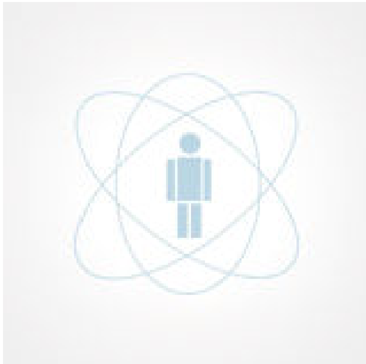
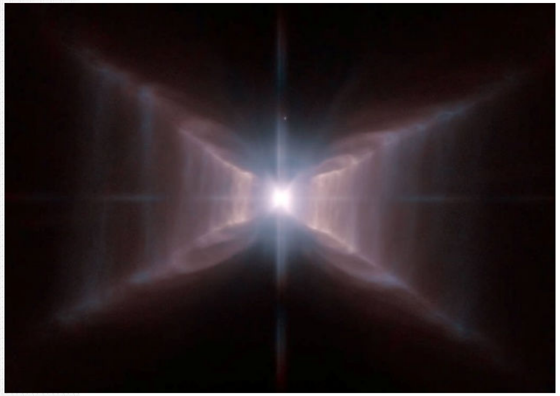
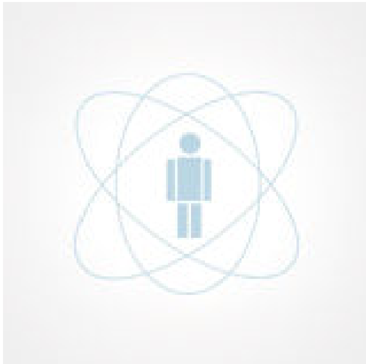
On Gauge Theories and Inhomogeneous Cosmologies

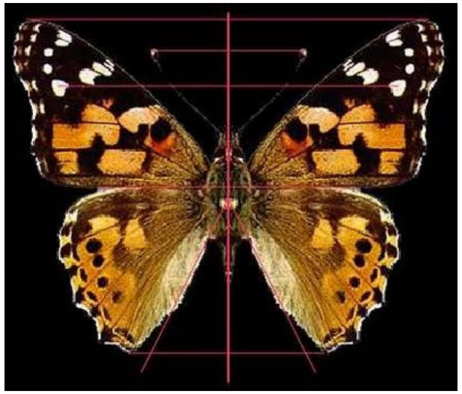


Nilo Serpa

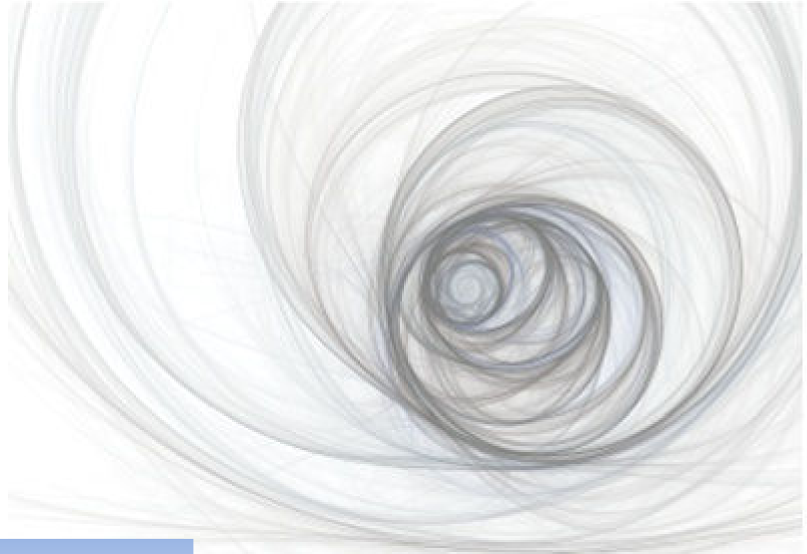
Why Symmetries?





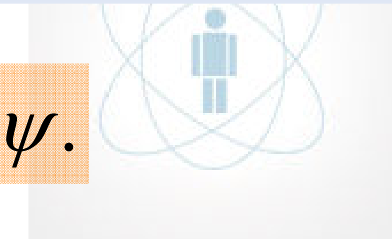


The Gauge Principle



Lagrangian is invariant for global gauge transformations:

$$\psi \rightarrow e^{i\theta}\psi.$$



It is required that this global property also holds locally. Thus, we get a gauge invariance which features a true dynamical principle:

$$\psi \rightarrow e^{i\theta(x)}\psi.$$

From the Downing to …

- The spacetime dependent length scales proposed by Weyl in 1918;

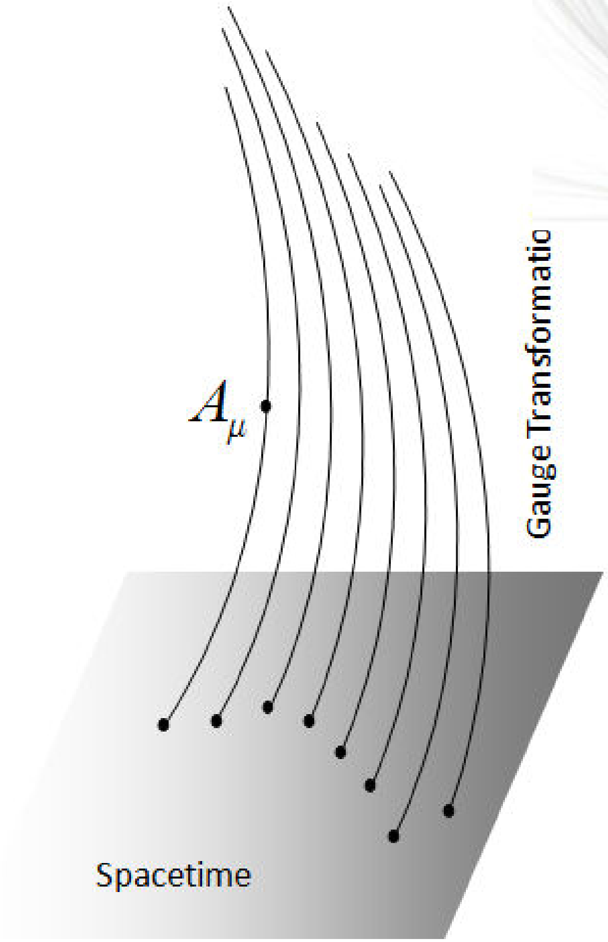
- Fock-Weyl global phases of wave functions with minimal coupling in 1927-1929;

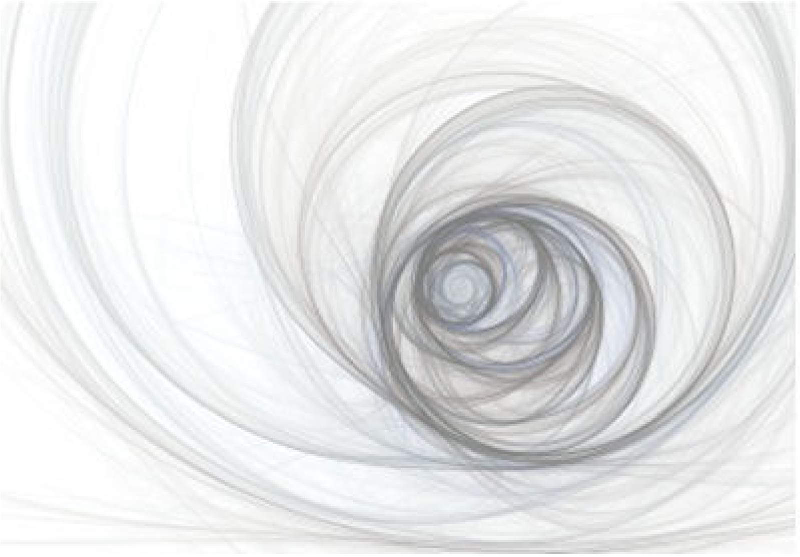
- Lyra's gauge approach of Riemannian geometry in 1951;

- Yang-Mills approach from Fock-Weyl model extended to non-Abelian groups in 1954;

- Faddeev and Popov/de Witt selfconsistent scheme for the quantization of massless Yang-Mills fields in 1967.

The Gauge Principle





The Implementation of a Gauge Theory in Classical Domain

Gauge Principle in Classical Field Approach

- Thermodynamics is a nice and profound theory;
- A beautiful thing in thermodynamics is its evolutionary approach in terms of the wear of the systems;
- Why not gauge principle in classical thermodynamics?

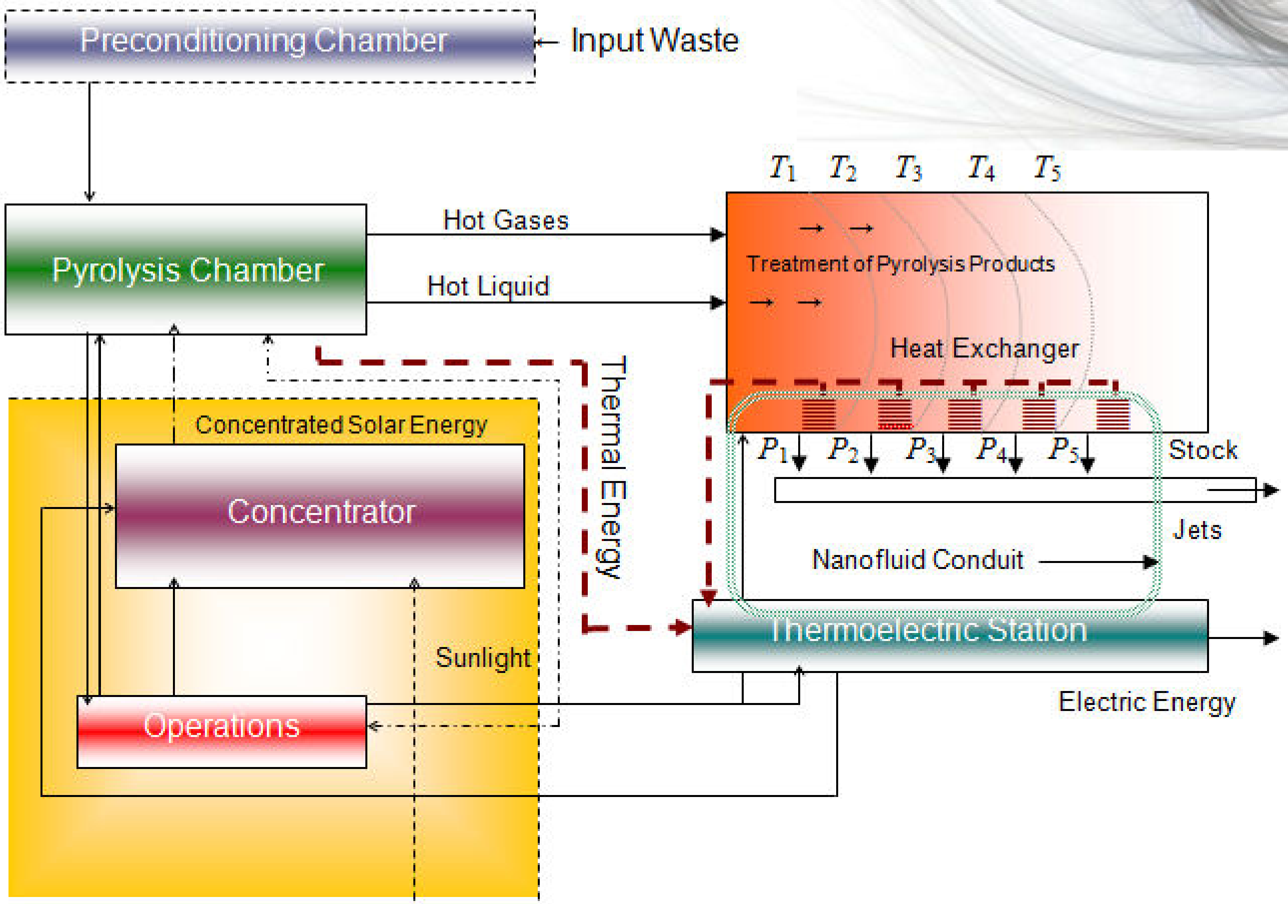
Fundamentals of the Proposal

- Efficiency as a direct result of controlled entropy
- Gauge field as a tool to evaluate entropy from interaction processes (condensed matter and thermal energy)
- Thermal energy representation by the construct of <<caloric field>>
- Thermodynamics as a gauge theory

Fundamentals of the Proposal

Entropy indicates incapacity to transform energy or to apply energy to transform matter.

The Plant in Process



Caloric Field (ξ) Equation

$$\partial_q \partial^q \xi + \left(1 - \gamma^2\right)\xi - 2\gamma^2 \xi \ln|\xi| = 0$$

$-2\gamma^2 \xi \ln|\xi|$: the entropy term

$\left(1 - \gamma^2\right)$: the luminothermic capacity

γ: the opacity

Total Lagrangian

$$\mathcal{L}_0' + \mathcal{L}_{gauge}' = \partial_q \xi^\dagger \partial^q \xi - |\xi|^2 + \gamma^2 |\xi|^2 \ln|\xi|^2 +$$

$$+ Q^2 A_q A^q \xi^\dagger \xi + iQ\left(A_q \xi^\dagger \partial^q \xi - A^q \partial_q \xi^\dagger \xi\right)$$

or

$$\mathscr{L} = \mathcal{L}_0' + \mathcal{L}_{gauge}' = \partial_q \xi^\dagger \partial^q \xi - |\xi|^2 + \gamma^2 |\xi|^2 \ln|\xi|^2 +$$

$$+ Q^2 A_q A^q \xi^\dagger \xi + iQ\left\{A_q \partial^q, A^q \partial_q\right\}_{\xi^\dagger, \xi}$$

Interesting Calculations

$$\boxed{\partial_q \left(\frac{\partial \mathscr{L}}{\partial_q \partial \xi^\dagger} \right) - \frac{\partial \mathscr{L}}{\partial \xi^\dagger} = 0}$$

$$\frac{\partial \mathscr{L}}{\partial \partial_q \xi^\dagger} = \left(\partial^q - iQA^q \right) \xi \text{ (is the covariant derivative)}$$

$$\partial_q \left(\partial^q - iQA^q \right) \xi \; = \partial_q \partial^q \xi - iQ \partial_q A^q \xi - iQA^q \partial_q \xi$$

Interesting Calculations

$$-\frac{\partial \mathscr{L}}{\partial \xi^\dagger} = -\left[-\xi + \gamma^2 \xi \ln|\xi|^2 + \gamma^2 \xi + Q^2 A_q A^q \xi + iQA_q \partial^q \xi\right]$$

$$\partial_q \left(\frac{\partial \mathscr{L}}{\partial \partial_q \xi^\dagger}\right) - \frac{\partial \mathscr{L}}{\partial \xi^\dagger} =$$

$$= \partial_q \partial^q \xi - iQ\partial_q A^q \xi - iQA^q \partial_q \xi + \xi - \gamma^2 \xi \ln|\xi|^2 -$$

$$-\gamma^2 \xi - Q^2 A_q A^q \xi - iQA_q \partial^q \xi =$$

$$= \boxed{\partial_q \partial^q \xi - 2iQA^q \partial_q \xi - iQ\partial_q A^q \xi - Q^2 A_q A^q \xi} + \left(1-\gamma^2\right)\xi - \gamma^2 \xi \ln|\xi|^2$$

$$\boxed{\mathscr{D}_q \mathscr{D}^q \xi}$$

New Field Equation

$$\mathscr{D}_q \mathscr{D}^q \xi + \left(1-\gamma^2\right)\xi = 2\gamma^2 \xi \ln\left|\xi\right|$$

Additional Equations

About the diffusion model: the combined equation Lane-Emden/Langmuir proposed for $n = 2$ (n is the polytropic index),

$$-\frac{1}{x^2}\left(2x\frac{d}{dx}y(x)+x^2\frac{d^2}{dx^2}y(x)\right)+3y(x)\frac{d^2}{dx^2}y(x)+\left(\frac{d}{dx}y(x)\right)^2+$$

$$+4y(x)\frac{d}{dx}y(x)-1=0;$$

this equation describes the evolution of the dimensionless caloric density y on a conductive cylindrical symmetry, keeping the density relatively stable over a longer radius measured by x; it serves to establish a curve which defines a useful level of energy balance.

Additional Equations

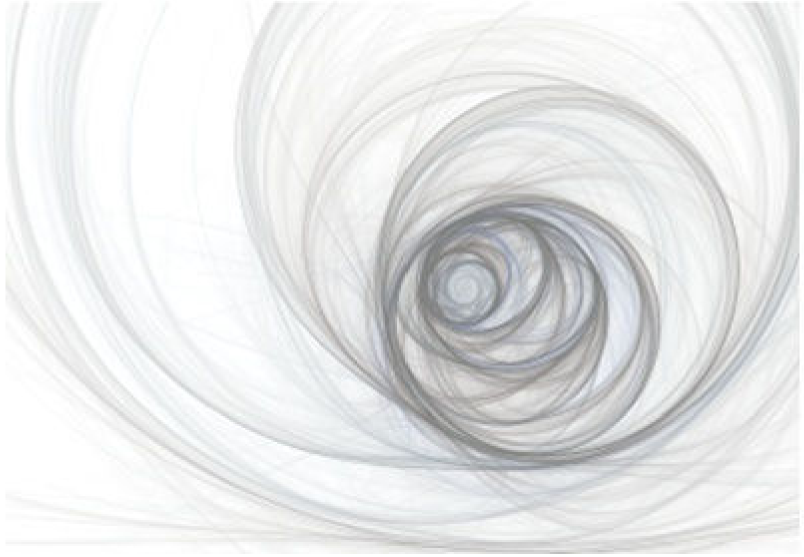
About the relationship between the thermodynamic variables *S*, P, T and V (entropy, pressure, temperature and volume): deduced equation in partial derivatives,

$$\tilde{c}\,\frac{\partial^2 V}{\partial S \partial P} = -\frac{1}{P}\frac{\partial T}{\partial P};$$

this second-order equation, with the parameter $\tilde{c} > 0$ corresponding to the polytropic index (n), describes the variation in volume due to entropy and pressure (assuming *P* parameterized with respect to *S*).

Additional Equations

These equations plus field equation are crucial to the construction of the production control algorithm to run up in the sector "Operations". These equations have in common the polytropic index, since it is always possible to give field ξ parameterized by n. It is also important to note that all we can do is to work on systems with a good approximation for $n = 2$.

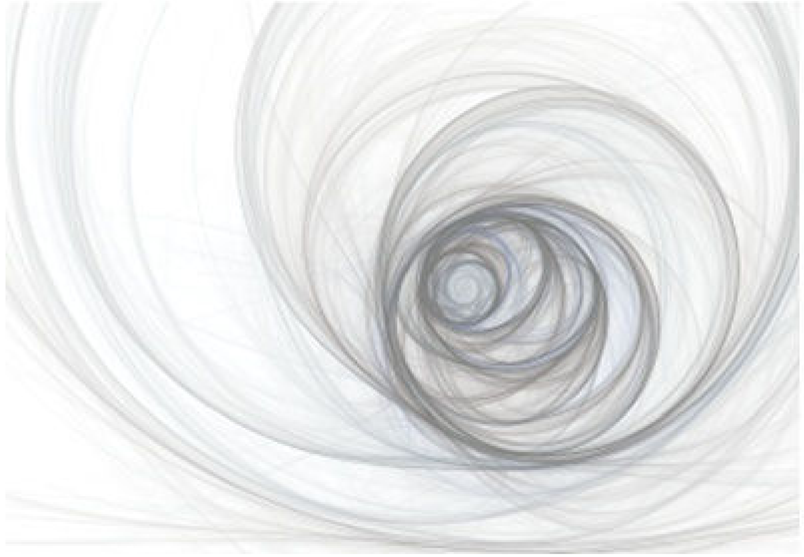


Quantum Gravity and Lyra Geometry

A New Approach on Quantum Gravity

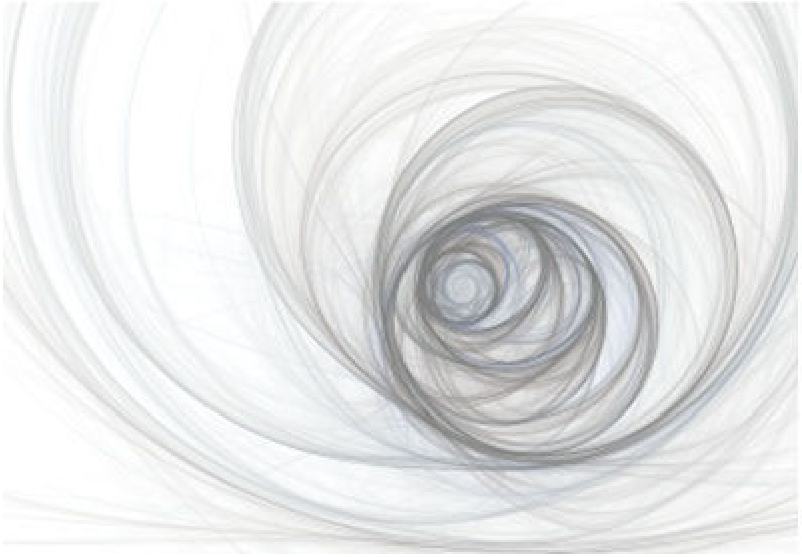
1. Attempts to reconciles QM with GR
2. Quantum spacetime ⟺ quantum Riemannian geometry
3. Spacetime is a granular structure (singularity functions)
4. 4-dimensional
5. *G-closure* (bubble of compressed spacetime)

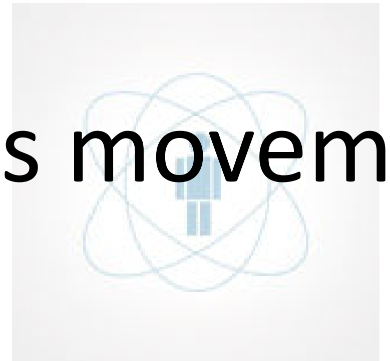
A Crucial Constraint



Perturbative quantum gravity is inconsistent on quantum level due to the infinite number of non-renormalizable ultraviolet divergences!!

Rethinking Space





What is movement??

What is Lyra Geometry?

Lyra’s geometry is a generalization of Riemannian geometry—initially taken in a manifold not endowed of a metric—with a positive definite function, the scalar field $\chi(x^k)$ for scale changes, in which the reference system is defined not only by the coordinates but also by including that scalar field, that is, the gauge function $\chi(x^k)$

What is Lyra Geometry?

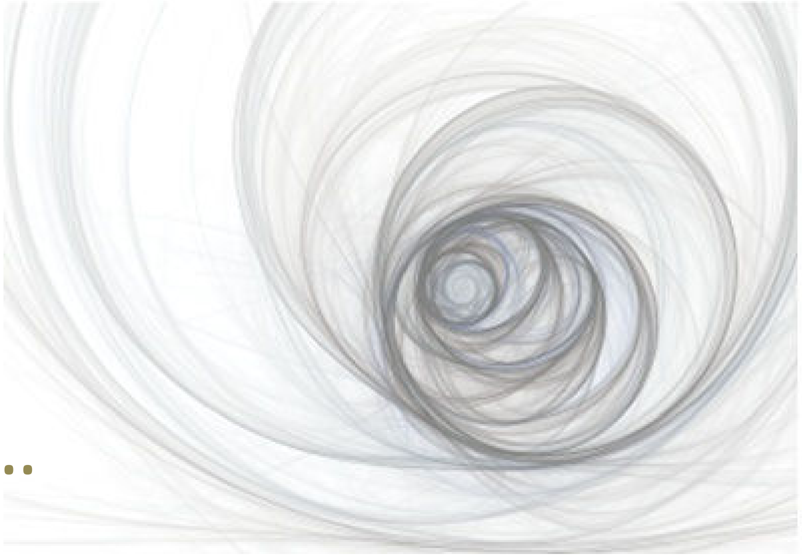
The connection:

$$ {}^{\dagger}\Gamma^{\alpha}_{\mu\sigma} = \chi^{-1}\Gamma^{\alpha}_{\mu\sigma} - \frac{1}{2}\left(\delta^{\alpha}_{\mu}\phi_{\sigma} + \delta^{\alpha}_{\sigma}\phi_{\mu} - g_{\mu\sigma}\phi^{\alpha}\right) $$

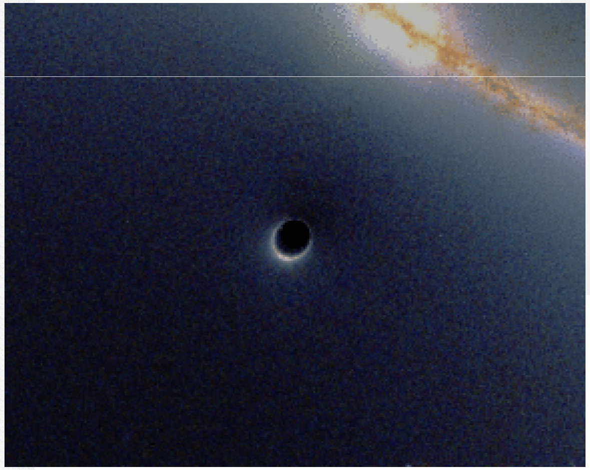
The metric:

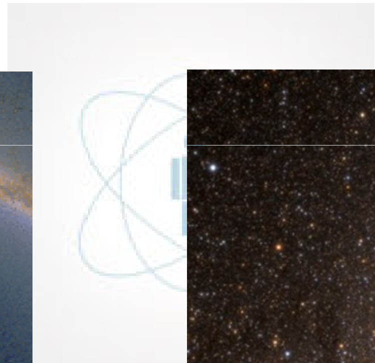
$$ ds^{2} = \chi^{2} g_{\mu\sigma} dx^{\mu} dx^{\sigma} $$

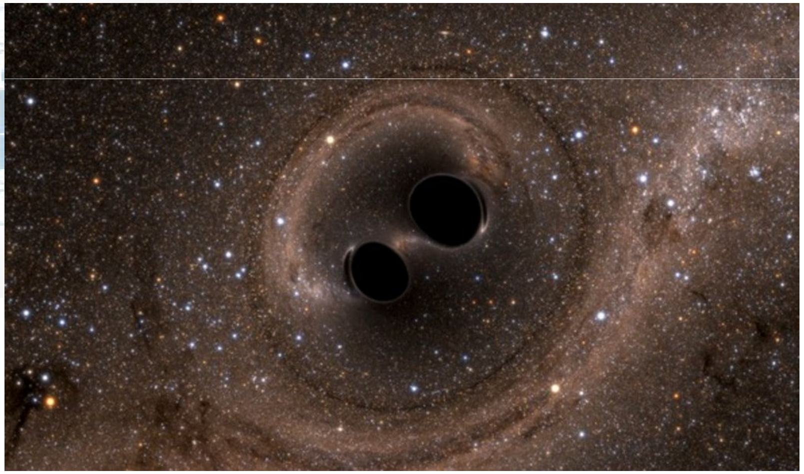
Motivations



Black Holes (singularities)…



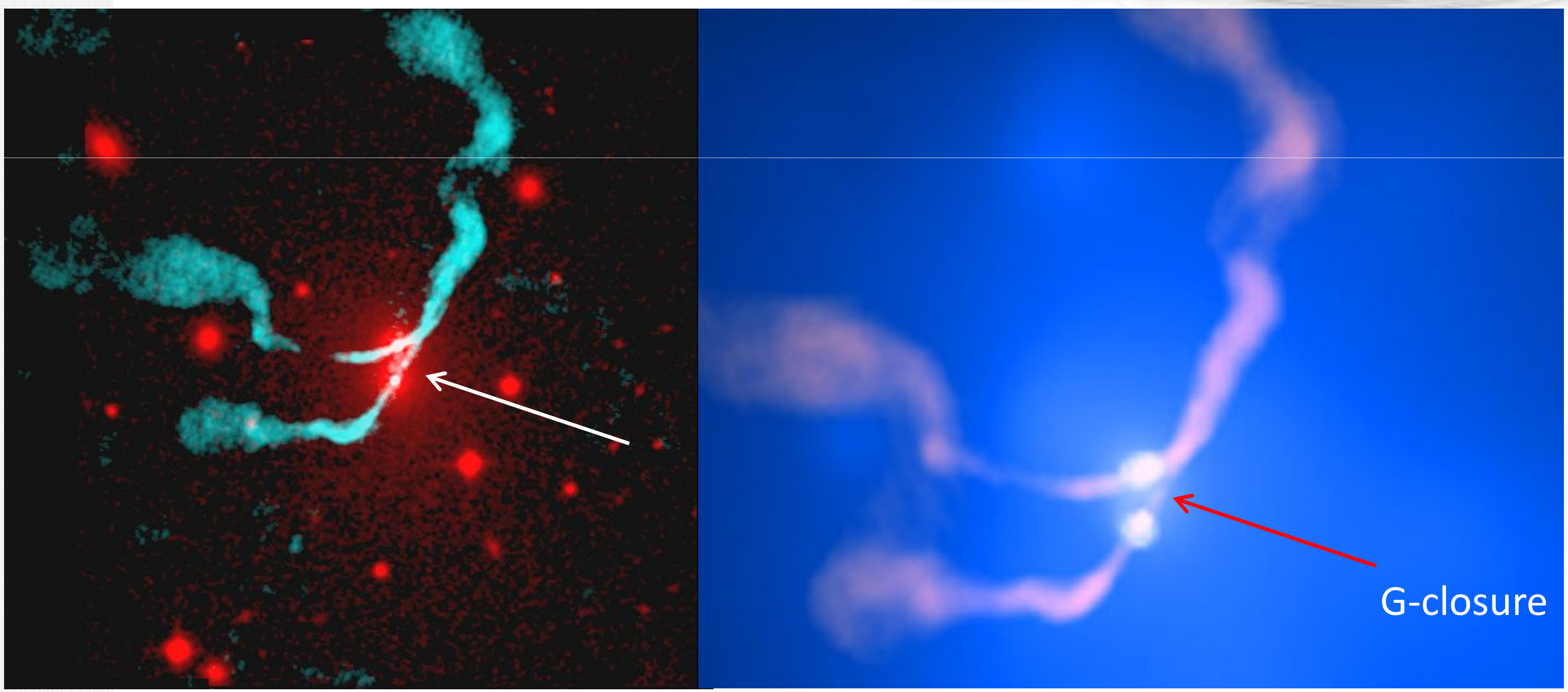




…as bridges between GR and QM

Motivations

The model (phenomenological)



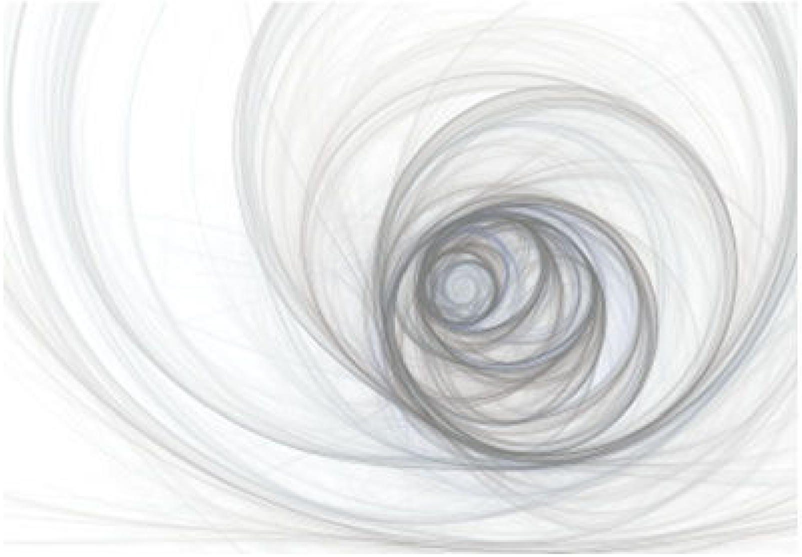
Co-orbiting supermassive black holes powering the giant radio source 3C75

Motivations

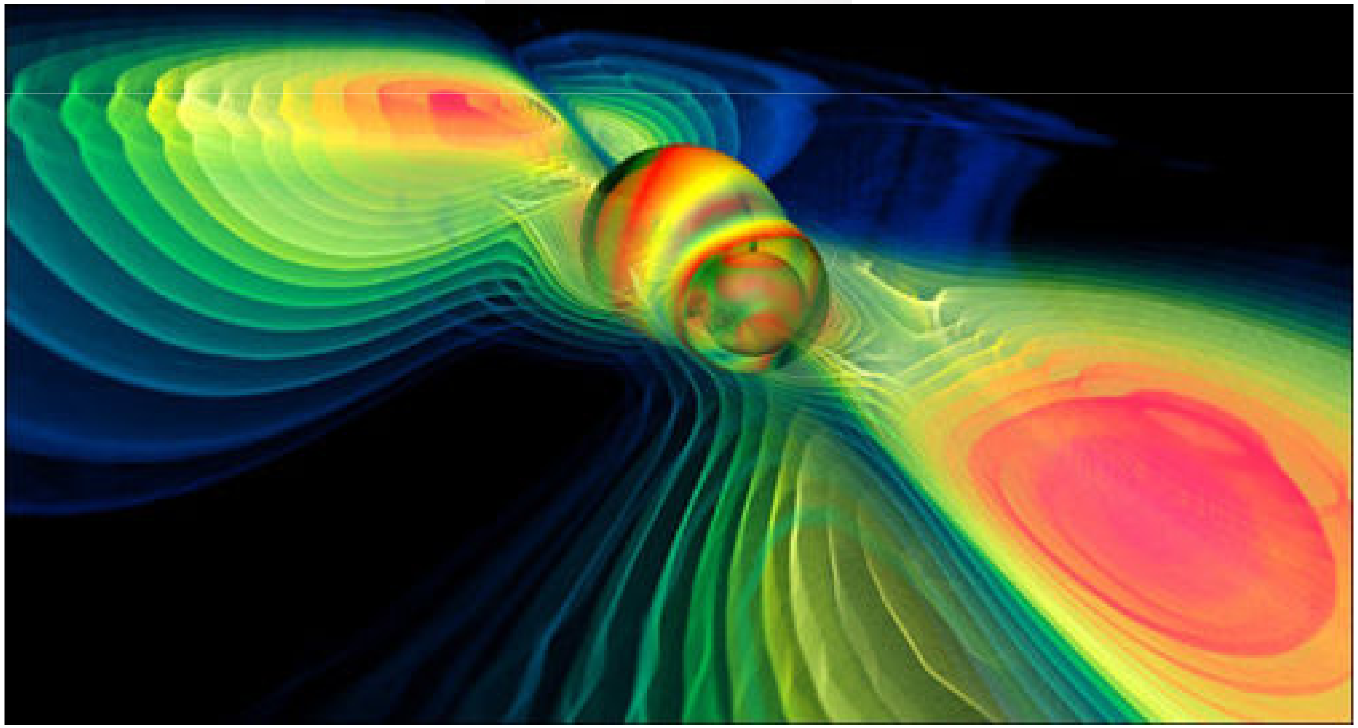
G-closure

1. Gravitational attraction between the bodies tends to squeeze or compress them as they pull one another toward their center points; so, the strong attraction between massive bodies leads to a compression of space contained among them.
2. <u>Proposition</u> : *under strong gravitational compression, time dilates and space ceases to be a degree of freedom in the direction of compression.*

Motivations



G-closure



Singularity Functions

$$\langle x-x_0\rangle^n = \begin{cases}(x-x_0)^n, x > x_0 \\ 0, x \le x_0\end{cases}$$

$$\int \langle x-x_0\rangle^n dX = \frac{1}{n+1}\langle x-x_0\rangle^{n+1} + C;$$

$$\frac{d\langle x-x_0\rangle^n}{dX} = n\langle x-x_0\rangle^{n-1}.$$

1. they describe phenomena based on geometry (very interesting for our proposal),
2. they capture any changes in time evolution,
3. they can include infinitely many spacetime segments in different states,
4. they can be rescaled by any factor,
5. they are independent of units, and
6. they are continuous, differentiable and integrable like common functions.

The measure operation on the spacetime invariant element

$$ds^2 = g_{\mu\nu} d\left\langle x_\mu - \varepsilon_\mu \right\rangle d\left\langle x_\nu - \varepsilon_\nu \right\rangle,$$

$\varepsilon_\mu, \varepsilon_\nu$ are fixed distances from a point on the spherical boundary of the G-closure

Fundamental correlation function:

$$\langle 0 | g_{\mu\nu} d\left\langle x - \varepsilon \right\rangle_\mu d\left\langle x - \varepsilon \right\rangle_\eta |0\rangle = -d\left\langle t - \varepsilon \right\rangle_0^2 +$$

$$+R^2_{\langle t-\varepsilon\rangle_0} d\left\langle \vec{x} - \vec{\varepsilon} \right\rangle d\left\langle \vec{x} - \vec{\varepsilon} \right\rangle,$$

The measure operation on the spacetime invariant element

The quantum spacetime was matched with quantum Riemannian metric in order to obtain the correlation function, that restricted to time paths is

$$\langle 0 | \, g_{\mu\sigma} d \left\langle x - \varepsilon \right\rangle_{\mu} d \left\langle x - \varepsilon \right\rangle_{\sigma} \left| 0 \right\rangle = -d \left\langle x - \varepsilon \right\rangle_{0}^{2}$$

The measure operation on the spacetime invariant element

The new geodesic equation in Lyra geometry

$$\frac{d^2\langle x-\varepsilon\rangle_\alpha}{d\tau^2}+\Gamma^{\alpha}_{\mu\sigma}\frac{d\langle x-\varepsilon\rangle_\mu}{d\tau}\frac{d\langle x-\varepsilon\rangle_\sigma}{d\tau}-$$

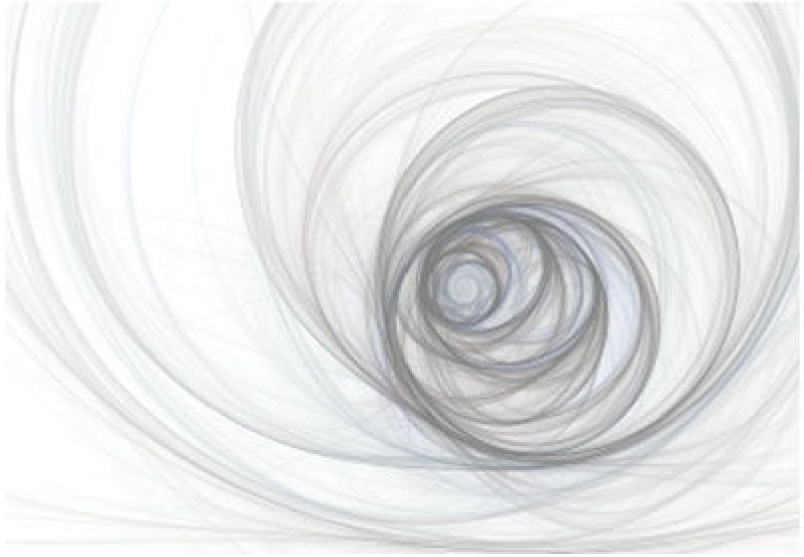
$$-\frac{\chi}{2}\left(\delta^{\alpha}_{\mu}\phi_{\sigma}+\delta^{\alpha}_{\sigma}\phi_{\mu}-g_{\mu\sigma}\phi^{\alpha}\right)\frac{d\langle x-\varepsilon\rangle_\mu}{d\tau}\frac{d\langle x-\varepsilon\rangle_\sigma}{d\tau}=0$$

The measure operation on the spacetime invariant element

The timelike geodesic in Lyra geometry

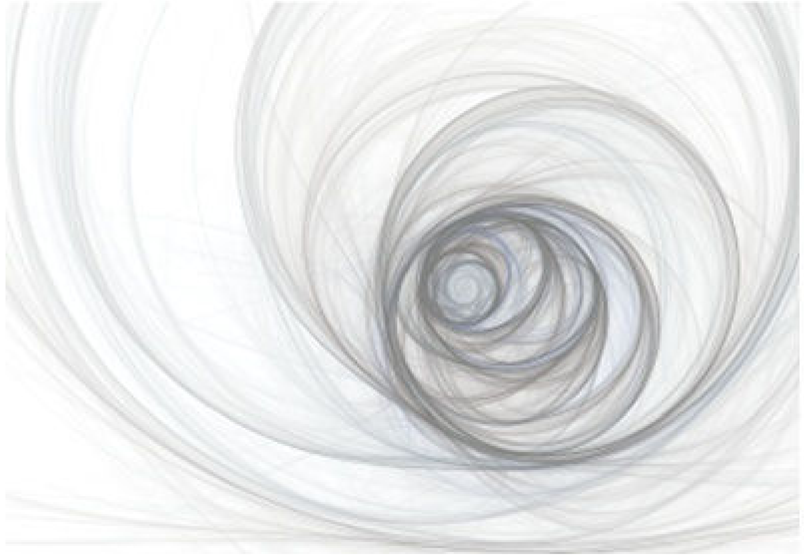
$$\frac{d^2\langle x-\varepsilon\rangle_\alpha}{d\tau^2}+\Gamma^\alpha_{00}\frac{d\langle x-\varepsilon\rangle_0}{d\tau}\frac{d\langle x-\varepsilon\rangle_0}{d\tau}-$$

$$-\frac{\chi}{2}\left(\delta^\alpha_0\phi_0+\delta^\alpha_0\phi_0\right)\frac{d\langle x-\varepsilon\rangle_0}{d\tau}\frac{d\langle x-\varepsilon\rangle_0}{d\tau}=0$$



Paleogravity: A Classical Supersymmetric Model

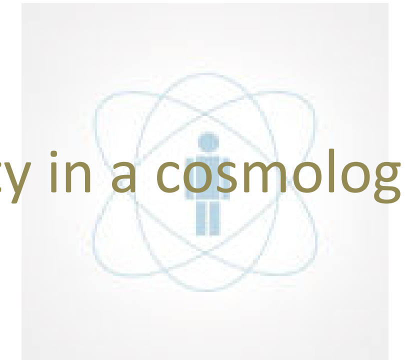
What is Paleogravity?



Paleogravity is “ancient gravity”, or, in a precise way, effects of massive systems accumulated all along in time. Based on this concept, any local gravitational effect is influenced by its own history. “Gravity takes a long time to manifest its strength”.

Motivations

- To study only symmetry relations between gravitons and gravitinos under Lagrangian formalism, abstracting all other features

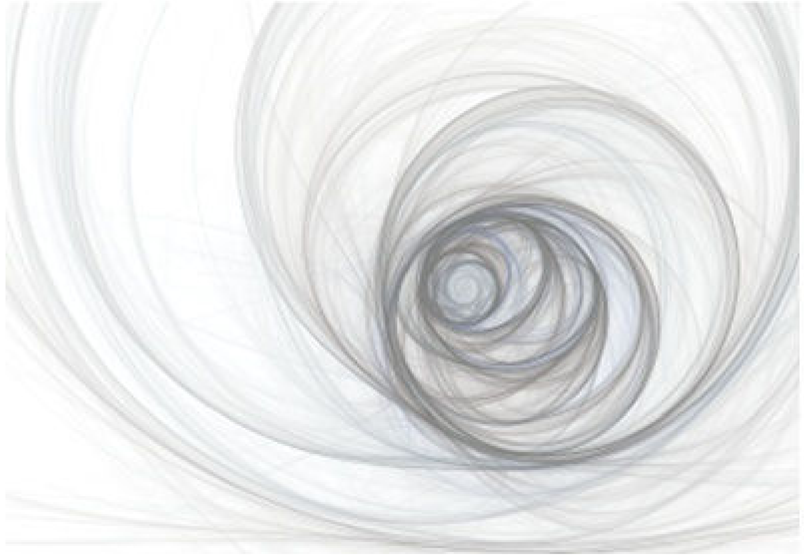


- To consider supergravity in a cosmological pre-quantum field approach of gravity

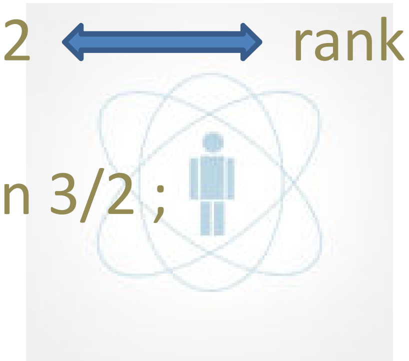
- To analyze geometric backgrounds of the symmetries

- To explain some features to be expected from gravitinos, if they exist

Generalities



Graviton (boson): spin 2 $\longleftrightarrow$ rank 2 $g_{\mu\nu}$;



Gravitino (fermion): spin 3/2 ;

Both not yet detected.

Generalities

Light gravitinos (10^{-6} eV to few keV range);

They provide additional supernova cooling mechanisms (light gravitinos emission);

States modeled by light gravitinos would appear to the observer as missing energy since gravitinos cannot be detected.

The Problem of Non-Locality

- Higher derivative Lagrangian models lead to immense difficulties and huge complications!!

- Non-locality evokes a real effort of philosophical reflection on macro and micro scales of the universe

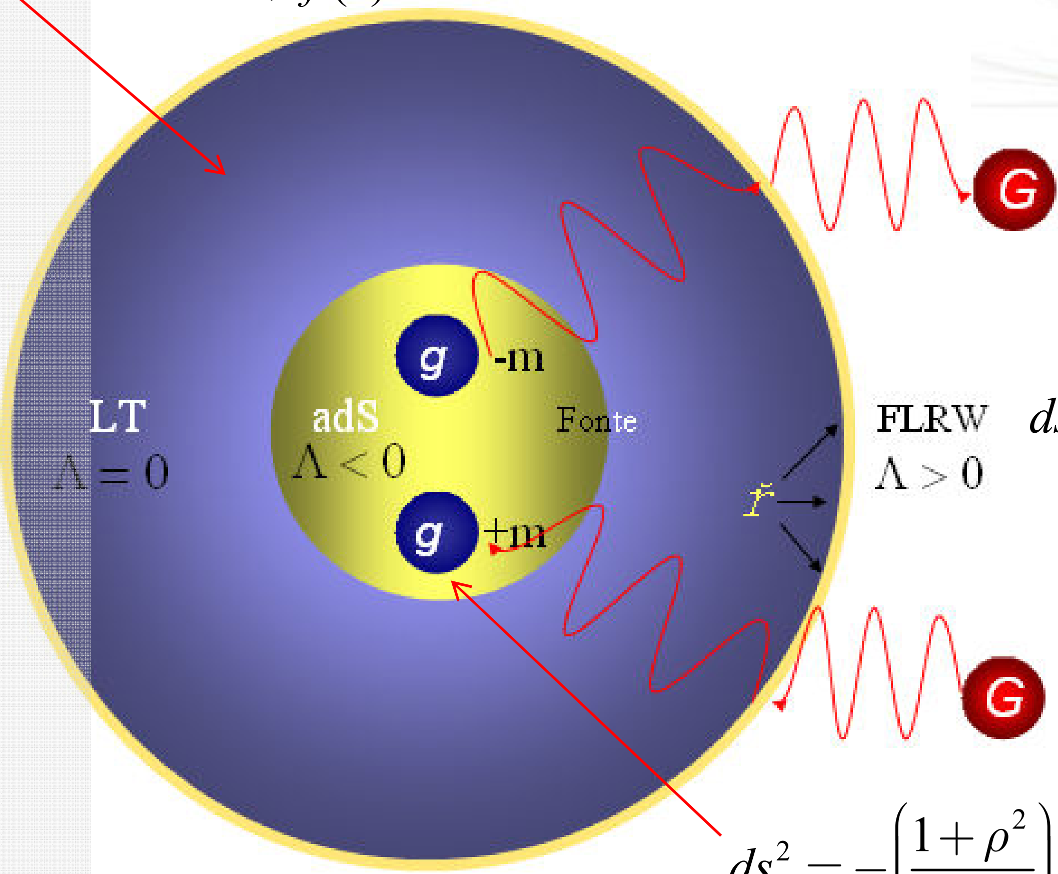
- Non-local inheritance factor as a device to include far-off interferences

- The myth of Lagrangians with no integrals

- Gravity as the cosmic agent of the cumulative results from the evolution of galactic clusters, superclusters and so on

The LT Bubble: The Swiss Cheese Model

$$ds^2 = -dt^2 + \frac{R'^2}{1+f(r)} dr^2 + R^2 d\widehat{\Omega}^2$$



$$ds^2 = -dt^2 + a^2 \left(\frac{dr^2}{1-kr^2} + r^2 d\widehat{\Omega}^2 \right)$$

$$ds^2 = -\left(\frac{1+\rho^2}{1-\rho^2} \right) dt^2 + \frac{4}{\left(1-\rho^2\right)^2} \left(d\rho^2 + \rho^2 d\widehat{\Omega}^2 \right)$$

The group $S(\gamma_\eta)$

$$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix},\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix};$$

$$\begin{pmatrix}0 & i\\ -i & 0\end{pmatrix},\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix},\begin{pmatrix}i & 0\\ 0 & i\end{pmatrix},\begin{pmatrix}-i & 0\\ 0 & -i\end{pmatrix};$$

$$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix},\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix},\begin{pmatrix}0 & i\\ i & 0\end{pmatrix},\begin{pmatrix}0 & -i\\ -i & 0\end{pmatrix};$$

$$\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix},\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix},\begin{pmatrix}i & 0\\ 0 & -i\end{pmatrix},\begin{pmatrix}-i & 0\\ 0 & i\end{pmatrix}.$$

The group $\bigcup$ of gravitors with elements

$$(\pm\mathbb{1}_2,\gamma_\eta),\ (\pm\mathbb{i}_2,\gamma_\eta)$$

is formed via direct product of their two matrix components; examples:

$$G_\mu=\left[\begin{pmatrix}\mathbb{1}_2\\ \sigma_1\end{pmatrix},\begin{pmatrix}\mathbb{1}_2\\ i\sigma_2\end{pmatrix},\begin{pmatrix}\mathbb{1}_2\\ i\sigma_3\end{pmatrix},\begin{pmatrix}\mathbb{1}_2\\ \mathbb{i}_2\end{pmatrix}\right],g_\mu=\left[\begin{pmatrix}\mathbb{i}_2\\ i\sigma_1\end{pmatrix},\begin{pmatrix}\mathbb{i}_2\\ -\sigma_2\end{pmatrix},\begin{pmatrix}\mathbb{i}_2\\ -\sigma_3\end{pmatrix},\begin{pmatrix}\mathbb{i}_2\\ -\mathbb{1}_2\end{pmatrix}\right];$$

$$\begin{pmatrix}\mathbb{i}_2\\ \sigma_3\end{pmatrix}^2=\begin{pmatrix}\mathbb{i}_2\\ \sigma_3\end{pmatrix}\circledast\begin{pmatrix}\mathbb{i}_2\\ \sigma_3\end{pmatrix}=\begin{pmatrix}\mathbb{i}_2\times\mathbb{i}_2\\ \sigma_3\times\sigma_3\end{pmatrix}.$$

The Clifford algebra adS $\mathbb{C}_{3,2}^{(\gamma_\mu)}$

$$\left(\gamma^0\right)^2 = \mathbb{1}, \gamma^0 = \gamma^{0\dagger}\text{(hermitian)},$$

$$\left(\gamma^a\right)^2 = -\mathbb{1}, (a = 1,2,3), \gamma^a = \gamma^{a\dagger}\text{(a-hermitian)},$$

$$\left(\gamma^4\right)^2 = \mathbb{1}, \gamma^4 = \gamma^{4\dagger}\text{(hermitian)},$$

$$\gamma^a\gamma^b = -\gamma^b\gamma^a, a \neq b.$$

Those gravitors were related by the action of the adS algebra $\mathbb{C}_{3,2}^{(\gamma_\mu)}$ according to

$$\begin{pmatrix} 0 & \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} \\ \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} & 0 \end{pmatrix} \begin{pmatrix} \mathbb{1}_2 \\ i\sigma_3 \end{pmatrix} = \begin{pmatrix} i\mathbf{i}_2 \\ -\sigma_3 \end{pmatrix},$$

and so on, that features Wick-rotations.

The phenomenological Lagrangian of interaction with non-local terms:

$$\mathcal{L}_{\text{int}} = -4\left(\partial_\tau \langle G| \int |g\rangle d\tau + \partial_\tau |g\rangle \int \langle G| d\tau\right) - \gamma\langle G|^2 - 2\langle G||g\rangle - \gamma^- |g\rangle^2$$

Euler-Lagrange

$$\langle G| = \gamma^- |g\rangle \qquad\qquad |g\rangle = \gamma \langle G|$$

Noether's theorem applied to the fields gives

$$\left[\!\left[\langle G|, |g\rangle \right]\!\right] = \partial_\tau \langle G| \int |g\rangle \, d\tau - \partial_\tau |g\rangle \int \langle G| \, d\tau = 0$$

In short:

- gravitinos live in adS regions (instable);

- If supersymmetry is broken, that is, gravitinos come up from adS region, they get mass (coupling to <<border gauge>> field), but they become gravitons as soon as they cross the junction between the two universes;

- gravitino and graviton are Wick-rotations of one another.

The phenomenological Lagrangian density exhibiting a time-integral and a <<border gauge>>:

$$\mathcal{L} = M^2 \,|\, g\rangle\langle\breve{G}\rangle\partial_\tau\langle\breve{G}\rangle\int |\, g\rangle d\tau + 1/3M^2\langle\breve{G}\rangle^3 + i\breve{r}\partial_\tau\breve{r}$$

gravitino field

border gauge field inhomogeneity

gravitino non-local factor

self-interaction term

junction term

border gauge field

The phenomenological Lagrangian density exhibiting a time-integral and a <<border gauge>>:

Euler-Lagrange

$$\left(\langle\breve{G}\rangle, \partial_\tau \langle\breve{G}\rangle\right)$$

$$\mathcal{L} = M^2 |g\rangle \langle\breve{G}\rangle \partial_\tau \langle\breve{G}\rangle \int |g\rangle d\tau + 1/3M^2 \langle\breve{G}\rangle^3 + i\breve{r}\partial_\tau \breve{r}$$

$$\langle\breve{G}\rangle = |g\rangle^2 + \partial_\tau |g\rangle \int |g\rangle d\tau$$

$$\boxed{\mathrm{A}' = \mathrm{A}^2 + \partial_\tau \mathrm{A} \int \mathrm{A}\, d\tau}$$

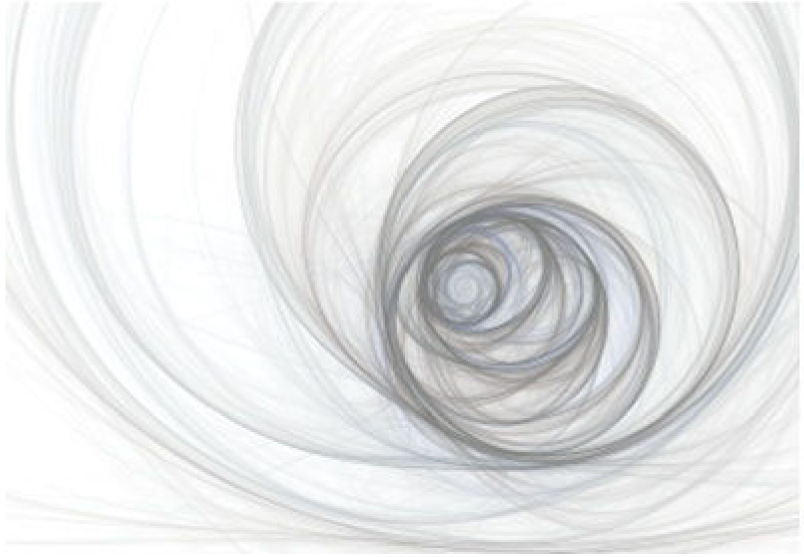
Moral of the story: field $\langle \breve{G} \rangle$ connects the gravitino field with its own "history" acting on its inhomogeneity ($\langle \breve{G} \rangle = | g \rangle^2 + \partial_\tau | g \rangle \int | g \rangle d\tau$).

In this sense, it is interesting to put this connection in the friendly form of a transformation as

$$\boxed{\mathrm{A}' = \mathrm{A}^2 + \partial_\tau \mathrm{A} \int \mathrm{A} d\tau}$$

Inhomogeneous Cosmologies

The pertinence



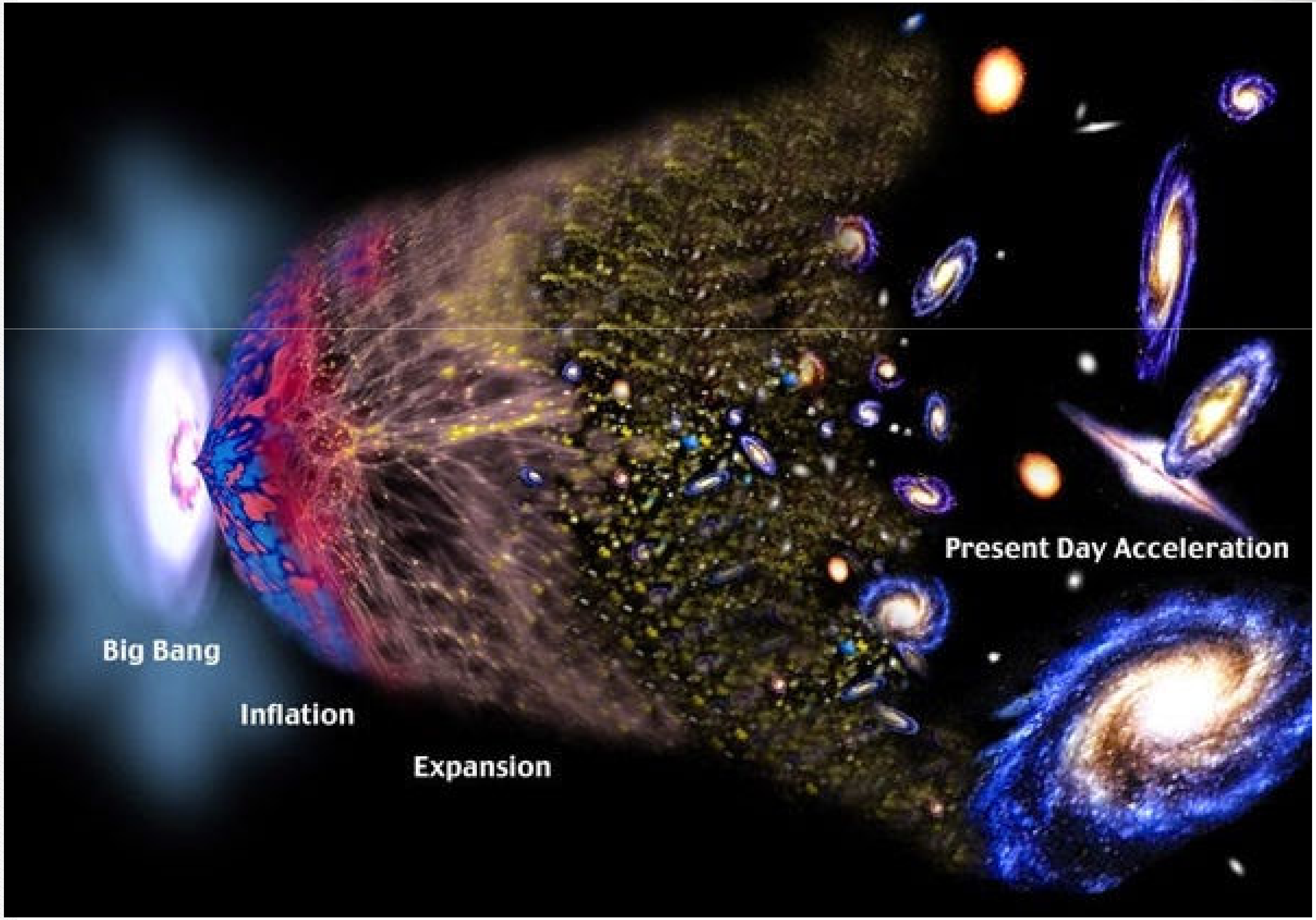
Physics

Astronomy

Cosmology

Astrophysical Cosmology

The usual conception

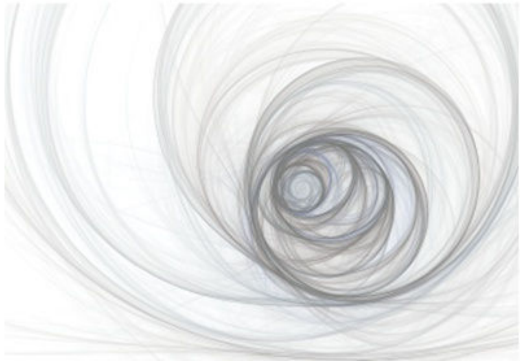


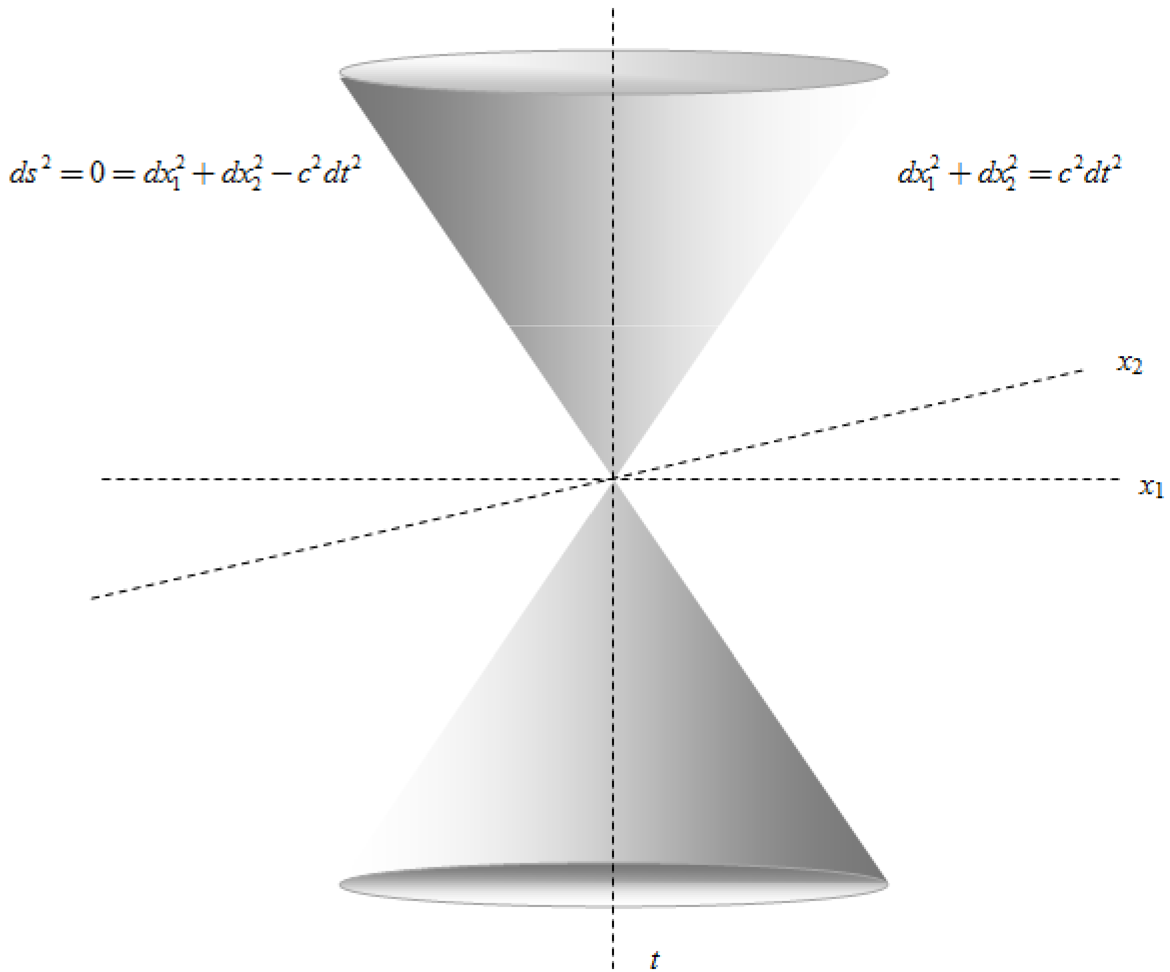
Fundamentals

- Spacetime is 4-dimensional, being homogeneous at *Gpc* scale, isotropic and in uniform expansion (ΛCDM)

- Light travels through null geodesics ($ds^2 = 0$)

Fundamentals





Fundamentals

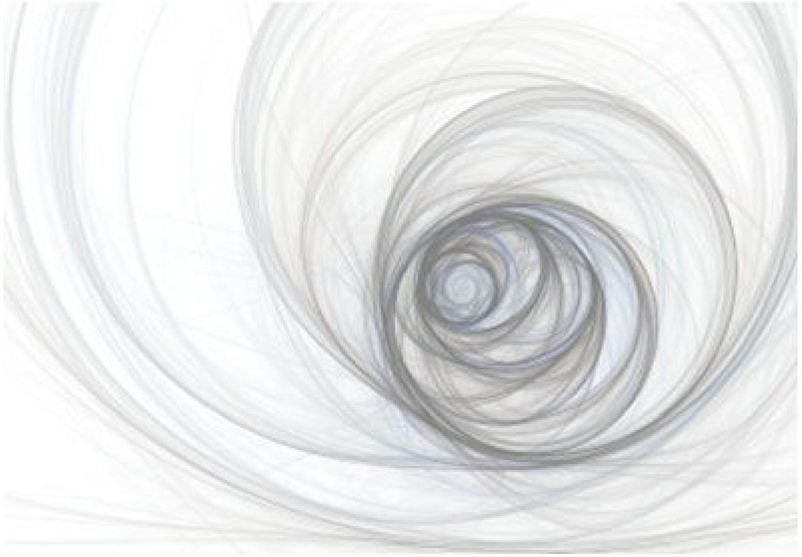
- *Ockham's razor* – if we deal observationally and satisfactorily with simple symmetric models, we do not need any complicated model.
- Isotropy is tested only in relation to one point (it does not prove homogeneity).
- We always look at the past.

Inhomogeneous cosmologies

- LTB Cosmology – inhomogeneous density (*dust shells*)

- Stephani Cosmology – inhomogeneous pressure (*gradient-of-pressure shells*)

LTB Modeling



- 3 arbitrary functions:

$M(r)$ — Energy || Baryonic gravitational mass (*active gravitational mass*)

$\beta(r)$ — Constant → simultaneous *Big-Bang*

$f(r)$ — Curvature

The LTB metric

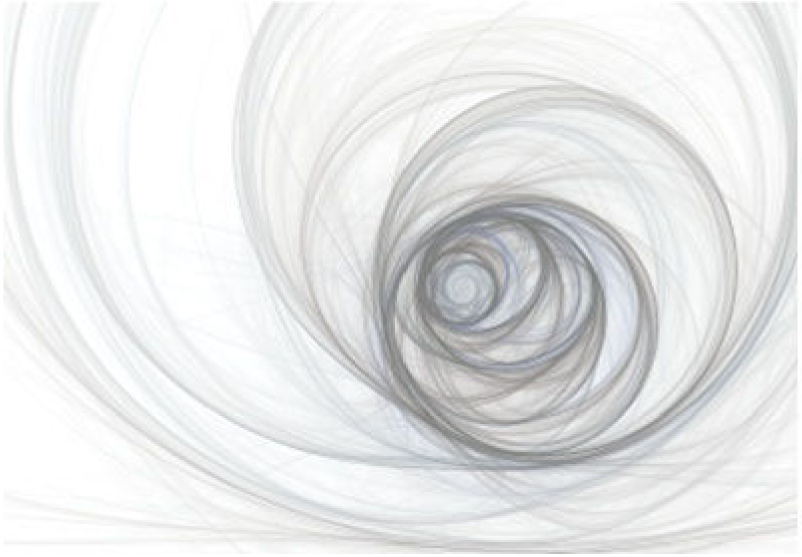
- General class of metrics

$$ds^2 = -dt^2 + b^2(r,t)\,dr^2 + R(r,t)^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

A family of solutions that describe inhomogeneous collapse of dust:

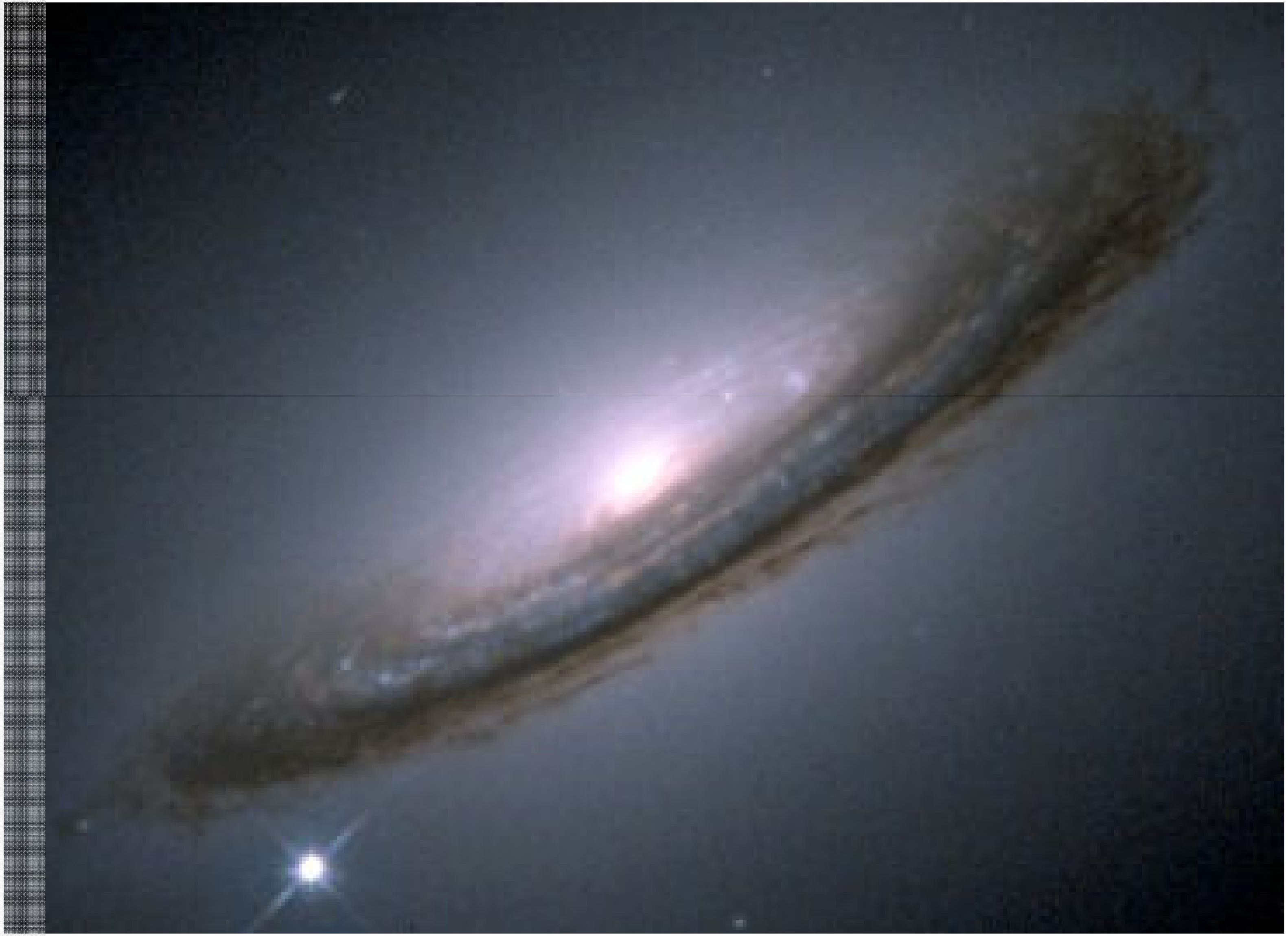
$$b^2 = \frac{R'(r,t)^2}{1+f(r)}$$

FLRW *X* LTB



MODEL	METRICS	SCALE FACTOR	DENSITY
FLRW	$ds^2 = -dt^2 + a^2\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]$	$a = a(t)$	$\rho = \rho(t)$
LTB	$ds^2 = -dt^2 + \frac{R'^2}{1+f(r)}dr^2 + R^2(r,t)d\Omega^2$ $R(r,t) = a(r,t)r$	$a = a(r,t)$	$\rho = \rho(r,t)$

Type Ia Supernova at Virgo, NGC 4526



Type Ia Supernovae

nickel → cobalt → iron

- Type Ia supernovae derive from carbon-oxygen white dwarf in binary systems. The dwarf absorbs mass from its companion, a red giant, achieving critical mass of explosion.

Type Ia Supernovae

advantages

- Extreme luminosity (10^9 - 10^{10} L_{Sun})
- High homogeneity

Standard candles

disadvantages

- Rare event
- Short duration

LT model

$$\text{Assuption}\begin{cases} R(r,t) = a(r,t)r \\ f = A(r)r^2 \\ M = \beta r^3 \end{cases}$$

$$\text{Einstein's equations}\begin{cases} \dot{R}^2 = f + \dfrac{M}{R} \\ \dot{a}^2 r^2 = Ar^2 + \dfrac{\beta r^3}{ar} \end{cases}$$

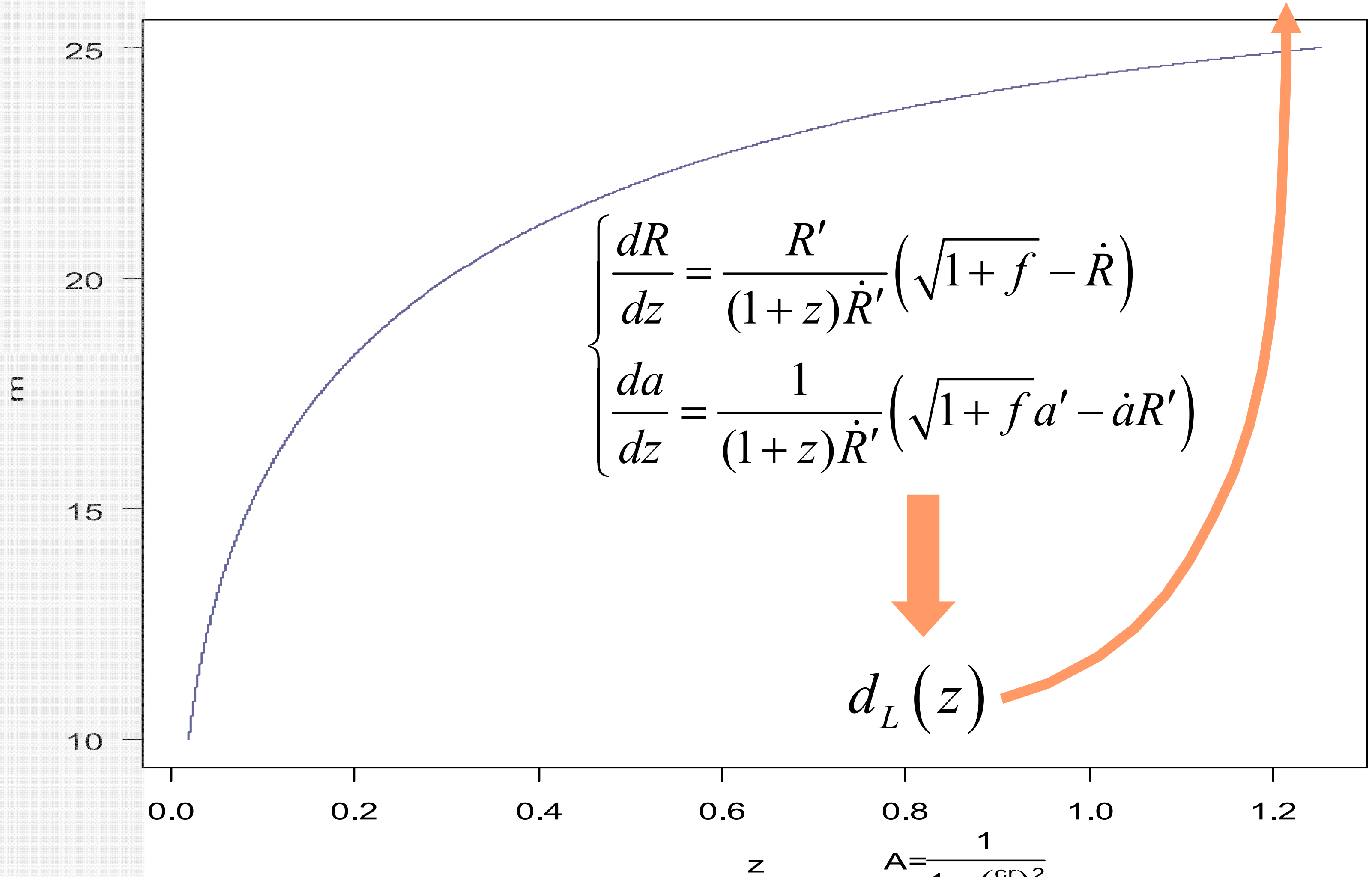
$$\text{Assuption}\left\{ A(r) = \frac{1}{1+(\mathrm{c}r)^2} \right.$$

$$\Omega_M = \left(1 + \frac{9a}{4}\right)^{-1}$$

Ref: Garfinkle D, arXiv:gr-qc/0605088 (2006).

LT model

LT plot: Ω=0.3 , c= 8.5 ; $m = M_B + 5\log(H_0 d_L)$



$$\begin{cases} \dfrac{dR}{dz} = \dfrac{R'}{(1+z)\dot{R}'}\left(\sqrt{1+f} - \dot{R}\right) \\ \dfrac{da}{dz} = \dfrac{1}{(1+z)\dot{R}'}\left(\sqrt{1+f}\,a' - \dot{a}R'\right) \end{cases}$$

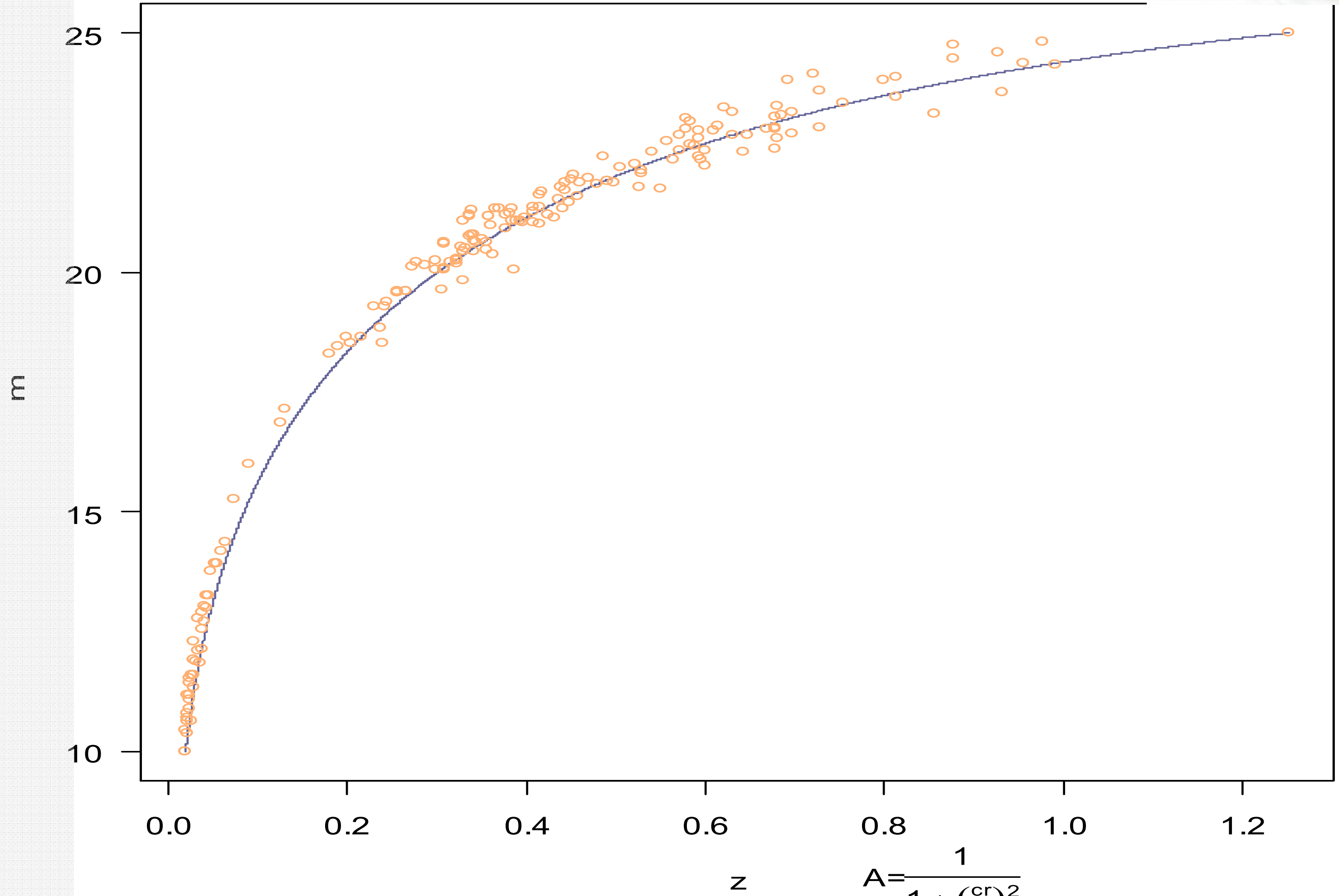
$$d_L(z)$$

$$A = \frac{1}{1+\left(\frac{cr}{a}\right)^2}$$

Ref: Garfinkle D, arXiv:gr-qc/0605088 (2006).

Riess table

LT plot: Ω=0.3 , c= 8.5



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Ref: Riess *et al*, arXiv: astro-ph/0611572v2 (2007).

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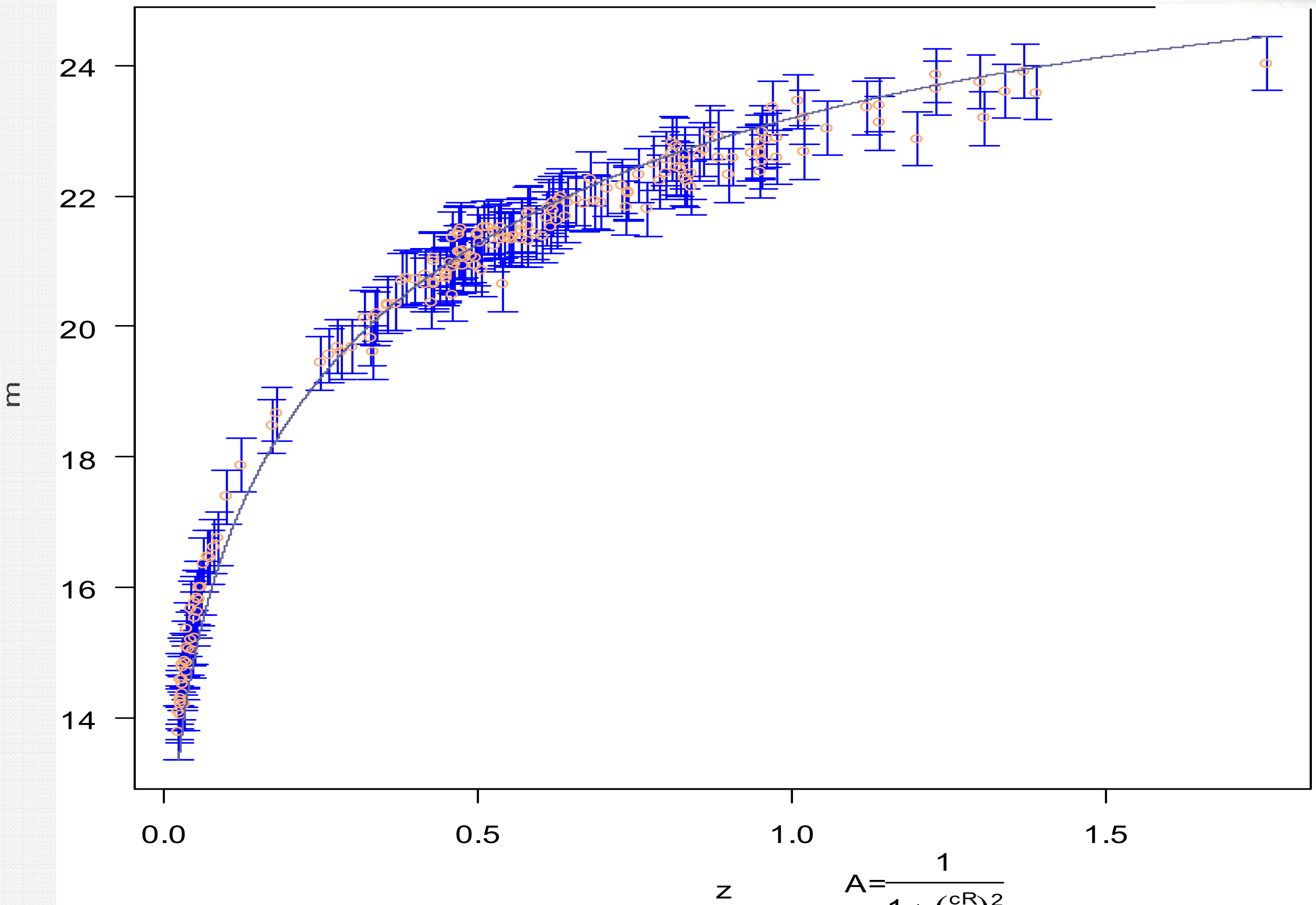

$$A = \frac{1}{1 + (\frac{cR}{a})^2}$$

Stephani cosmology

$$ds^2 = -D^2 dt^2 + V^2 \left[dr^2 + f^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right],$$

$$D = \frac{1 + F^2 \left(k - a k_{,a} \right)}{1 + kF^2},$$

$$V = \frac{a}{1 + kF^2}.$$

$$f = r, \qquad F = r/2;$$

$$f = \sin r, \qquad F = \sin\left(r/2\right);$$

$$f = \sinh r, \qquad F = \sinh\left(r/2\right).$$

Field equation in Stephani cosmology

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{(k+k_0)}{a^2} = \frac{8\pi G}{3}\rho$$

The age of the universe is very larger in this cosmology!! This is a problem for the standard model!!

What now?

The physical image of the world depends on which part of the world we want to describe and in which circumstances we are observing the world.

Thanks a lot!!