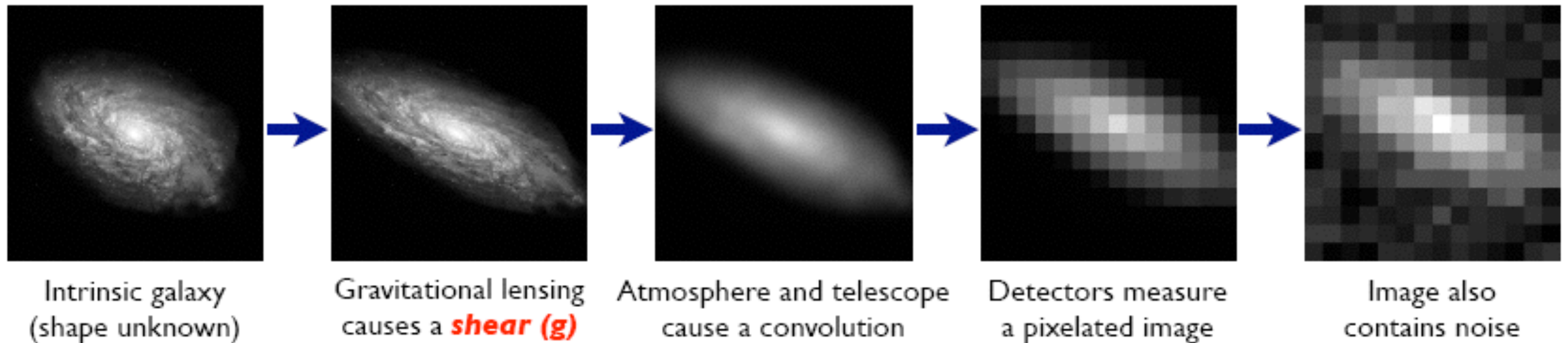
- Sinal $\langle \varepsilon_I \rangle = \gamma$ ($\varepsilon_I = \varepsilon_S + \gamma$)
- Ruído $\sigma_\varepsilon = \left\langle \left| \varepsilon_S^2 \right| \right\rangle^{1/2} \sim 0.3 \gg \gamma$
- Ganhar do ruído fazendo uma média em um grande número de galáxias

Regime	γ	$\gamma/\sigma_\varepsilon$	N_{gal} for $S/N \sim 1$
weak lensing by clusters	0.03	0.1	10^2
galaxy-galaxy lensing	0.003	0.01	10^4
cosmic shear	0.001	0.003	10^5

Much more galaxies for precision measurements needed.

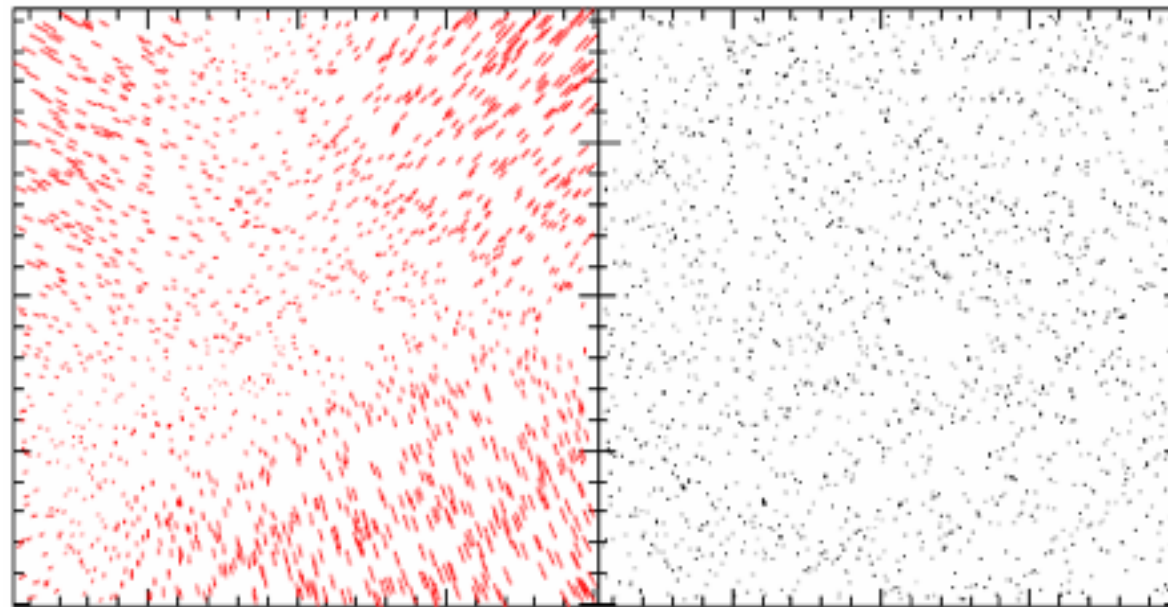
(de Kilbinger)

Desafios Técnicos



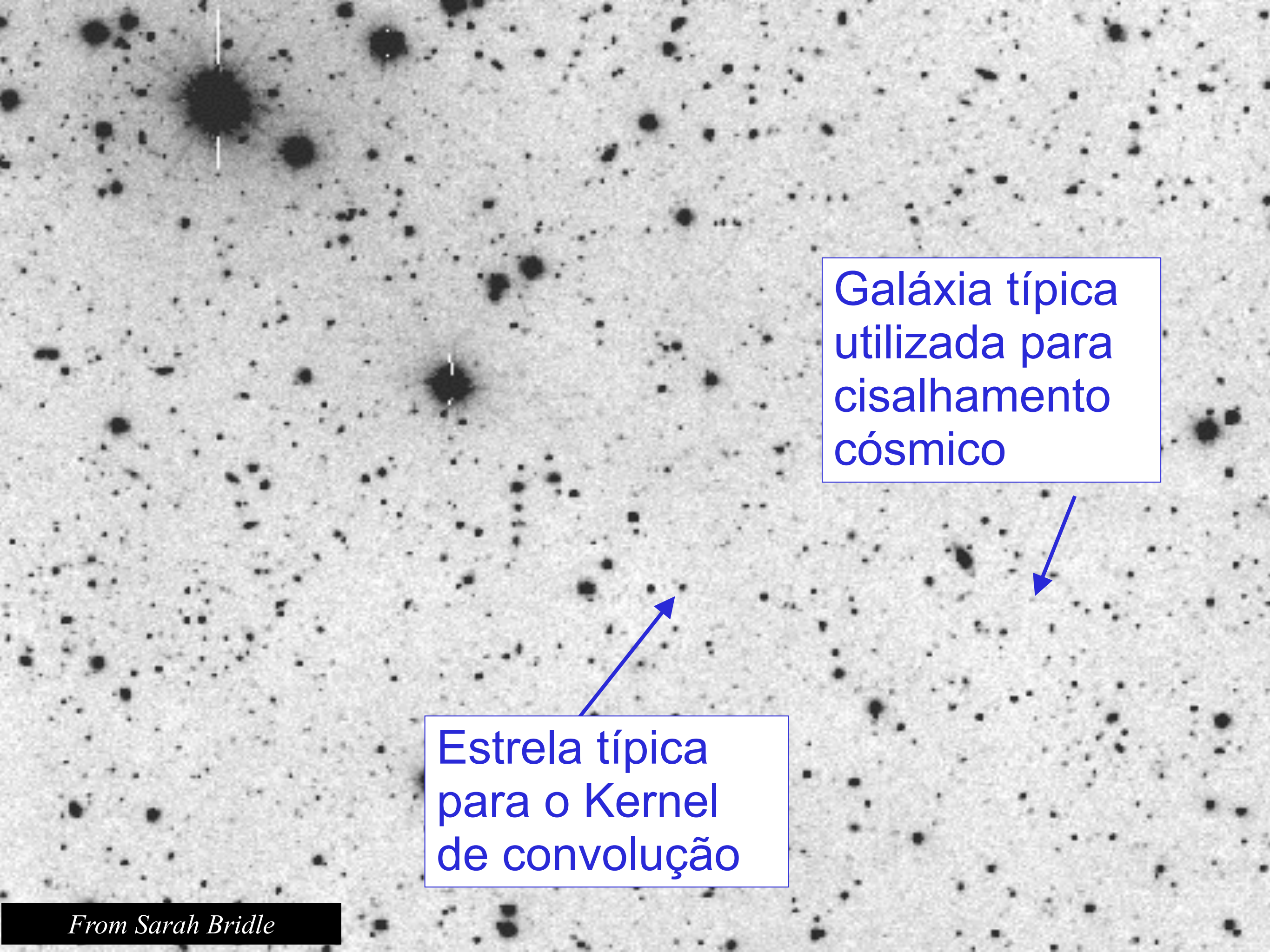
Sarah Bridle

- Modelar as distorções da PSF (muitas fontes)
 - medir um grande número de estrelas no campo



CFHTLS Deep 1
Gavazzi, Soucail 2006

- Dificuldades: estrelas saturadas, eficiência de transferência de carga, halos, acompanhamento...



Galáxia típica
utilizada para
cisalhamento
cósmico

Estrela típica
para o Kernel
de convolução

Caminho

1) Obter as medidas de forma das galáxias

- Identificar as estrelas
- Modelar as distorções da PSF (muitas fontes)
- Medir forma das galáxias
- Aplicar a PSF nas medidas de forma das galáxias
- Selecionar as galáxias (z , S/N, tamanho/magnitude, etc.)

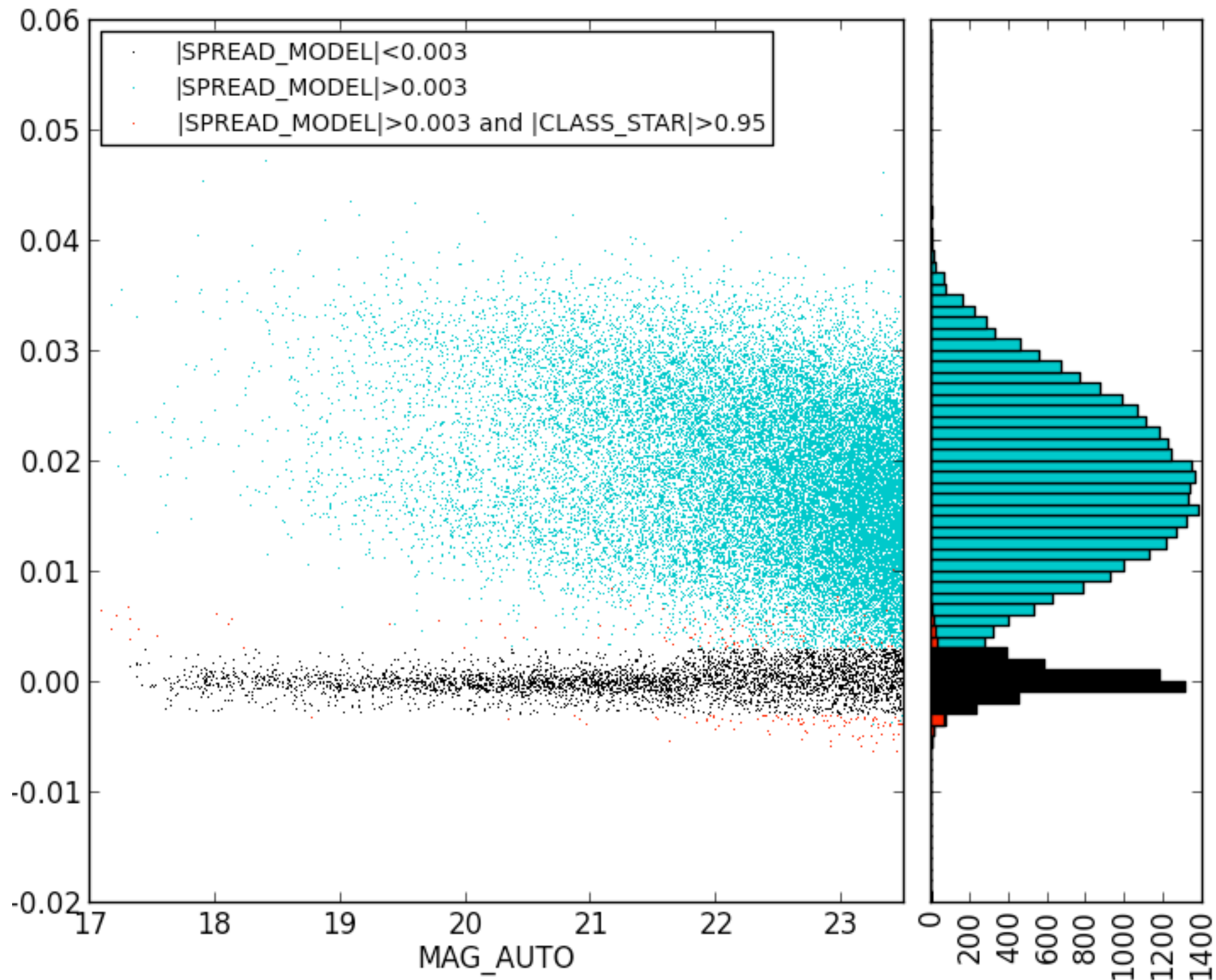
O domínio
da
estatística

2) Realizar as medidas das quantidades relevantes

- Miríade de observáveis e de estimadores estatísticos
- Obter estimadores óptimos, levando em conta efeitos instrumentais e observacionais
- Determinar incertezas

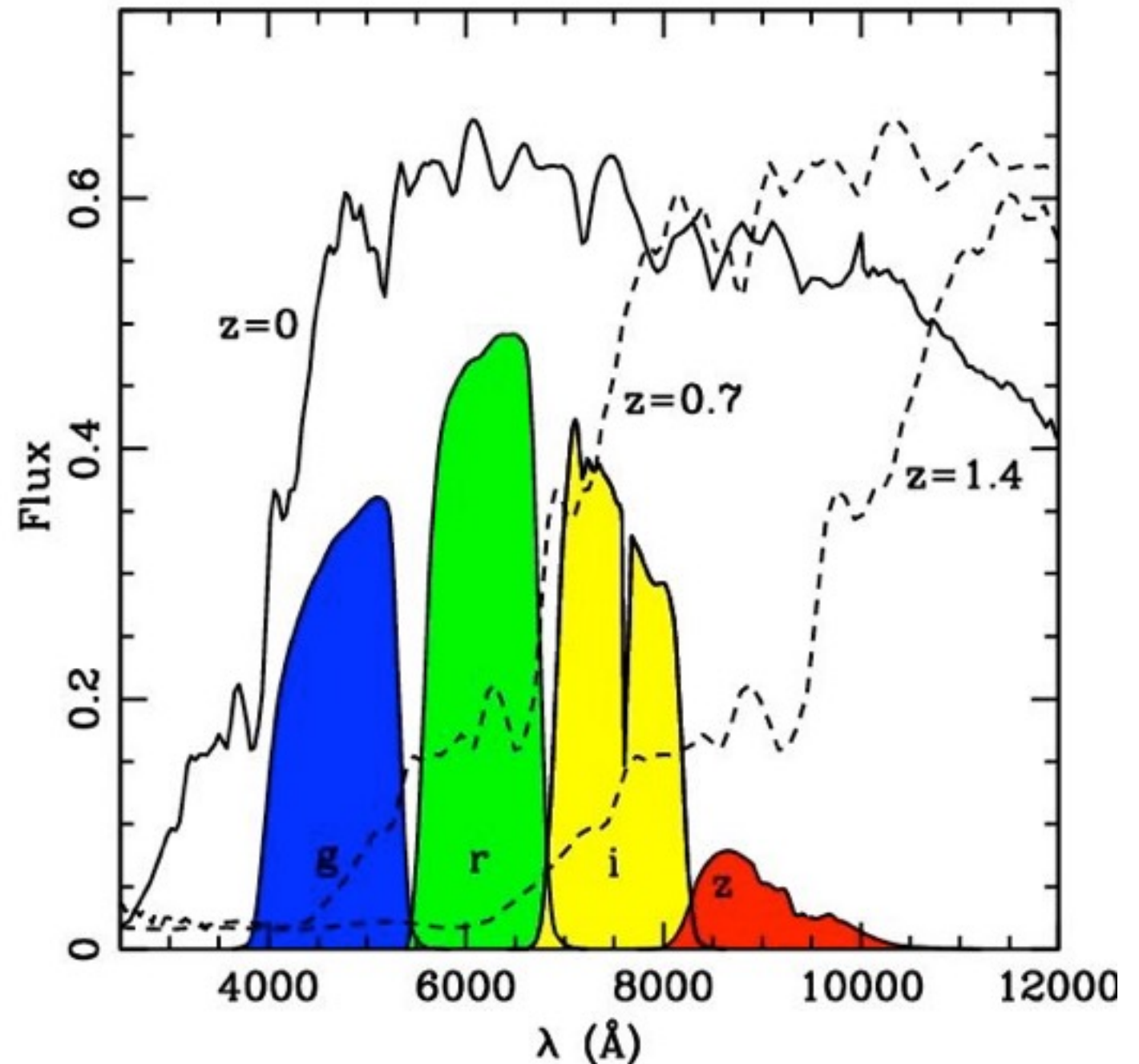
Exemplo:

separação estrela-galáxia



Desvios para o vermelho fotométricos

- Características marcantes do espectro (quebra de 4000 Å)
- Diferença no fluxo através dos filtros quando a galáxia é desviada para o vermelho



2 Regimes e Métodos

Lenteamento por galáxias e aglomerados

- Maior sinal
- Centro de referência
- Ajuste de modelos
- Objetos individuais ou *Stacking* do sinal

Estrutura em grande escala

- Mapas de convergência (também em aglomerados)
- Correlações:
 - espectro de potências, função de correlação
 - vários pontos, entre bins de z , com massa, CMB, etc.