Which quantum states are dual to classical spacetimes?

Based on arXiv: 1512.07850, 1703.02384, 1703.03483 [hep-th]

Marcelo Botta-Cantcheff, Guillermo Silva, PJM

CBPF - 09/02/2018

Outline

- Why AdS/CFT? (from the bottom up)
- Standard (Euclidean) Prescription: GKPW
  - Short Example: massless field
- Real-time and Excited states
  - Example: In-Out path
  - N-modes: Causality and Excited states
- Applications
  - Building up geometries
  - Multiple "filling" geometries
  - States with geometrical dual (Finally!)
- Conclusions

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Why AdS/CFT?

- I won't concentrate on $\mathrm{SYM}\ (N=4) \Leftrightarrow \mathrm{ST\ IIB}$ though it is useful as an example with a complete map [1]

- CFT correlators through classical computations in AdS
  - Correlators $\Leftrightarrow$ Solve theory
  - Strong coupling $\Rightarrow$ No perturbations
  - Symmetry protected quantities

  - CFT does not generally have an action principle

- Strong/Weak dualities

[1] Maldacena [hep-th/9711200v3]

Why AdS/CFT?

- CFT does not generally have an action principle
  - Conformal Group $\Longrightarrow$

$$\langle \mathcal{O}(x) \rangle = 0$$

$$\langle \mathcal{O}(x)\mathcal{O}(y) \rangle = |x-y|^{-\Delta}$$

$$\langle \mathcal{O}(x)\mathcal{O}(y)\mathcal{O}(z) \rangle = C_\Delta |x-y|^{-\Delta}|y-z|^{-\Delta}|z-x|^{-\Delta}$$

$$\langle \mathcal{O}(x)\mathcal{O}(y)\mathcal{O}(z)\mathcal{O}(w) \rangle = \ldots$$

  - Conformal Bootstrap fixes the rest

Why AdS/CFT?

- CFT does not generally have an action principle
  - This impedes the canonical perturbative framework

$$\text{Action} \Rightarrow \text{EOM} \Rightarrow \text{Perturb} \Rightarrow \text{Diagrams}$$

  - We will try to recover this

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GKPW

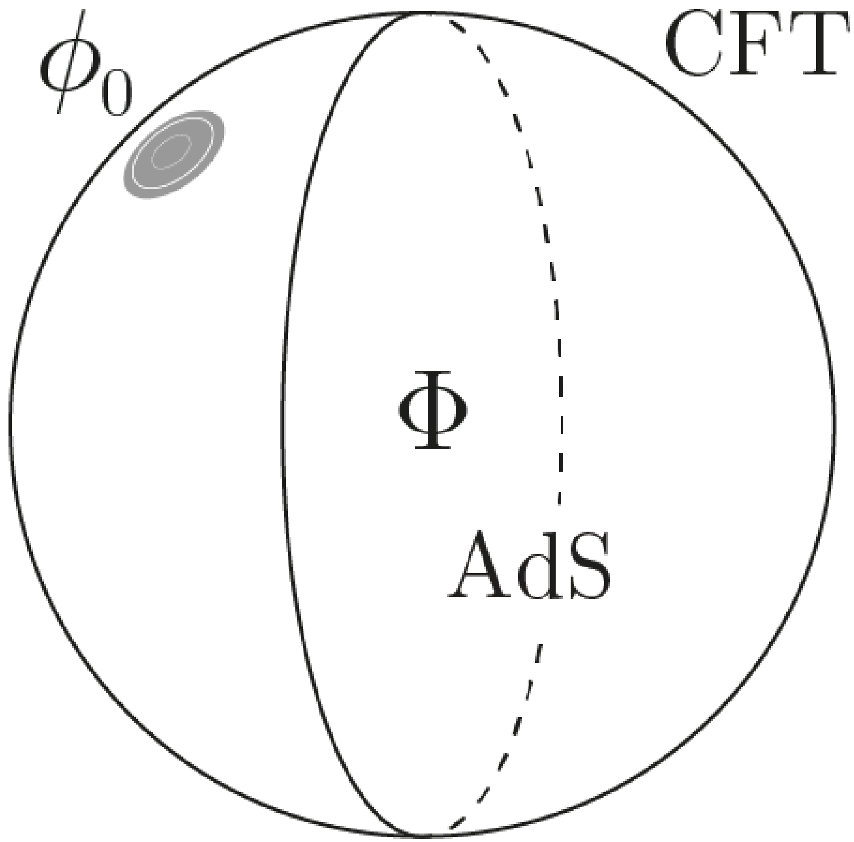
- GKPW is an Euclidean prescription to calculate n-point functions of CFT local operators through calculations in the dual bulk theory [2]:

$$\mathcal{Z}^{d}_{\mathrm{CFT}}[J = \phi_0] \equiv \mathcal{Z}^{d+1}_{\mathrm{AdS}}[\Phi|_\partial = \phi_0]$$

[2] Gubser, Klebanov & Polyakov [hep-th/9802109] + Witten [hep-th/9802150]

GKPW

- GKPW [2]: $\mathcal{Z}^{d}_{\mathrm{CFT}}[J = \phi_0] \equiv \mathcal{Z}^{d+1}_{\mathrm{AdS}}[\Phi|_\partial = \phi_0]$



[2] Gubser, Klebanov & Polyakov [hep-th/9802109] + Witten [hep-th/9802150]

GKPW

- $\mathcal{Z}^{d}_{\mathrm{CFT}}[J=\phi_0] \equiv \mathcal{Z}^{d+1}_{\mathrm{AdS}}[\Phi|_\partial = \phi_0]$

  - CFT side:

$$\mathcal{Z}^{d}_{\mathrm{CFT}}[J=\phi_0] \equiv \langle e^{-\int_\partial \mathcal{O}\phi_0} \rangle$$

  - $\frac{\delta^n}{\delta\phi_0^n}\left(\ln\left(\cdots\right)\right)_{\phi_0=0} =$ Connected n-point functions
  - No perturbative or exact methods available

GKPW

- $\mathcal{Z}^{d}_{\mathrm{CFT}}[J=\phi_0] \equiv \mathcal{Z}^{d+1}_{\mathrm{AdS}}[\Phi|_\partial = \phi_0]$
  - AdS side:

$$\mathcal{Z}^{d+1}_{\mathrm{AdS}}[\Phi|_\partial = \phi_0] \equiv \int [\mathcal{D}\Phi]_{\Phi|_\partial=\phi_0} e^{-S_E[\Phi]}$$

    - Hard to compute in general, but…

GKPW

- $\mathcal{Z}^{d}_{\mathrm{CFT}}[J=\phi_0] \equiv \mathcal{Z}^{d+1}_{\mathrm{AdS}}[\Phi|_\partial = \phi_0]$
  - Weakly coupled AdS:

$$\mathcal{Z}^{d+1}_{\mathrm{AdS}}[\Phi|_\partial = \phi_0] \equiv \int [\mathcal{D}\Phi]_{\Phi|_\partial=\phi_0} e^{-S_E[\Phi]}$$

$$\sim e^{-S_E^0[\phi_0]}$$

  - CFT side:

$$\mathcal{Z}^{d}_{\mathrm{CFT}}[J=\phi_0] \equiv \langle e^{-\int_\partial \mathcal{O}\phi_0} \rangle$$

GKPW

- (Operational) Master equation:

$$\left(\frac{\delta^n}{\delta \phi_0^n} \ln \langle e^{-\int_\partial \mathcal{O} \phi_0} \rangle \right)_{\phi_0 = 0} \equiv - \left(\frac{\delta^n}{\delta \phi_0^n} S_E^0 [\phi_0] \right)_{\phi_0 = 0}$$

GKPW

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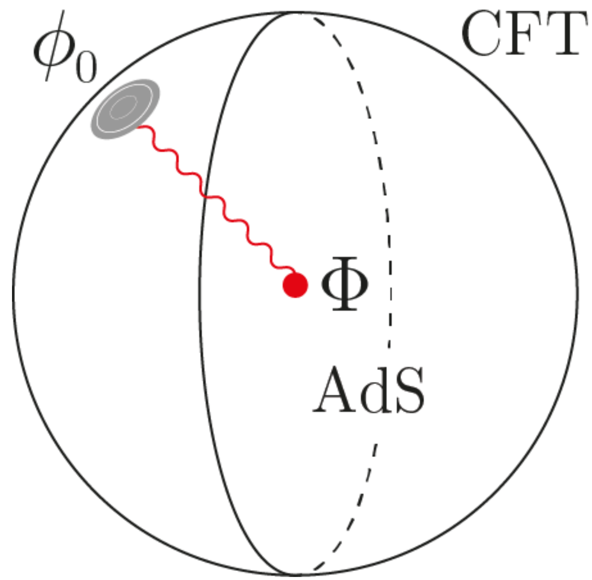
- Example: Massless Scalar in Pure AdS

GKPW

- Example: Massless Scalar in Pure AdS

$$S_E^0 = \int \sqrt{g}\, \partial_\mu \Phi \partial^\mu \Phi$$

- The solution to the EOM's is unique given the source ϕ_0



$$\Phi = \int \mathcal{K} \phi_0$$

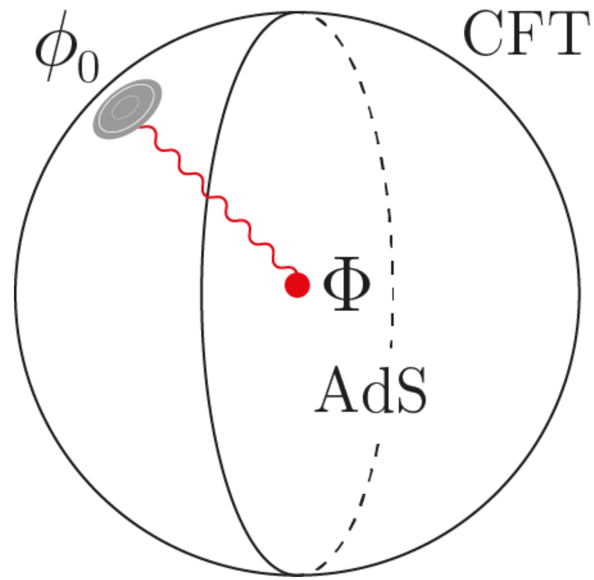
GKPW

- Example: Massless Scalar in Pure AdS

$$S_E^0 = \int \sqrt{g}\,\partial_\mu \Phi \partial^\mu \Phi = (\text{EDM}) + \int \partial_\mu \left(\sqrt{g}\Phi\partial^\mu\Phi\right)$$

$$= \int_\partial \sqrt{\gamma}\phi_0 \left[n^\mu \partial_\mu \Phi\right]_\partial$$

$$= \int_\partial \int_\partial \phi_0 \left[\sqrt{\gamma} n^\mu \partial_\mu \mathcal{K}\right] \phi_0$$



$$\Phi = \int \mathcal{K}\phi_0$$

GKPW

- Example: Massless Scalar in Pure AdS
  - Master Eq. $\Longrightarrow$ Correlators

$$\left(\frac{\delta^n}{\delta \phi_0^n} \ln \langle e^{-\int_\partial \mathcal{O}\phi_0} \rangle \right)_{\phi_0=0} \equiv - \left(\frac{\delta^n}{\delta \phi_0^n} S_E^0[\phi_0] \right)_{\phi_0=0}$$

$$S_E^0 = \int_\partial \int_\partial \phi_0 \left[\sqrt{\gamma} n^\mu \partial_\mu \mathcal{K} \right] \phi_0$$

GKPW

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$$S_E^0 = \int_\partial \int_\partial \phi_0 \left[\sqrt{\gamma} n^\mu \partial_\mu \mathcal{K} \right] \phi_0$$

$$\langle \mathcal{O}(x) \rangle = 0$$ ✓

$$\langle \mathcal{O}(x)\mathcal{O}(y) \rangle = |x-y|^{-\Delta}$$ ✓ $$\Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2}$$

$$\langle \mathcal{O}(x)\mathcal{O}(y)\mathcal{O}(z) \rangle = 0$$ ✗

GKPW

- Example: Massless Scalar
  - Master Eq. $\Longrightarrow$ Correlators

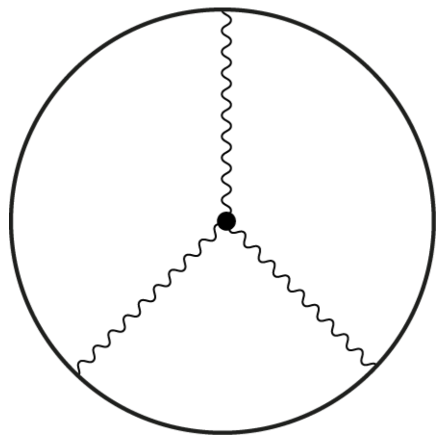
$$\left(\frac{\delta^n}{\delta\phi_0^n}\ln\langle e^{-\int_\partial \mathcal{O}\phi_0}\rangle\right)_{\phi_0=0} \equiv -\left(\frac{\delta^n}{\delta\phi_0^n}S_E^0[\phi_0]\right)_{\phi_0=0}$$

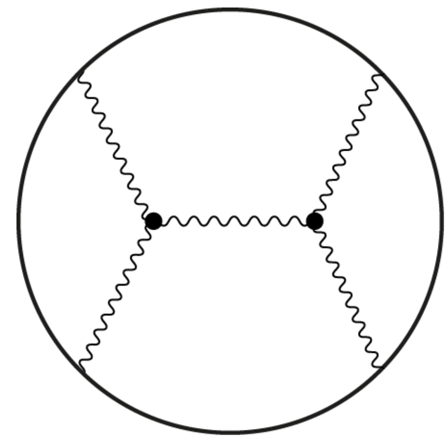
- $\langle\mathcal{O}(x)\mathcal{O}(y)\mathcal{O}(z)\rangle = 0$ ✘
  - Interactions in AdS [3]

[3] Freedman, Mathur, Matusis & Rastelli [hep-th/9804058v2]

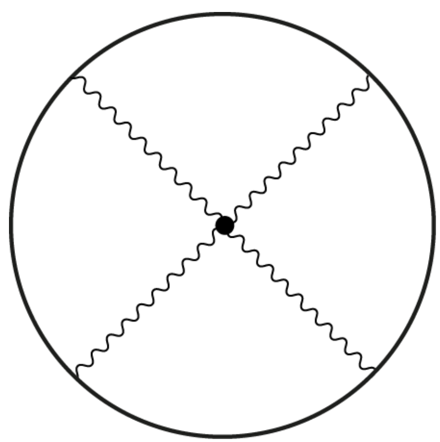
GKPW

- Witten diagrams!
  - $\alpha\Phi^3$





  - $\lambda\Phi^4$



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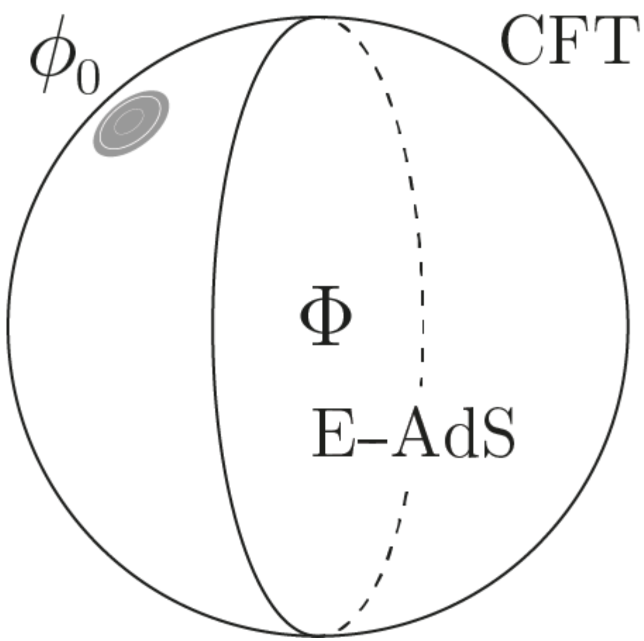
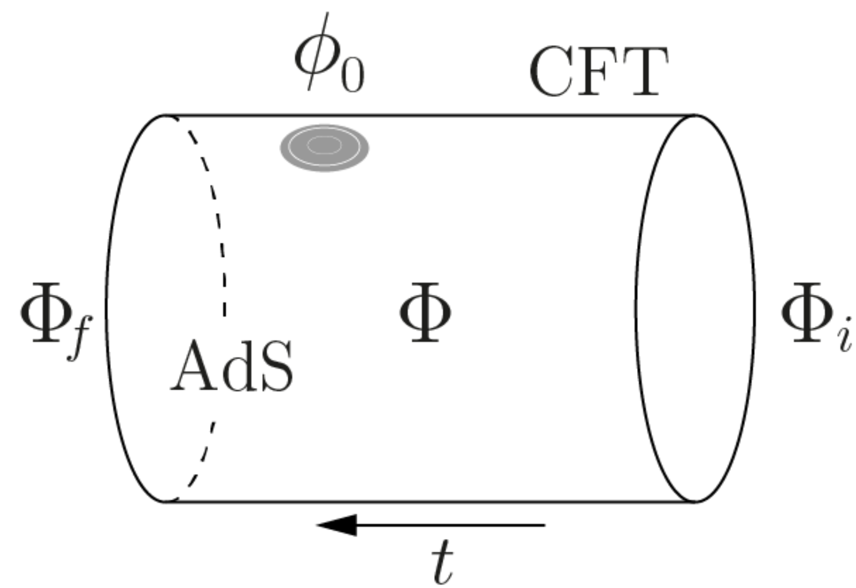
Real-time

- $¿\tau \rightarrow it?$: Holographic Problems

$$\langle e^{-\int_\partial \mathcal{O}\phi_0} \rangle \equiv e^{-S_E^0[\phi_0]}$$



$$\langle e^{-i\int_\partial \mathcal{O}\phi_0} \rangle \equiv e^{iS^0[\phi_0;\dots]}$$



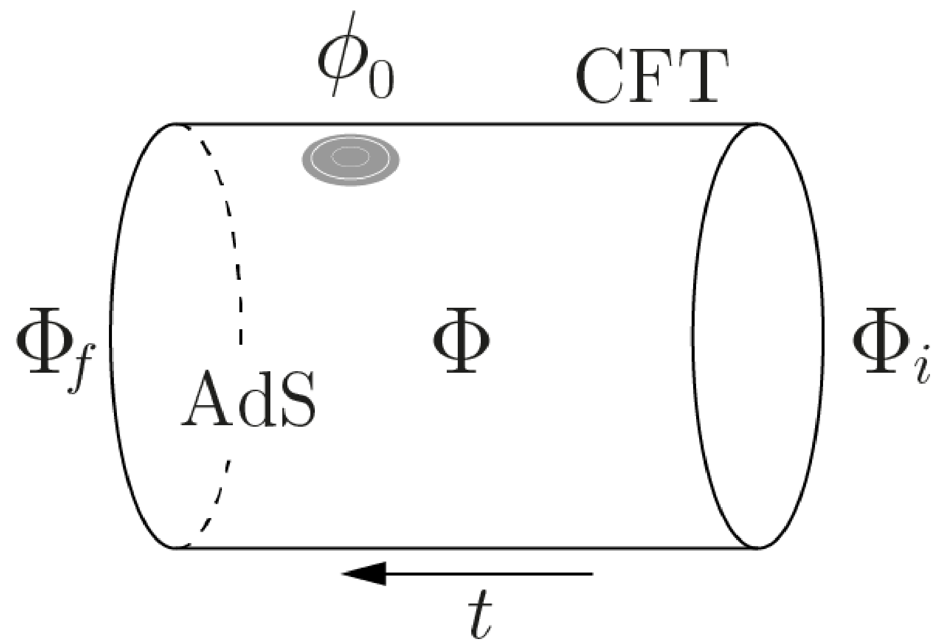
Real-time

- $¿\tau \to it?$: Holographic Problems

$$\langle \Phi_f | e^{-i\int_\partial \mathcal{O}\phi_0} | \Phi_i \rangle \equiv e^{iS^0[\phi_0;\Phi_i;\Phi_f]}$$

- EOM solution is not unique

$$\Phi = \int \mathcal{K}\phi_0 + \mathrm{N}(i,f)$$



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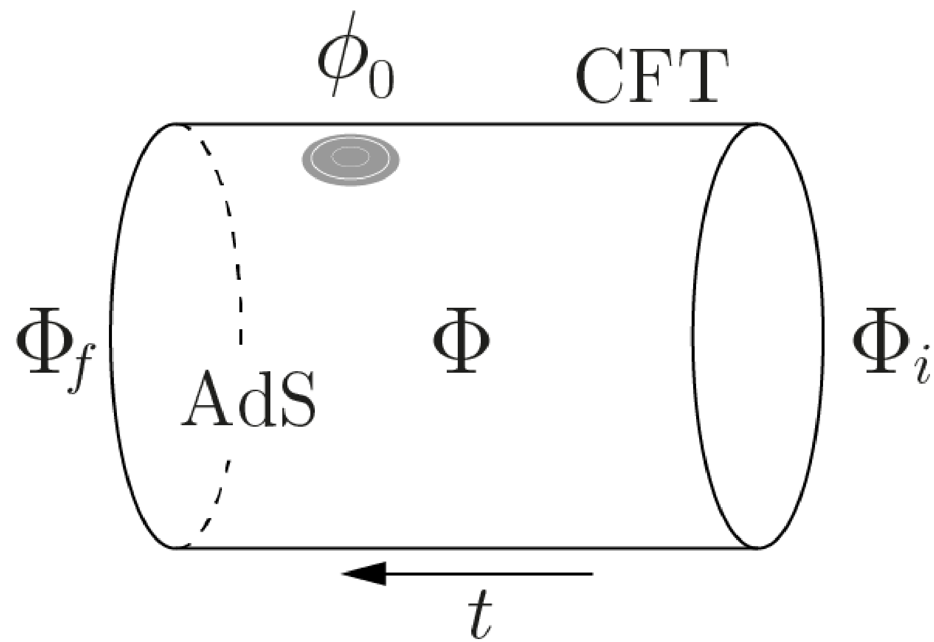
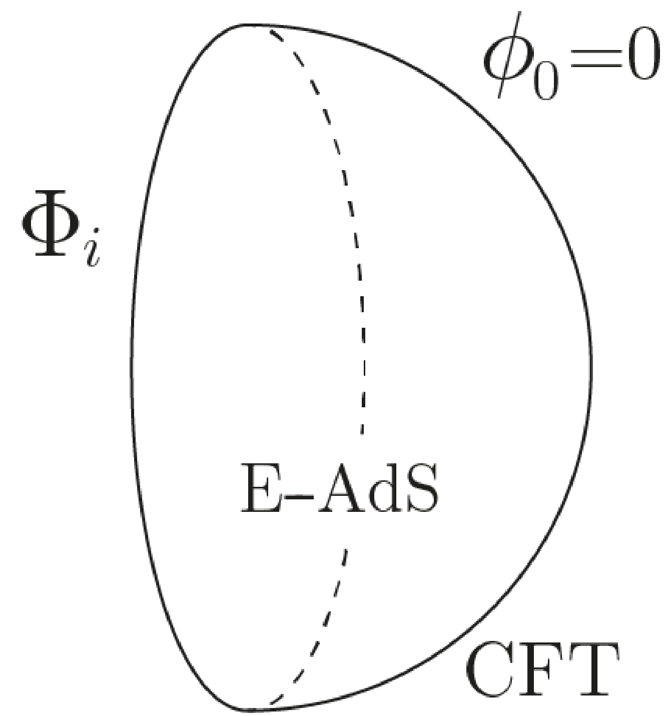


- What information does $\mathrm{N}(i,f)$ hide?

Real-time

- Borrowing the HH state [4],
  - $\langle \Phi_i | 0 \rangle = \int [\mathcal{D}\Phi]_{\Phi_i;0} \, e^{-S_E[\Phi]}$

$$\sim e^{-S_E^0[\Phi_i;0]}$$

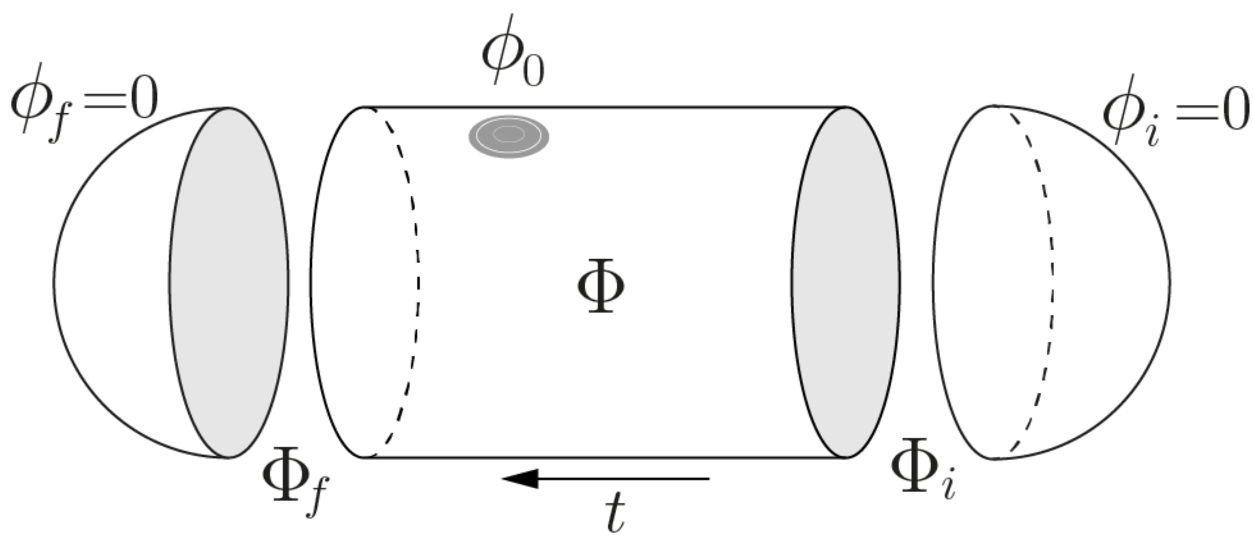


[4] Hartle & Hawking , Phys. Rev. D 28 (1983) 2960

Real-time

- SvR prescribed the vev's as [5],

$$\sum_{\Phi_i;\Phi_f} \langle 0|\Phi_f\rangle\langle\Phi_f|e^{-i\int_\partial \mathcal{O}\phi_0}|\Phi_i\rangle\langle\Phi_i|0\rangle = \langle 0|e^{-i\int_\partial \mathcal{O}\phi_0}|0\rangle$$



$$\langle 0|e^{-i\int_\partial \mathcal{O}\phi_0}|0\rangle \equiv e^{-S_f^0[0]}e^{iS^0[\phi_0]}e^{-S_i^0[0]}$$

[5] Skenderis & van Rees [arXiv:0805.0150] + [arXiv:0812.2909]

Excited States

- Immediate system to study:
  - $\phi_{i/f} \neq 0$ [6]

$$\langle \Phi_f | e^{-i\int_\partial \mathcal{O}\phi_0} | \Phi_i \rangle \equiv e^{-S_f^0[\phi_f]} e^{iS^0[\phi_0]} e^{-S_i^0[\phi_i]}$$

- EOM solution is unique again!

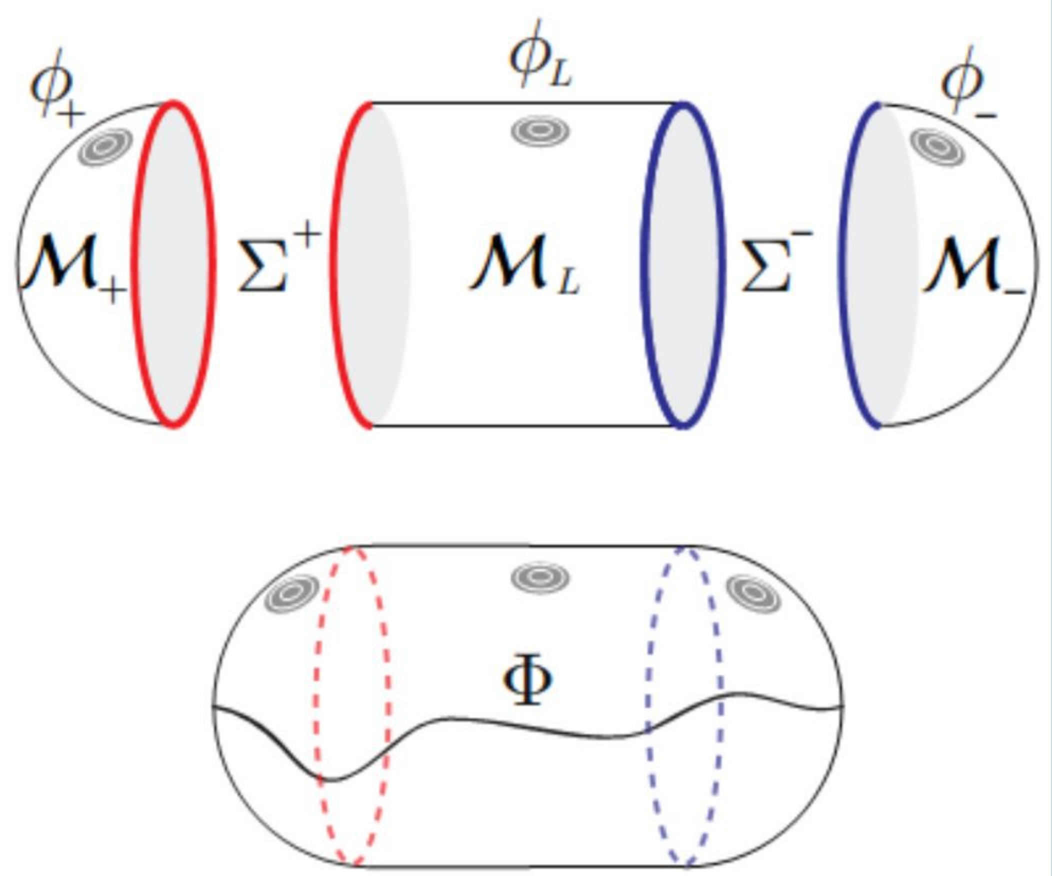
$$\Phi = \int \mathcal{K}\phi_0 + \mathrm{N}(i, f)$$

- What information does $\mathrm{N}(i, f)$ hide?
  - Causality
  - Excited States information

[6] MBC, GS , PJM [hep-th/1512.07850]

In-Out Formalism

- Steps
  - Solve the field EOM
    - Lorentz region
    - Euclidean regions
  - Match solution
  - On-shell action
  - Differentiate!



In-Out Formalism

- Lorentz region
  - Metric and EOM

$$ds^2 = -(1+r^2)dt^2 + (1+r^2)^{-1}dr^2 + r^2 d\varphi^2$$

$$\left(\Box - m^2\right)\Phi_L = 0 \implies \Phi_L(r,t,\varphi) \propto e^{-i\omega t + il\varphi}\, f(\omega,l,r)$$

$$f(\omega,l,r) = (1+r^2)^{\sqrt{\omega^2}/2}\; r^{|l|}\;{}_2F_1\left(\frac{\sqrt{\omega^2}+|l|+\Delta}{2}, \frac{\sqrt{\omega^2}+|l|-\Delta+2}{2}; 1+|l|; -r^2\right)$$

  - For frequencies $\pm\omega_{nl}^R$ one can build N solutions $g_{nl}(r)|_{r=R} = 0$
  - Solution

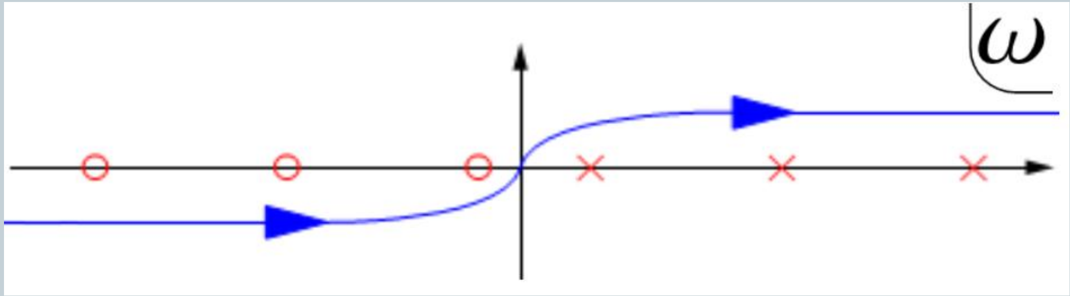
$$\Phi_L(r,t,\varphi) = \frac{R^{\Delta-2}}{4\pi^2}\sum_{l\in\mathbb{Z}}\int_{\mathcal{F}} d\omega dt' d\varphi' e^{-i\omega(t-t')+il(\varphi-\varphi')}\phi_L(t',\varphi')\frac{f(\omega,l,r)}{f(\omega,l,R)} +$$

$$\sum_{\substack{n\in\mathbb{N}\\ l\in\mathbb{Z}}}\left(L_{nl}^{+}\, e^{-i\omega_{nl}^R t} + L_{nl}^{-}\, e^{+i\omega_{nl}^R t}\right)e^{il\varphi} g_{nl}(r)$$

Feynman propagator

- The NN modes integrand as an infinite number of single poles at $\pm\omega_{nl}^{R}$

$$\frac{1}{f(\omega,l,R)}$$



- This forces to choose a complex integration path in the frequency integral.

In-Out Formalism

- Lorentz region
  - Metric and EOM

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$$\sum_{\substack{n\in\mathbb{N}\\ l\in\mathbb{Z}}}\left(L^+_{nl}\, e^{-i\omega^R_{nl}t} + L^-_{nl}\, e^{+i\omega^R_{nl}t}\right)e^{il\varphi}g_{nl}(r)$$

In-Out Formalism

- Euclidean regions
  - Metric and EOM

$$ds^2 = +(1+r^2)d\tau^2 + (1+r^2)^{-1}dr^2 + r^2 d\varphi^2$$

$$\left(\Box - m^2\right)\Phi_\pm = 0 \implies \Phi_\pm(r,\tau,\varphi) \propto e^{i\omega\tau + il\varphi}\, f(-i\omega, l, r)$$

$$f(\omega,l,r) = (1+r^2)^{\sqrt{\omega^2}/2}\; r^{|l|}\; {}_2F_1\left(\frac{\sqrt{\omega^2}+|l|+\Delta}{2}, \frac{\sqrt{\omega^2}+|l|-\Delta+2}{2}; 1+|l|; -r^2\right)$$

  - For frequencies $\pm i\omega_{nl}^R$ one can still build N solutions!
  - Solution for $\mathcal{M}_+$

$$\Phi_+(r,\tau,\varphi) = \frac{R^{\Delta-2}}{4\pi^2}\sum_{l\in\mathbb{Z}}\int d\omega d\tau' d\varphi' e^{i\omega(\tau-\tau')+il(\varphi-\varphi')}\phi_+(\tau',\varphi')\frac{f(-i\omega,l,r)}{f(-i\omega,l,R)} +$$

$$\sum_{\substack{n\in\mathbb{N}\\ l\in\mathbb{Z}}} E_{nl}^+\, e^{-\omega_{nl}^R(\tau+iT)+il\varphi} g_{nl}(r)$$

In-Out Formalism

- Matching the solution
  - Carrying out the ω integral for each solution, one finds that the continuity equations are satisfied if

$$L_{nl}^{\pm} = R^{\Delta-2} Res_{nl}^{R}\, \phi_{\mp;n(-l)}$$

$$E_{nl}^{+} = R^{\Delta-2} Res_{nl}^{R} \left(i\phi_{L;nl}^{*} + \phi_{-;n(-l)} \right)$$

$$E_{nl}^{-} = R^{\Delta-2} Res_{nl}^{R} \left(i\phi_{L;n(-l)} + \phi_{+;n(-l)} \right)$$

Excited States

- What have we learned? What were $\mathrm{N}(i, f)$ hiding!

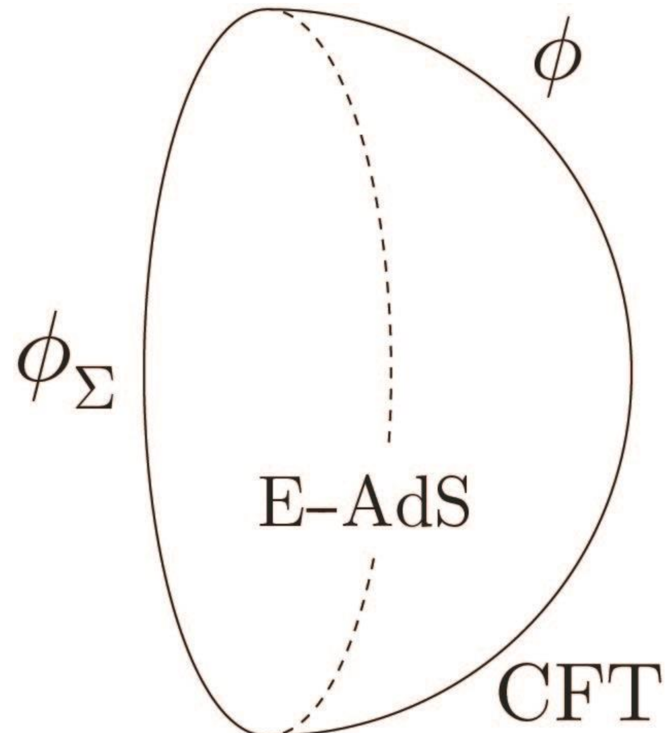
  - $L^{\pm}_{nl} \not\propto \tilde{\phi}_L$
    - Dependence on the same region source modifies the propagator itself by modifying the ω integration path we chose.

  - $L^{\pm}_{nl} \propto \tilde{\phi}_{\mp}$
    - Provides information on the excited state's nature
      - Coherent in large N

Excited States

- Our excited states are [6,7]
  Exact for all N values
  - CFT

$$|\phi\rangle_{\mathrm{CFT}} \equiv e^{-\int \mathcal{O}\phi}|0\rangle$$



  - AdS (inspired in [3])

$$\Psi^{\phi}[\phi_\Sigma] = \int [\mathcal{D}\Phi]_{\phi;\phi_\Sigma}\, e^{-S_E[\Phi]}$$

  - We will study the large N (coherent limit) later.

[4] Hartle & Hawking , Phys. Rev. D 28 (1983) 2960
[6] MBC, GS , PJM [hep-th/1512.07850], [7] MBC, GS , PJM [hep-th/1703.02384]

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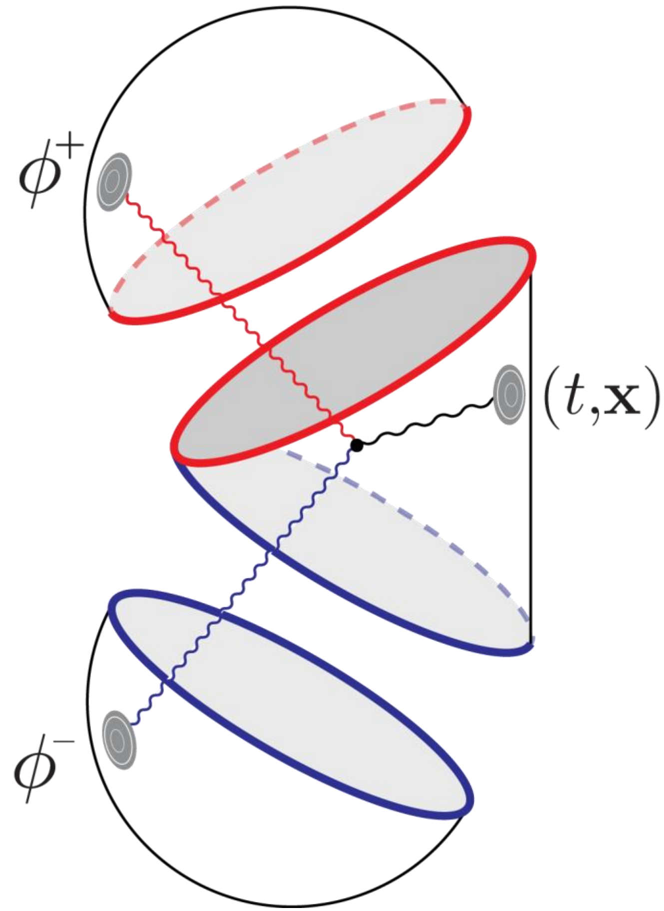
Applications

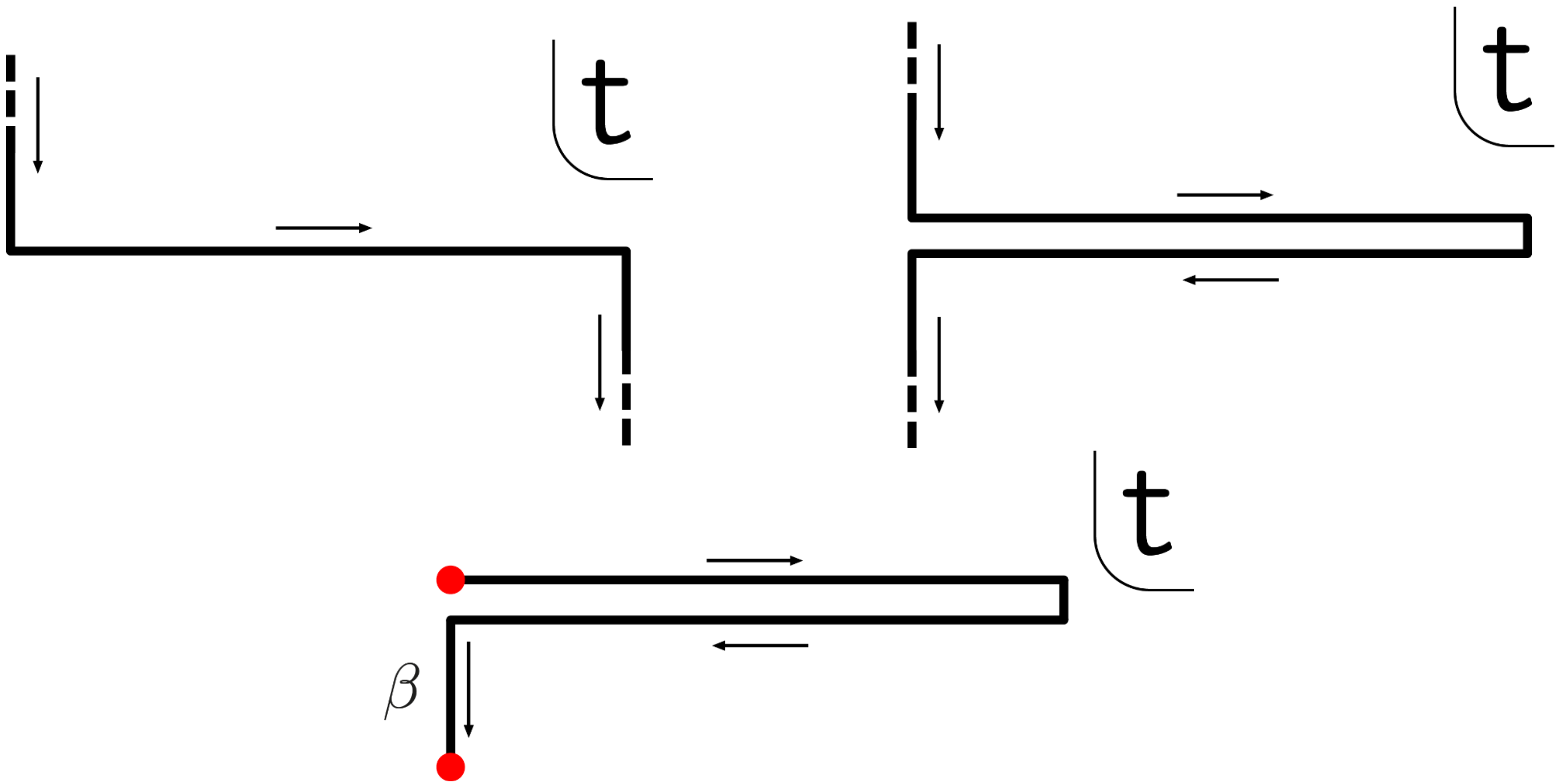
- Our second work [7] gave us a more intuitive look at our complex-signature manifolds
  - Diagrams on closed manifolds.
  - Interacting theories lead to modifications on our state.
  - Euclidean and Lorentzian sources powers are connected but their physical interpretation is different.



[7] MBC, GS , PJM [hep-th/1703.02384]

Applications

- Many different (Schwinger-Keldysh) paths are of interest:
  - Causality games



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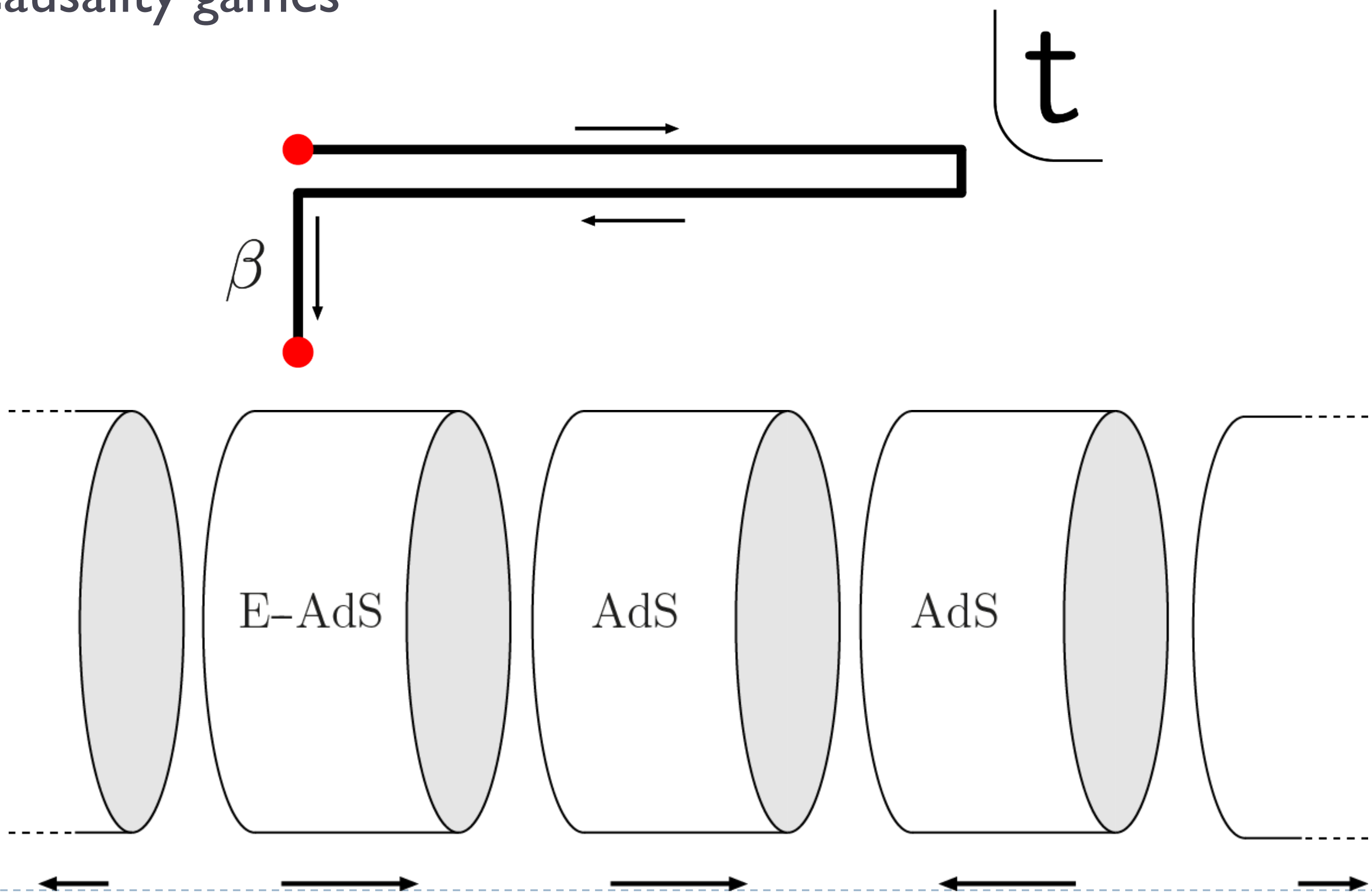
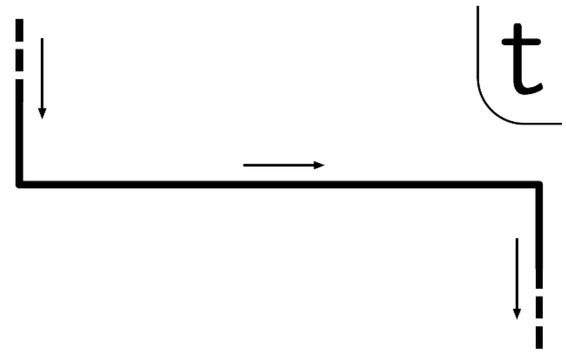


Applications

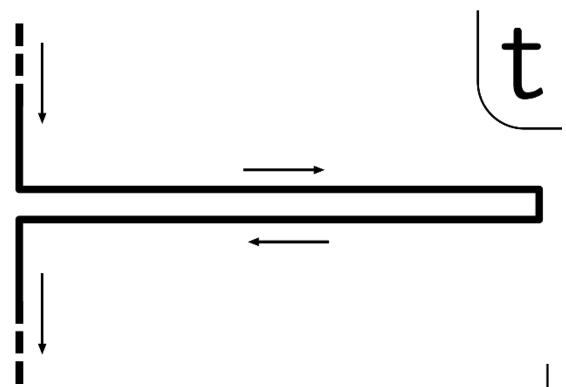
- Causality and N modes

$$L^{\pm}_{nl} \not\propto \tilde{\phi}_L \qquad L^{\pm}_{nl} \propto \tilde{\phi}_{\mp}$$

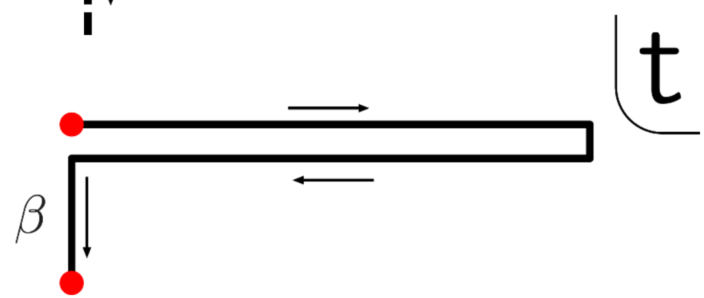


$$L2^{\pm}_{nl} \propto \tilde{\phi}_{L1} + \tilde{\phi}_{L2}$$

$$L2^{\pm}_{nl} \propto \tilde{\phi}_{\mp}$$



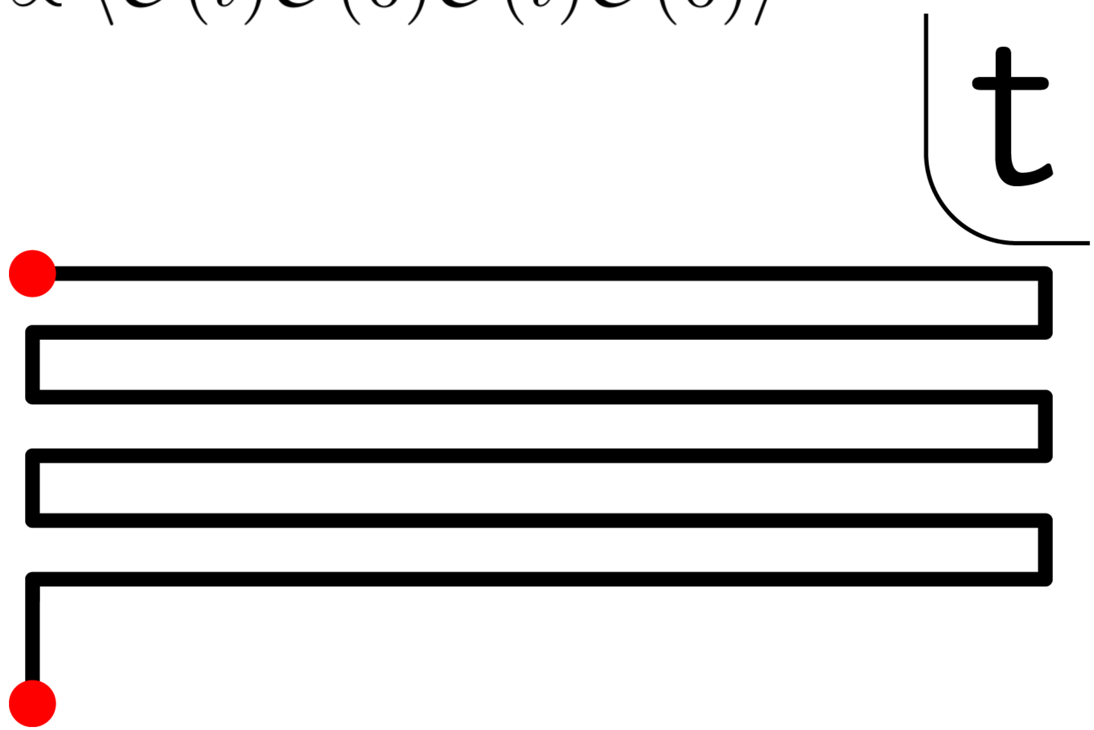
$$L^{\pm}_{nl} \propto (\tilde{\phi}_L + \tilde{\phi}_{\pm}) f(\beta)$$



Applications

- Many different (Schwinger-Keldysh) paths are of interest:
  - OTOC's related to Quantum Chaos [8]

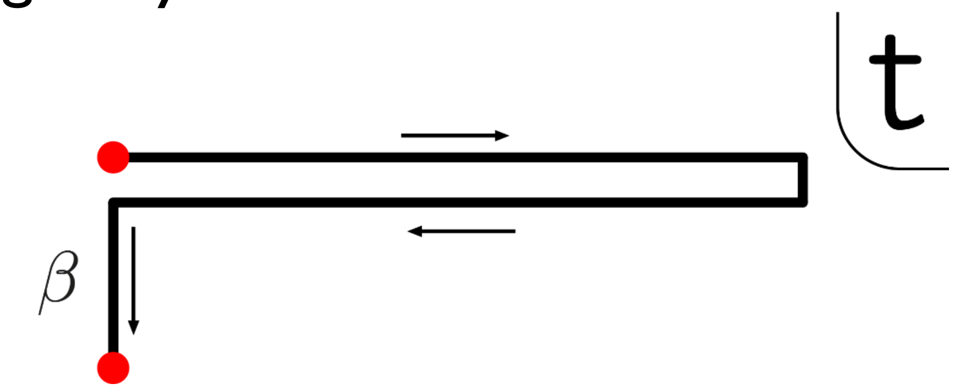
$$\langle [\mathcal{O}(t), \mathcal{O}(0)]^2 \rangle \propto \langle \mathcal{O}(t)\mathcal{O}(0)\mathcal{O}(t)\mathcal{O}(0) \rangle$$



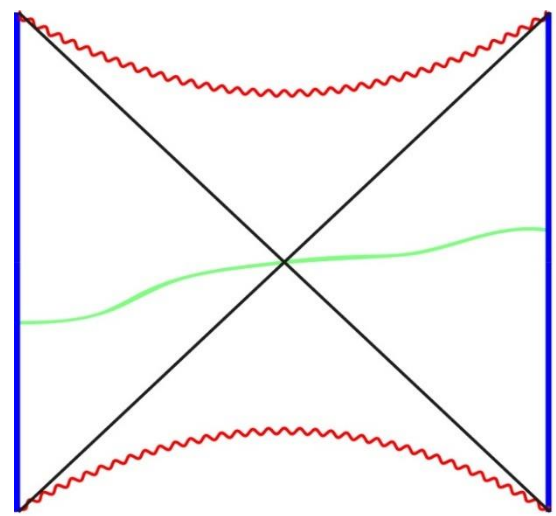
[8] Murata, et al [hep-th/1703.09435]

Applications

- Thermal Path: 2 possible gravity duals
  - Thermal AdS
  - AdS Black Hole [9]



- It is interesting to see how this holographic prescription behaves in bulk duals with horizons [10]
  - We are currently working on this!



[9] Maldacena [hep-th/0106112v6], [10] Son-Herzog [hep-th/0212072v4]

Outline

- Why AdS/CFT? (from the bottom up)
- Standard (Euclidean) Prescription: GKPW
  - Short Example: massless field
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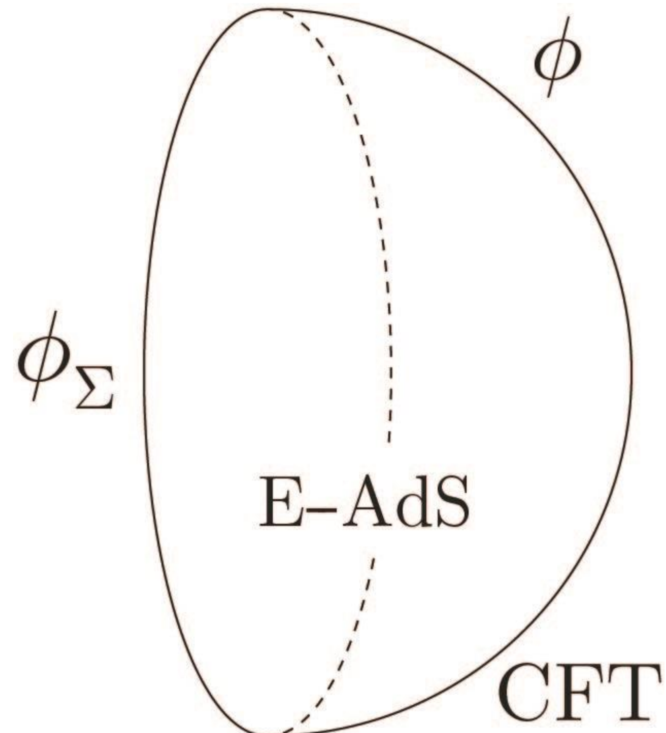
- Conclusions

States with Geometrical Dual

- Our excited states are [6,7]
  Exact for all N values
  - CFT

$$|\phi\rangle_{\mathrm{CFT}} \equiv e^{-\int \mathcal{O}\phi}|0\rangle$$



  - AdS (inspired in [4])

$$\Psi^{\phi}[\phi_\Sigma] = \int [\mathcal{D}\Phi]_{\phi;\phi_\Sigma}\; e^{-S_E[\Phi]}$$

  - Of course this extends to other fields of higher spin.

[4] Hartle & Hawking , Phys. Rev. D 28 (1983) 2960
[6] MBC, GS , PJM [hep-th/1512.07850], [7] MBC, GS , PJM [hep-th/1703.02384]

States with Geometrical Dual

- The states themselves:
  - Don't seem like natural states to study
    - Momentum basis
    - Non orthogonal
  - But they truly are the most natural states to reach
    - Coherent basis is the most "classical" basis: Heisenberg Principle
    - Topological equal to vacuum considered
    - They expand the dictionary and go continuously to standard examples:
      - ☐ Zero T° vacuum $\rightarrow$ Pure AdS
      - ☐ Finite T° vacuum $\rightarrow$ Thermal or BH AdS

$$|\phi\rangle_{\mathrm{CFT}} \equiv e^{-\int \mathcal{O}\phi}|0\rangle$$

Coherent States

- Consistency with BDHM prescription [11]

$$\hat{\mathcal{O}}(t,\Omega) \equiv \lim_{r\to\infty} r^{\Delta}\,\hat{\Phi}(t,r,\Omega) = \sum_k \hat{a}_k^\dagger F_k^*(t,\Omega) + \hat{a}_k F_k(t,\Omega)$$

reinforces this interpretation

$$|\phi\rangle_{\mathrm{CFT}} \propto e^{-\int \mathcal{O}\,\phi}|0\rangle \propto e^{\sum_k \lambda_k a_k^\dagger}|0\rangle$$

$$\lambda_k = -\int_{\partial_r \mathcal{M}} d\tau d\Omega\, F_k^*(-i\tau,\Omega)\,\phi(\tau,\Omega)$$

although not orthogonal, they form a complete basis.

[11] Banks, Douglas, Horowitz & Martinec [hep-th/9808016]
Harlow & Stanford [hep-th/1104.2621] A. Fitzpatrick & J. Kaplan [hep-th/1104.2597]

States with Geometrical Dual

- By "Geometrical" Dual we mean that they are described by a classical geometry, i.e. it can be reached by a saddle-point approximation.
  - Quantum states are generally not of this nature
  - The standard example of a state without geometrical dual is a sum of states which have a geometrical dual themselves [12]

$$|\phi_1\rangle_{\mathrm{CFT}} + |\phi_2\rangle_{\mathrm{CFT}}$$

[12] Van Raamsdonk, et. al. [hep-th/1709.10101]

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$$|\phi_1\rangle_{\mathrm{CFT}} + |\phi_2\rangle_{\mathrm{CFT}}$$

  - But…

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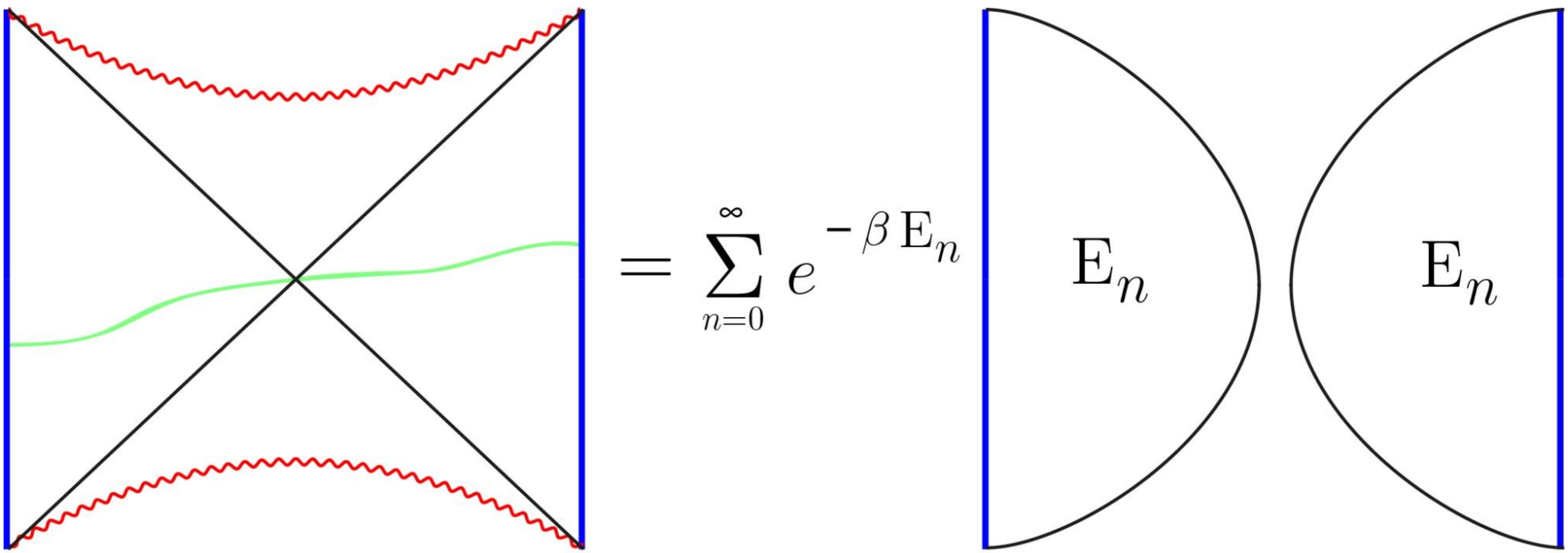
States with Geometrical Dual

- This has recently interested us [13] as a re-expansion of the TFD state as written in [14]



$$|\mathrm{TFD}\rangle = \sum_n e^{-\frac{\beta}{2}E_n} |E_n\rangle \otimes |E_n\rangle$$

[13] MBC, PJM [hep-th/1703.03483] , [14] Van Raamsdonk [hep-th/1005.3035]

States with Geometrical Dual

- This has recently interested us [13] as a re-expansion of the TFD state as written in [14]
  - It would be very interesting to have a more geometrical interpretation of the rhs, for no simple known geometry describe E-eigenstates.
  - We have proven that $|E_n\rangle$ do not belong to our basis except vacuum: coherent states do not commute with H
  - Our states provide a basis with good geometrical dual!

$$|\mathrm{TFD}\rangle = \sum_n e^{-\frac{\beta}{2}E_n} |E_n\rangle \otimes |E_n\rangle$$

[13] MBC, PJM [hep-th/1703.03483] , [14] Van Raamsdonk [hep-th/1005.3035]

States with Geometrical Dual

- In particular one could write E-eigenstates as

$$|E_n\rangle \sim \int d\phi \, C_\phi \, |\phi\rangle_{\mathrm{CFT}}$$

$$|\phi\rangle_{\mathrm{CFT}} \propto e^{\sum_k \lambda_k^- a_k^\dagger}|0\rangle$$

States with Geometrical Dual

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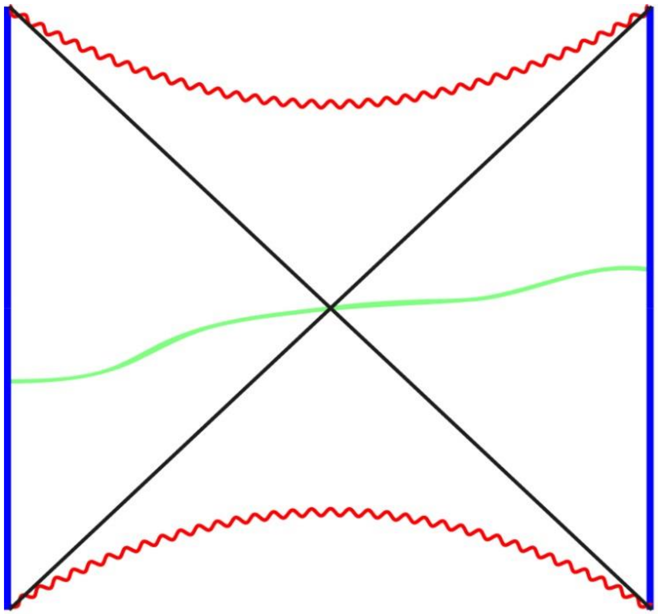
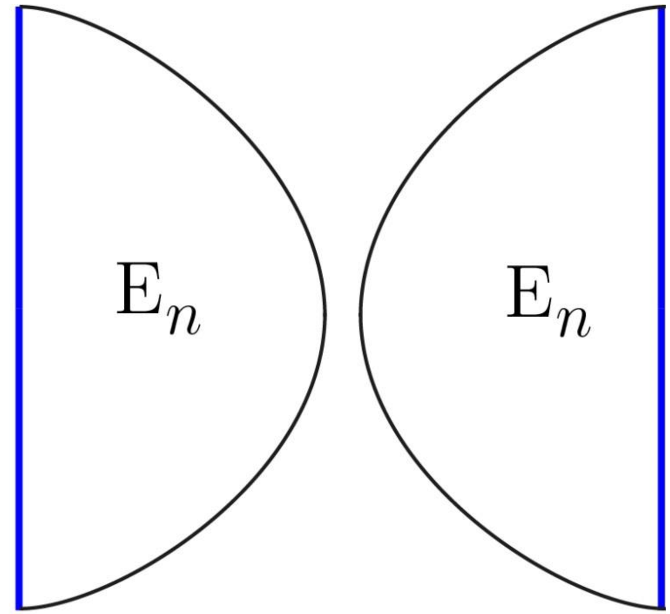


$$= \sum_{n=0}^{\infty} e^{-\beta \mathrm{E}_n}$$



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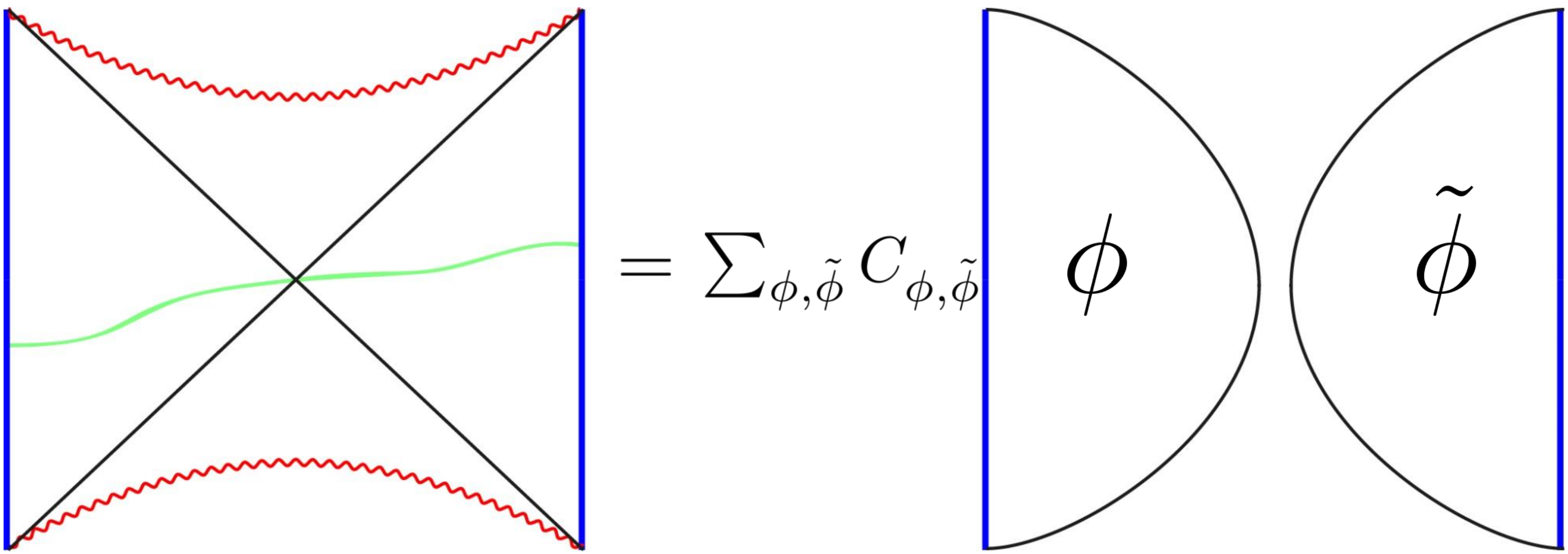
States with Geometrical Dual

- Where now each term in the expansion does have a "simple" geometric interpretation [13]

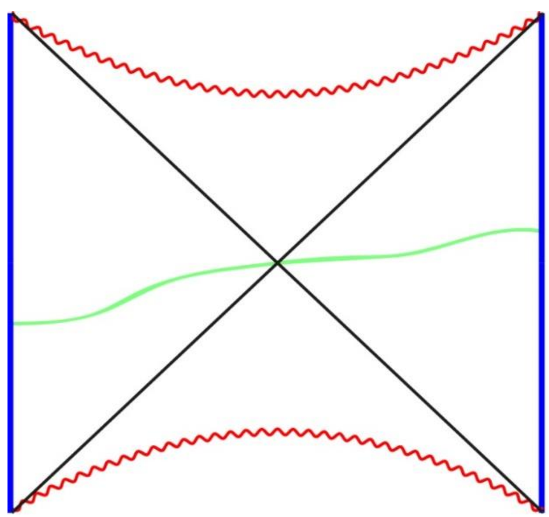


$$|\mathrm{TFD}\rangle = \sum_{\phi,\tilde{\phi}} C_{\phi,\tilde{\phi}} |\phi\rangle \otimes |\tilde{\phi}\rangle$$

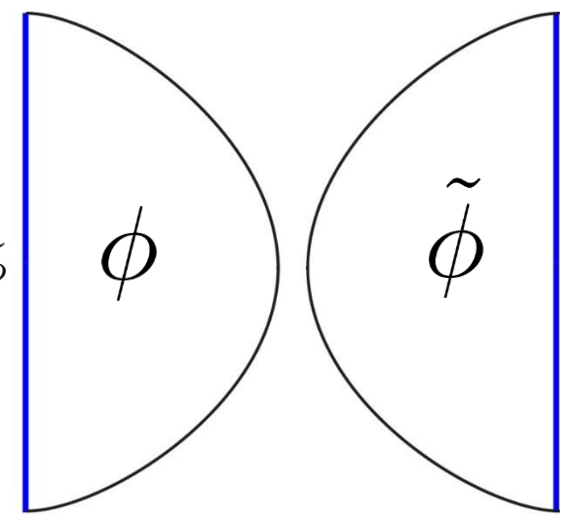
[13] MBC, PJM [hep-th/1703.03483]

States with Geometrical Dual

- This turns out to be quite interesting:
  - We found a sum of well known geometries which re-sum as topologically different classical geometry



$$= \sum_{\phi,\tilde{\phi}} C_{\phi,\tilde{\phi}}$$



$$|\mathrm{TFD}\rangle = \sum_{\phi,\tilde{\phi}} C_{\phi,\tilde{\phi}} |\phi\rangle \otimes |\tilde{\phi}\rangle$$

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Conclusions

- We understood GKPW as a tool
  - Bulk and Boundary
- We understood (and partially solved) real-time problems
  - Non Holographic time-like boundaries: N modes
  - Path: Causality, Temperature and Excited States
- We can now start filling the paths of our interest
  - Different fillings
- Studied the nature of the states themselves
  - Non orthogonal but natural in holography context
  - Basis of excited states

¡Muchas Gracias!

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  - [4] Hartle & Hawking , Phys. Rev. D 28 (1983) 2960
  - [5] Skenderis & van Rees [arXiv:0805.0150] + [arXiv:0812.2909]
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  - [7] MBC, GS , PJM [hep-th/1703.02384]
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  - [9] Maldacena [hep-th/0106112v6],
  - [10] Son-Herzog [hep-th/0212072v4]
  - [11] Banks, Douglas, Horowitz & Martinec [hep-th/9808016]
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That's all Folks!