Deformações Cônicas do Espaço Tempo em Extra Dimensões: Motivações e Aplicações

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- Vortex Configuration and Some Aplications
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Topological Defects in Relativistic Quantum Field Theories

- Topological defects are stable configurations of matter formed at phase transitions in the very early Universe or matter.

- There are a number of possible types of defects, such as domain walls, cosmic strings, monopoles, textures and other 'hybrid' creatures.

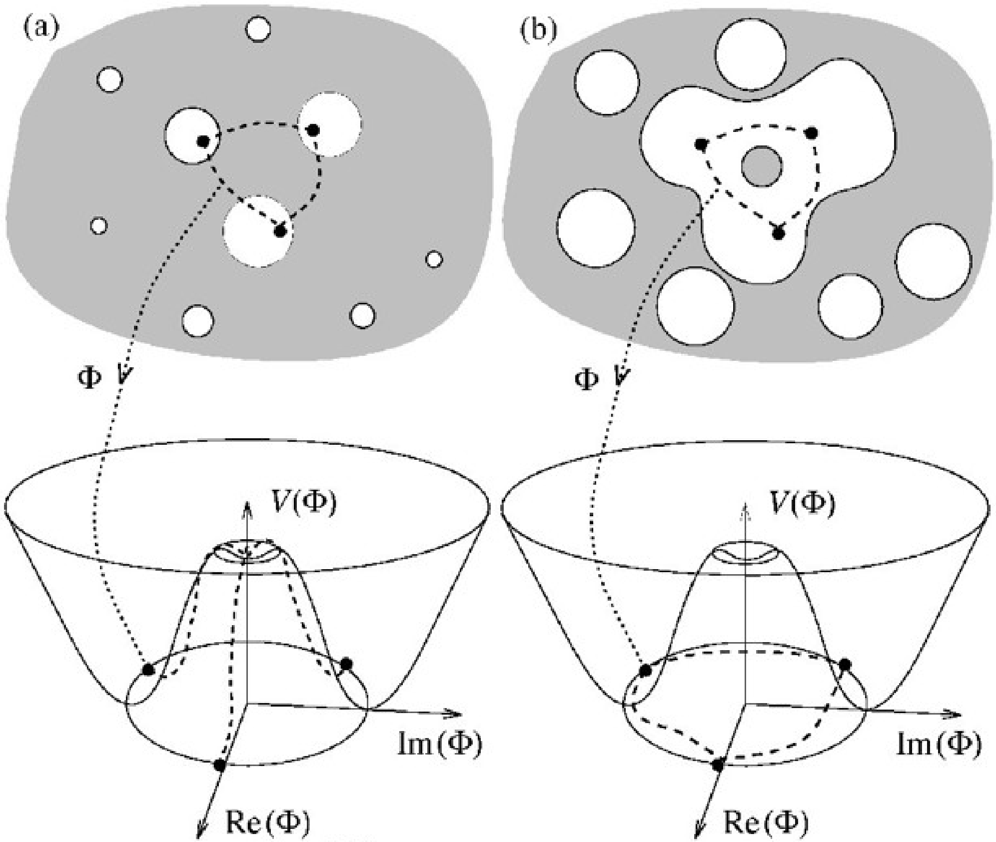
Cosmic Strings

- These are one-dimensional (that is, line-like) objects which form when an axial or cylindrical symmetry is broken.
- Strings can be associated with grand unified particle physics models, or they can form at the electroweak scale.
- They are very thin and may stretch across the visible universe. A typical GUT string has a thickness that is less then a trillion times smaller that the radius of an Hydrogen atom.

Cosmic Strings

- In quantum systems they appear as topological line defects;
- In relativistic quantum field theories they are known as cosmic strings;
- In superconductors as quantified flux lines;
- In superfluids and low-density Bose–Einstein condensates as quantified vortex lines.

The Kibble Mechanism

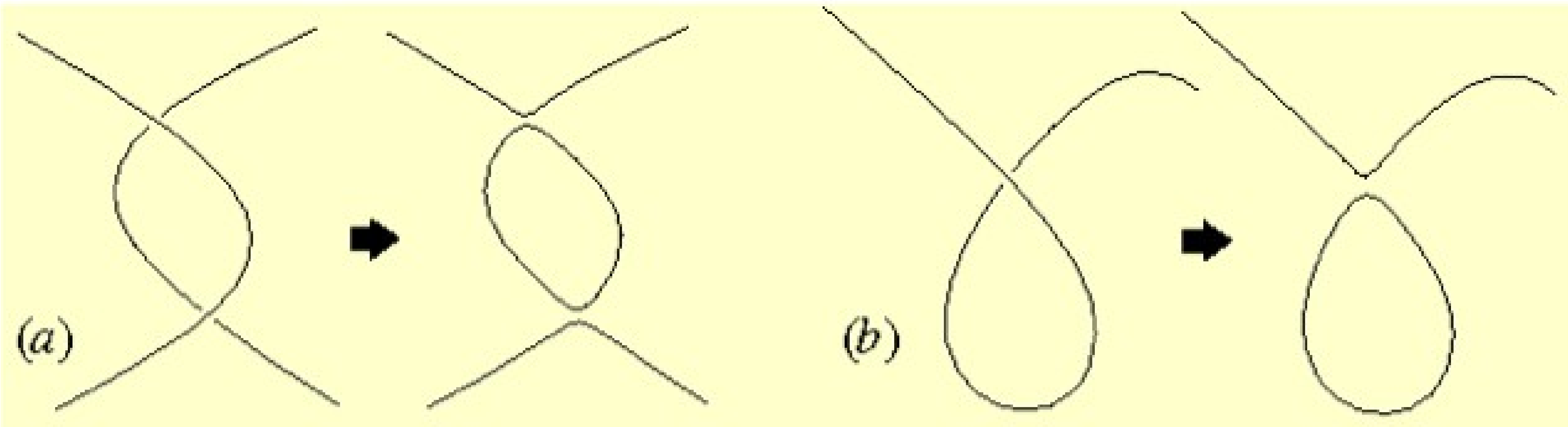


U(1) symmetry breaking of a complex scalar field produces cosmic strings.

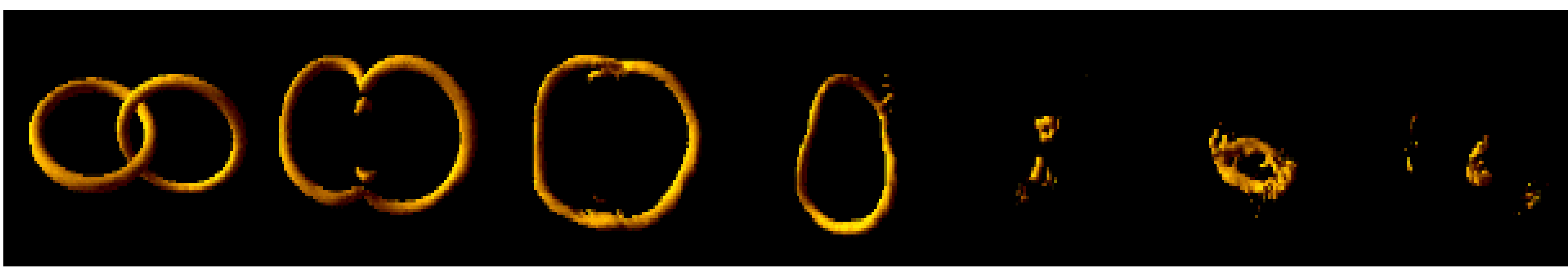
(a) patches with true vacuum energies start growing as the symmetry is broken. gray region represents false vacua;

(b) As the patches with true vacua merge, false vacuum regions are squeezed and form topological defects

Stability of Cosmic String

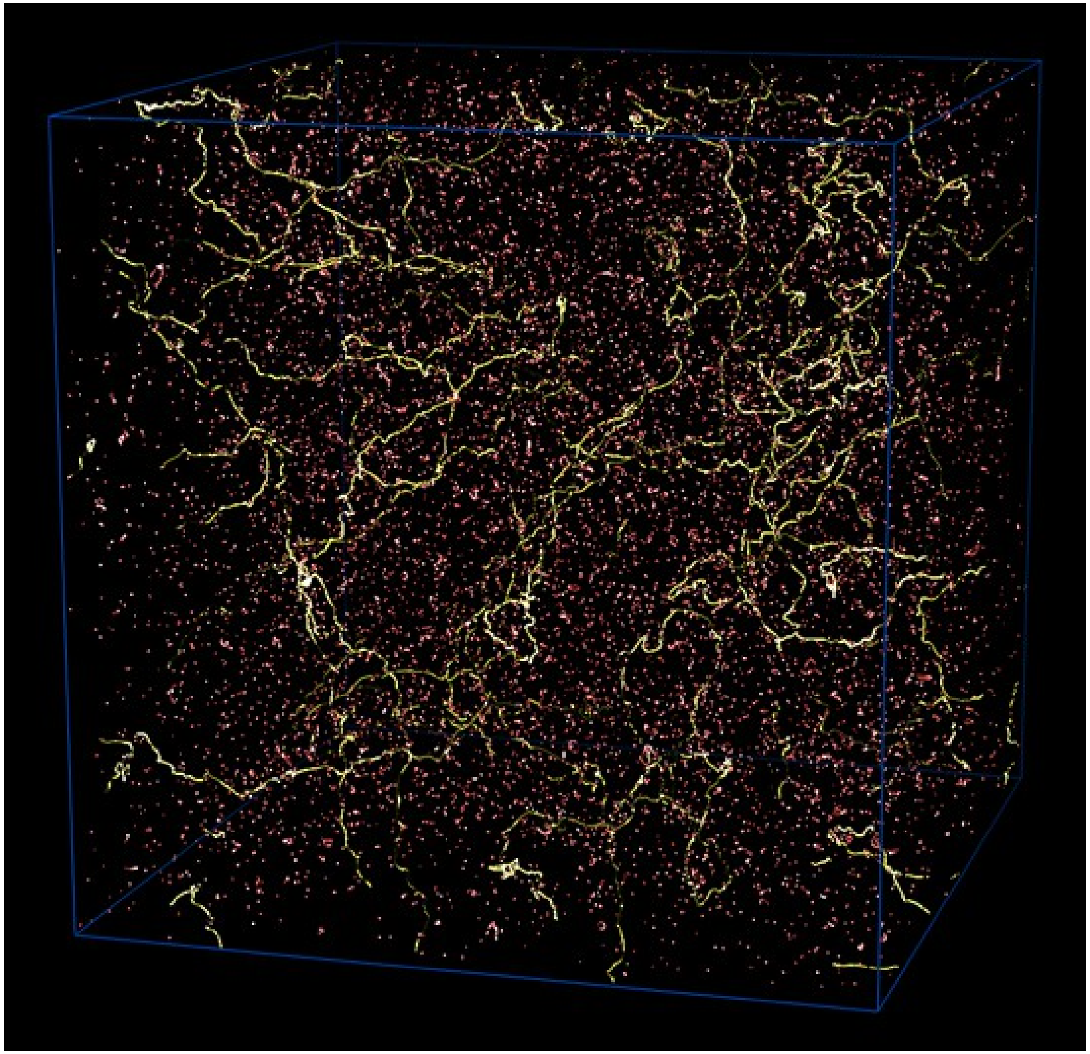


Cosmic string loop formation. A loop forms (a) when two strings interacts in 2 separate points or (b) when a string crosses itself.
© Cambridge cosmology group

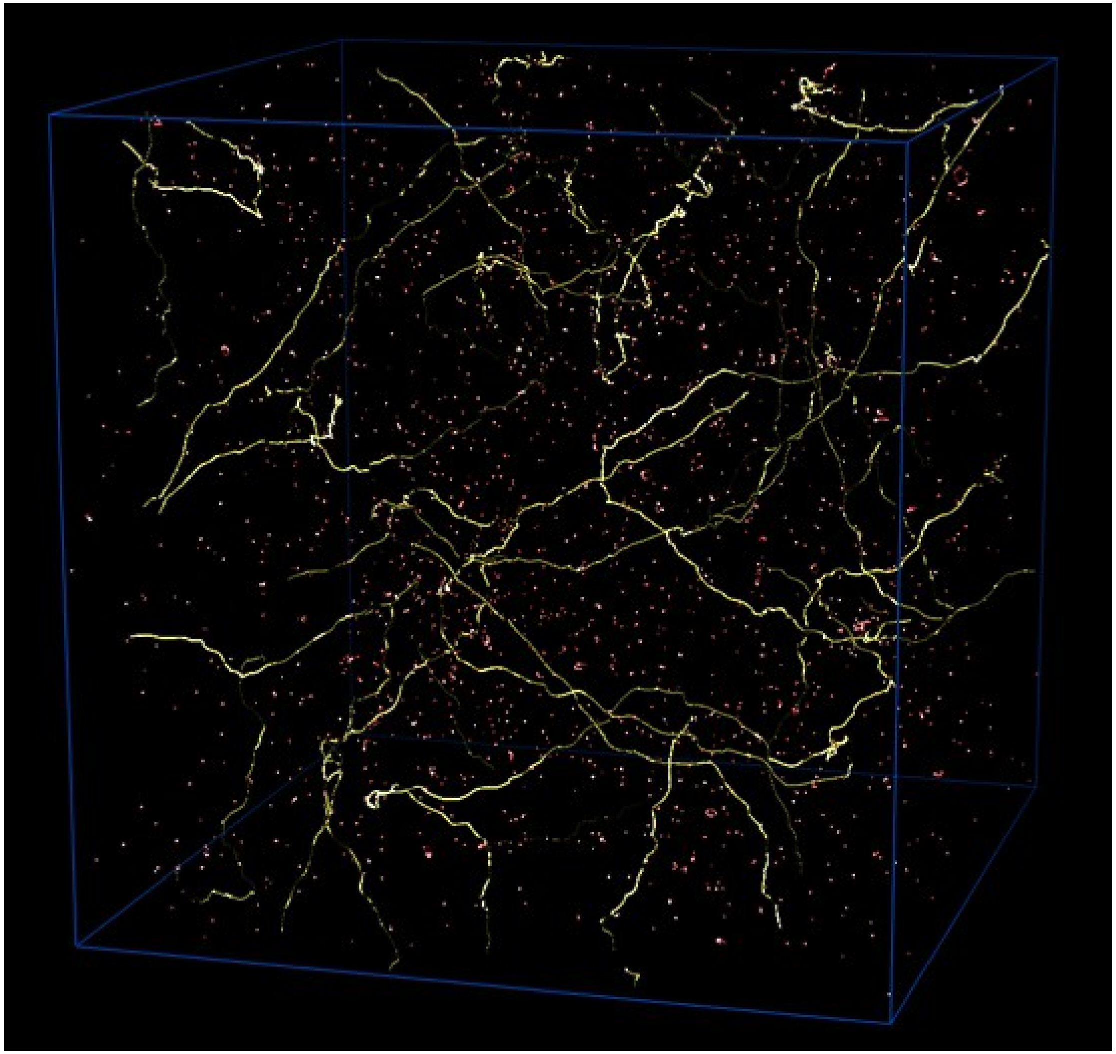


Snapshots from the two loop decay process.

Snapshot of a string network in the radiation era

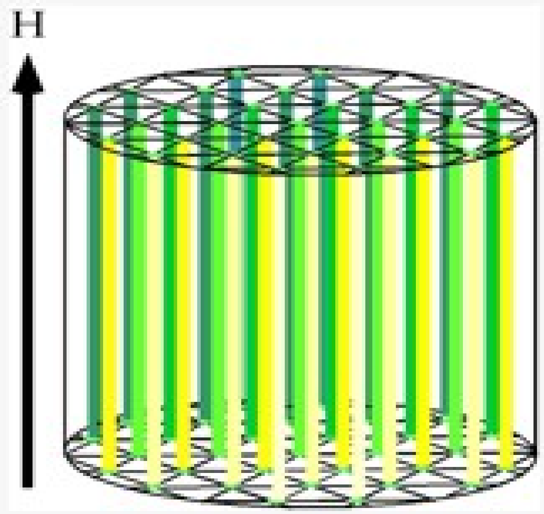


Snapshot of a string network in the matter era



Quantified Flux lines

- In type II superconductors magnetic flux penetrates the sample in the form of vortices of quantized flux.



The vortex lattice in conventional type II superconductors

- In the simplest case this form an ordered hexagonal lattice, but in exotic superconductors, such as the high-Tc cuprate materials, a whole zoo of vortex phases exist, including vortex glasses and vortex liquids.

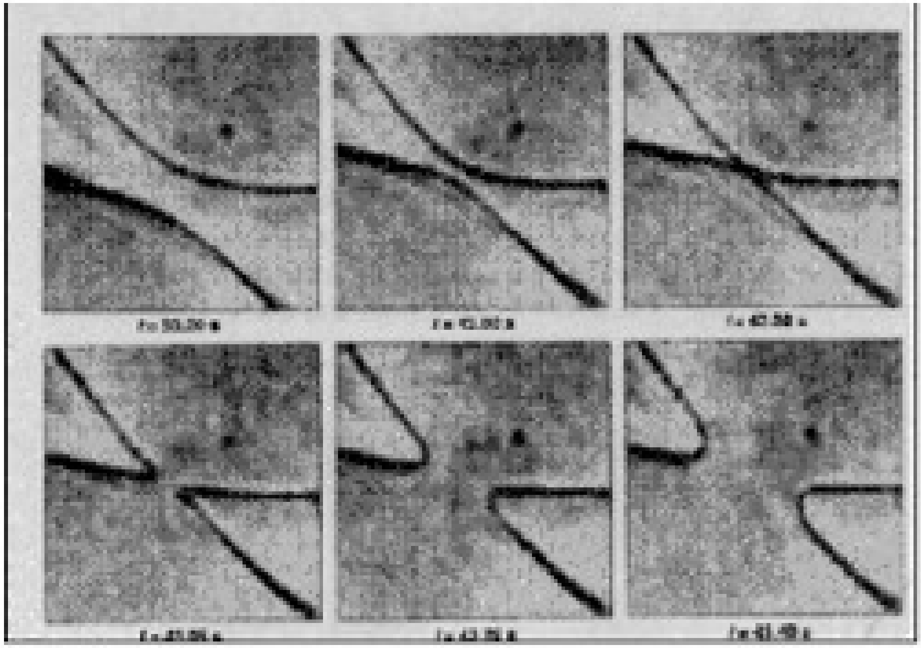


Fig.1: Long-string intercommuting in a liquid crystal. The two strings exchange ends at the crossing

Cosmic String and Gravity

A cosmic string has very interesting physical effects.

All these effects are originated from its unique way to deform the space-time around it.

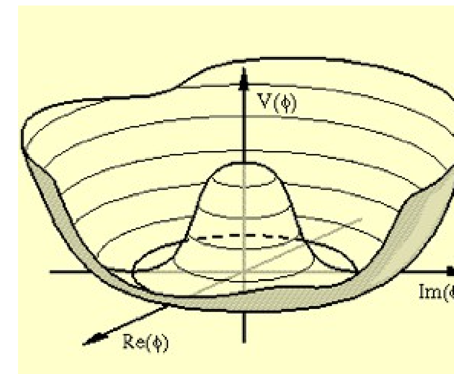
A model of an infinitely long, straight cosmic string is one of the exactly solvable systems in General Relativity.

Since it is made of vacuum energy rather than ordinary matter, it has a different solution of the Einstein equations, a conic space-time!

Theoretical Model and Symmetry

- The cosmic string arises from a spontaneous breakdown of the symmetry of the gauge model U(1) and the action is described by

$$S_m = \int d^4x\sqrt{-g}\left[D_\mu\varphi D^\mu\varphi^* - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\varphi)\right]$$



- where the covariant derivative and the potential are

$$D_\mu\varphi \equiv (\partial_\mu + ieA_\mu)\varphi \qquad V(\phi) = \frac{\lambda_\varphi}{4}(\varphi\varphi^* - \eta^2)^2$$

- The ansatz for the cosmic string fields are

$$\varphi = R(\rho)e^{in\theta} \qquad A_\mu = \frac{1}{e}[P(\rho) - 1]\delta^\theta_\mu$$

- The boundary conditions are

$$R(0) = 0 \quad P(0) = 1 \qquad \lim_{\rho\to\infty} R(\rho) = \eta \quad \lim_{\rho\to\infty} P(\rho) = 0$$

Einstein Equations

- The cosmic string has a gravitational coupling. The general metric in cylindrical coordinates is given by

$$ds^2 = -e^{2\psi}dt^2 + e^{2(\gamma-\psi)}(d\rho^2 + dz^2) + \beta^2 e^{-2\psi}d\theta^2$$

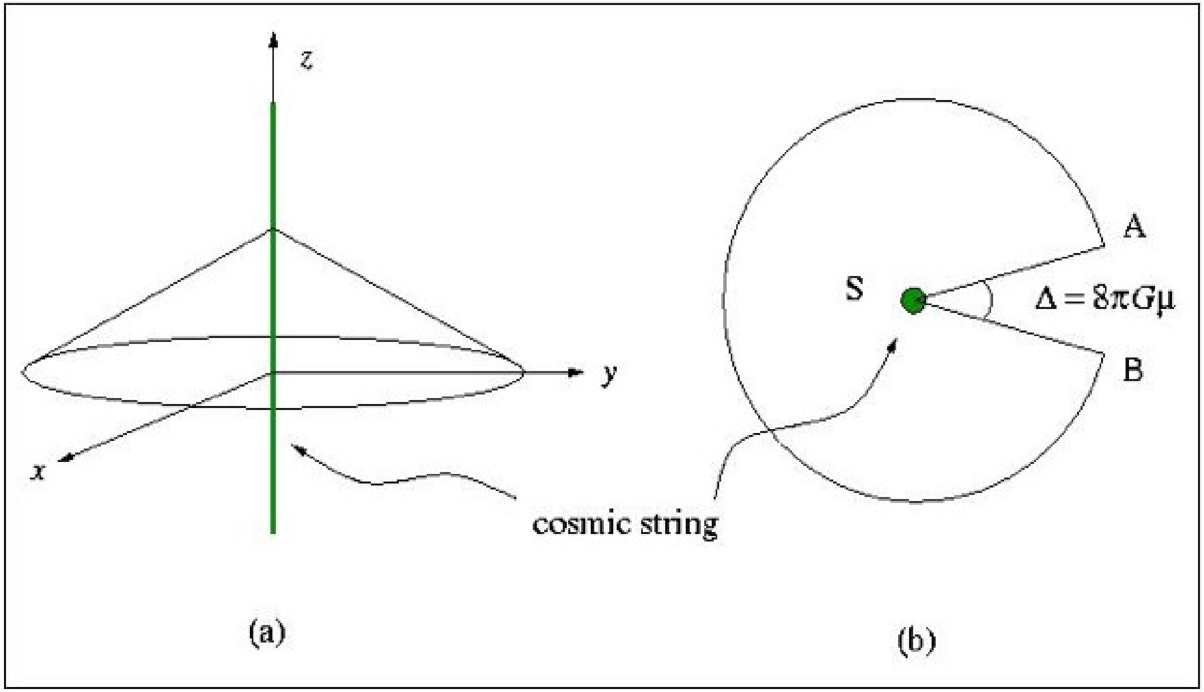
- The Einstein equations in the ordinary cosmic string has the constraint $\psi = \gamma/2$ that reduces to

$$ds^2 = e^{\gamma}(-dt^2 + d\rho^2 + dz^2) + \beta^2 e^{-\gamma}d\theta^2$$

- The Einstein equations are

$$\beta'' = 8G\beta(T_t^t + T_\rho^\rho)e^{\gamma}$$

$$(\beta\gamma')' = 8\pi G\beta(T_\rho^\rho + T_\theta^\theta)e^{\gamma}$$



Conic space-time formed around a straight string. One complete turn around the string makes up less than 2π by the deficit angle Δ=8πGμ

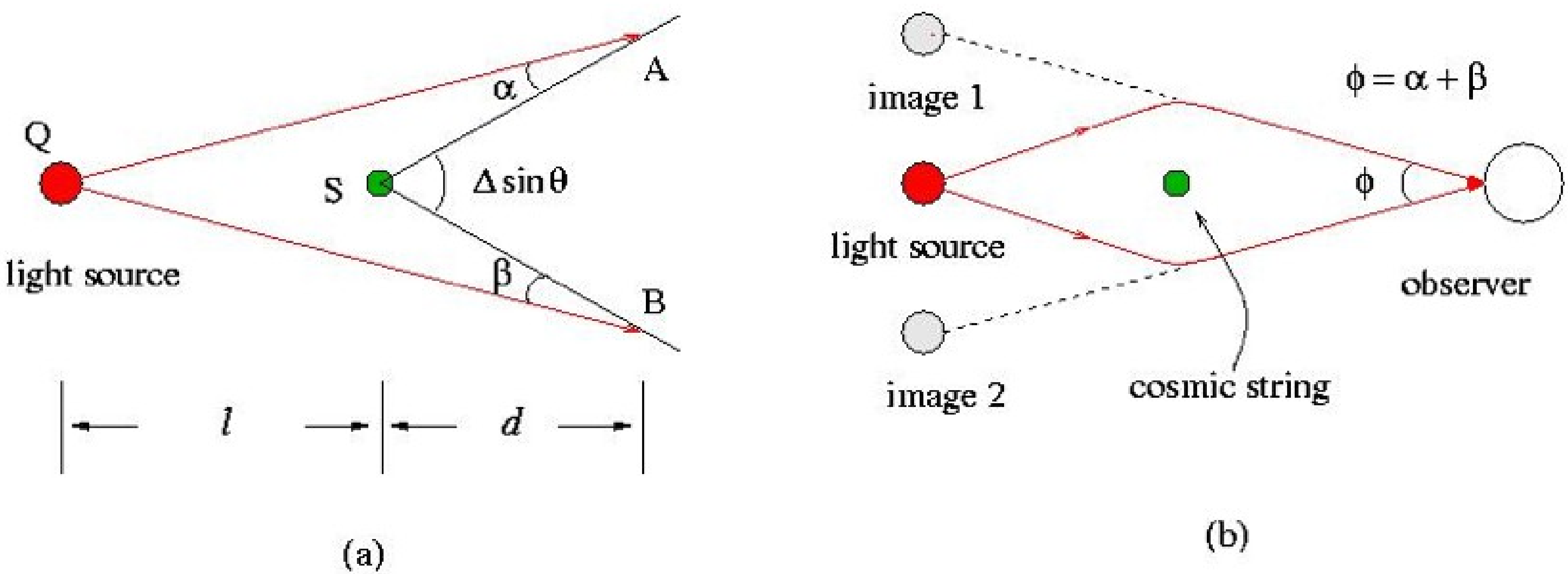
$$ds^2 = -dt^2 + d\rho^2 + (1-4G\mu)^2\rho^2 d\theta^2 \qquad T_t^t = T_z^z = 8G\mu\,\delta^2(\rho)$$

$$0 \le \bar{\theta} \equiv (1-4G\mu)\theta \le 2\pi(1-4\pi G\mu) \qquad T_\theta^\theta = T_\rho^\rho = 0$$

$$r = \frac{\rho^{1-4G\mu}}{(1-4G\mu)} \qquad 1-8G\mu\ln(r) \sim r^{-8G\mu} \qquad \Phi_B = \oint \vec{A}.\vec{d\ell} = \frac{2\pi n}{q}$$

$$ds^2 = -dt^2 + (1-8G\mu\ln(r))(dr^2 + r^2 d\theta^2)$$

String Dynamics



Double Image

Cosmic String Detection

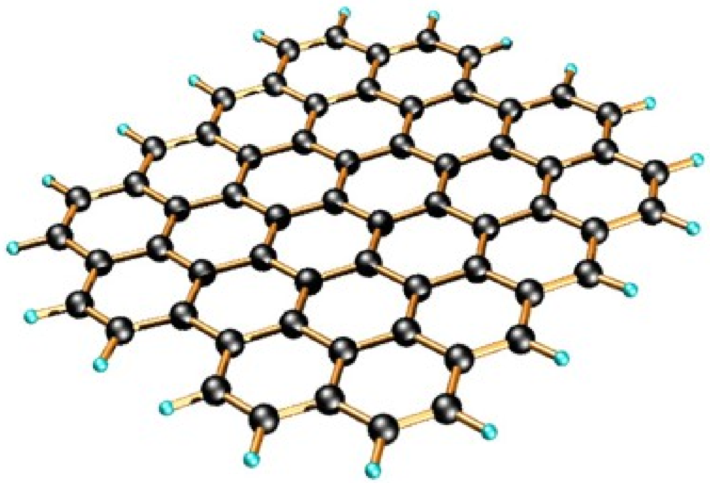
- Until now the cosmic strings were not detected yet, but there are a lot of researchers improving the techniques of the detection.
- Today, the cosmic strings are very interesting to string theory in direction of the comprovation of string theory measurement. arXiv:1011.2640
- There are strong studies about the relation of the cosmic string and the primordial magnetic field.
- The topological defects like cosmic string are important to the AdS/CFT correspondence and others low dimensional reduced mechanisms as Randall Sundrum like prescription and Calabi Yau compactifications.

Topological Defect Like Cosmic String in Condensed Matter Systems

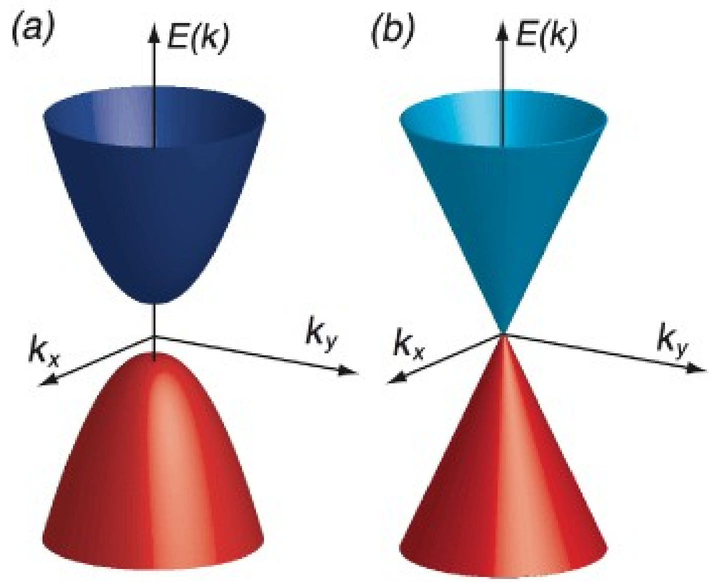
- The idea is to include the condensed matter effects that is related with the holographic principle where the most useful is the Ads/CFT correspondence;
- Supergravity theory in AdS corresponding to Conformal Field Theory on the boundary;
- The topological defect like cosmic string in four dimensions can be described with AdS_4 (3+1) that corresponds to a CFT in (2+1) with defect;
- The importance of the models in (2+1) dimensions is the fact that there are materials like graphene that can be described by Dirac theory.

Graphene Like Structures

- Graphene is the two-dimensional crystalline form of carbon: a single layer of carbon atoms arranged in hexagons,



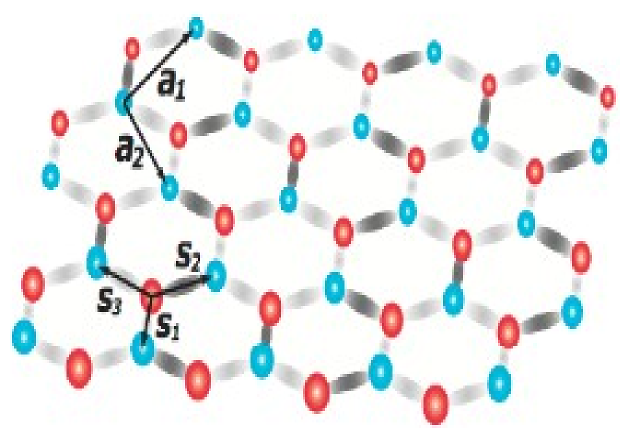
- The graphene was created in laboratory by physicists at the University of Manchester (2004).



Energy dispersion of

(a) a typical two-dimensional semiconductor and

(b) that of graphene, which is a zero-gap semiconductor.



In the graphene like structures we have two triangular sub-lattices: A (red) and B(blue)

Theoretical Description

- The graphene have a linear dispersion and can be described by a continuous model which reduces to the Dirac equation in two dimensions

$$\mathcal{H}_{0i} = \hbar v_{\mathrm{F}} \bar{\Psi}_i(\mathbf{r})(\sigma_x \partial_x + \sigma_y \partial_y)\Psi_i(\mathbf{r})$$

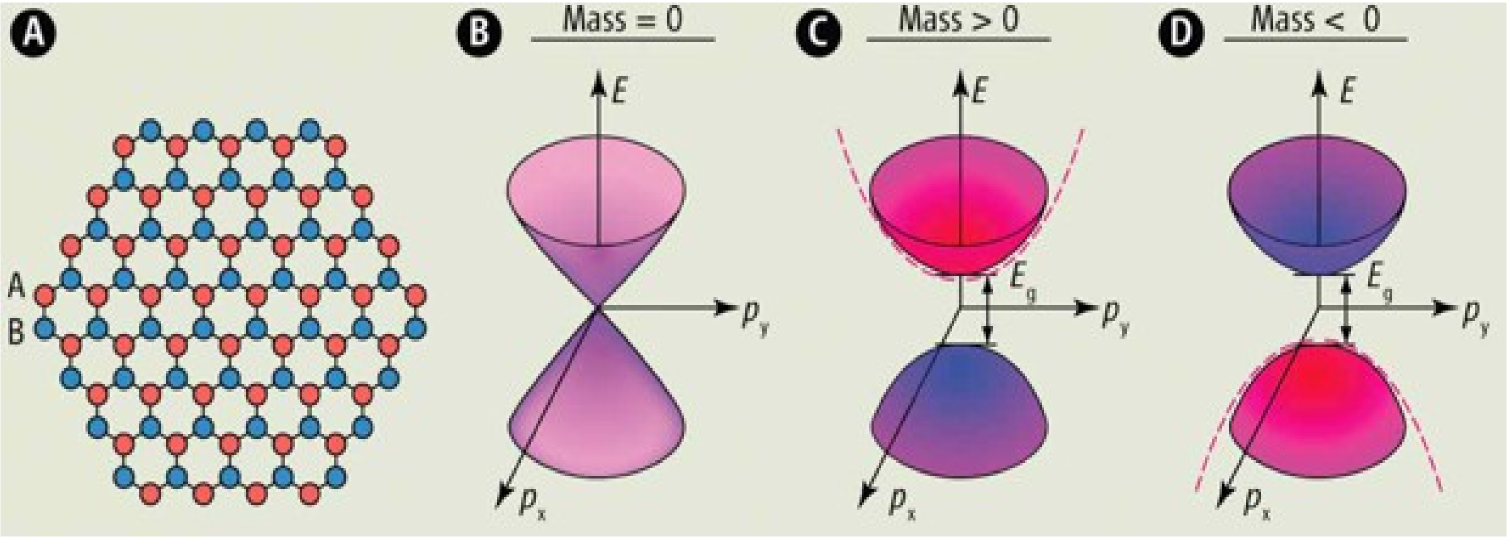
$$\Psi_i(\mathbf{r}) = \begin{pmatrix} \psi_A(\mathbf{r}) \\ \psi_B(\mathbf{r}) \end{pmatrix}$$

- The equation that described is the Dirac equation

$$i\gamma^{\mu}\nabla_{\mu}\Psi_i = 0$$

Topological Defect Like Cosmic String and Graphene like Structures

The importance of the topological defects like cosmic string in the grafene like structures is that they are responsible for the mass gap



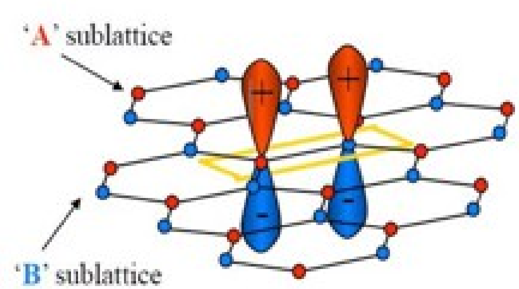
Kekulé distortion in a honeycomb array of carbon atom

- Here let us understand the mass gap model to the vortex mass gap prescription. The Hamiltonian can be written as

$$\mathcal{H} = \int d^2 r \Psi(r)^\dagger K \Psi(r)$$

- where ψ(r) is the four components spinor given by

$$\Psi(\mathbf{r}) = \begin{pmatrix} \psi_+^A(\mathbf{r}) \\ \psi_-^A(\mathbf{r}) \\ \psi_+^B(\mathbf{r}) \\ \psi_-^B(\mathbf{r}) \end{pmatrix}$$



$$K = \begin{pmatrix} 0 & -2i\partial_z & g\varphi(\mathbf{r}) & 0 \\ -2i\partial_z * & 0 & 0 & g\varphi(\mathbf{r}) \\ g\varphi^*(\mathbf{r}) & 0 & 0 & 2i\partial_z \\ 0 & g\varphi^*(\mathbf{r}) & 2i\partial_z * & 0 \end{pmatrix}$$

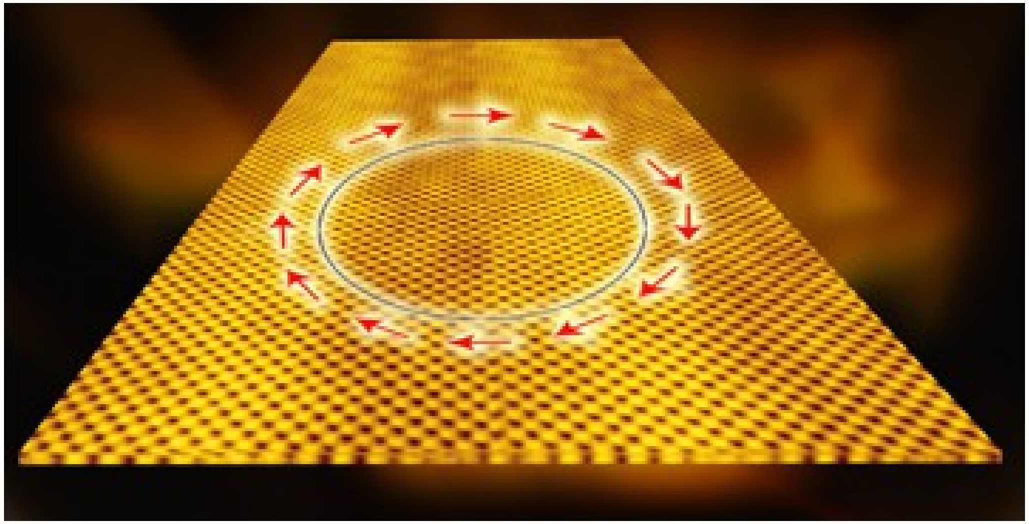
$$-2i\partial_z = \tfrac{1}{i}\,(\partial_x - i\,\partial_y)$$

- where the indices a and b refers to the two triangular sublattices of the honeycomb array and the indices $+$ and $-$ refers to the two inequivalent points (Dirac points) of the first Brillouin zone.

Topological Insulators

• Theorized and then discovered by researchers at the U.S. Department of Energy (Lawrence Livermore Lab) and their colleagues in other institutions.

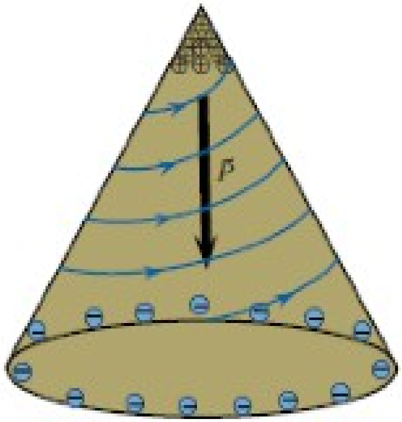
• These "strong 3-D topological insulators" – TIs for short – are seemingly mundane semiconductors with startling properties. For starters, picture a good insulator on the inside that's a good conductor on its surface.



cond-mat/0612374

Conical Topological Insulator

- The band structure of the surface states of a topological insulator like Bi_2Se_3 appears as two cones that meet at a point.
- There's no gap at all between the valence and conduction bands, only a smooth transition with increasing energy.
- This is similar to the band structure of the fascinating material graphene, a single sheet of carbon atoms, the thinnest possible surface.



Electric charge polarization in conical topological insulators: Wider TI cones (upper), with $\delta > 30^{o}$ tend to accumulate negative charges (electrons) near the apex

Conical Defect in Bosonic String Framework

The bosonic massless fields to the string in the combined action

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^D x \sqrt{g}\, \mathscr{L}_{\mathrm{B}} + S_\sigma \,,$$

with the bosonic lagrangian given by

$$\mathscr{L}_{\mathrm{B}} = R - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{12}\mathrm{e}^{-2\alpha\phi} H^2 \,,$$

and the sigma-model action by

$$S_\sigma = -\tfrac{1}{2}\mu \int \mathrm{d}^2\sigma \left(\sqrt{\gamma}\, \gamma^{mn}\, \partial_m X^\mu\, \partial_n X^\nu\, g_{\mu\nu}\, \mathrm{e}^{\alpha\phi} + \epsilon^{mn}\, \partial_m X^\mu\, \partial_n X^\nu\, B_{\mu\nu} \right)$$

The equations of motion that follow from the above action are

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = \kappa^2 T_{\mu\nu} ,$$

$$\nabla_\rho (\mathrm{e}^{-2\alpha\phi} H^{\mu\nu\rho}) = 2\mu\kappa^2 \int \mathrm{d}^2\sigma\, \epsilon^{mn} \partial_m X^\mu \partial_n X^\nu \frac{\delta^{(D)}(x - X)}{\sqrt{g}} ,$$

$$\nabla_\mu (g^{\mu\nu} \nabla_\nu \phi) = -\tfrac{1}{6} \alpha\, \mathrm{e}^{-2\alpha\phi} H^2$$

$$+ \mu\kappa^2\alpha \int \mathrm{d}^2\sigma \sqrt{\gamma}\, \gamma^{mn} \partial_m X^\mu \partial_n X^\nu g_{\mu\nu}\, \mathrm{e}^{\alpha\phi} \frac{\delta^{(D)}(x - X)}{\sqrt{g}} .$$

The energy momentum tensor for the antisymmetric tensor and dilaton fields as well as the energy—momentum tensor corresponding to the string source:

$$2\kappa^2 T_{\mu\nu} = \partial_\mu \phi\, \partial_\nu \phi - \tfrac{1}{2} g_{\mu\nu} (\partial\phi)^2 + \tfrac{1}{2}\left(H_{\mu\rho\sigma} H_\nu{}^{\rho\sigma} - \tfrac{1}{6} g_{\mu\nu} H^2 \right) \mathrm{e}^{-2\alpha\phi}$$

$$- 2\mu\kappa^2 \int \mathrm{d}^2\sigma \sqrt{\gamma}\, \gamma^{mn} \partial_m X_\mu \partial_n X_\nu\, \mathrm{e}^{\alpha\phi} \frac{\delta^{(D)}(x - X)}{\sqrt{g}} .$$

This is a complicated, nonlinear system of equations, and it would seem miraculous to be able to find an exact solution.

There is however a remarkably simple ansatz that reduces these equations to one linear differential equation for a single scalar function E(r):

$$A = \frac{D-4}{D-2}E(r), \qquad B = -\frac{2}{D-2}E(r), \qquad \phi = \alpha E(r), \qquad B_{01} = -e^{E(r)}.$$

Here the string runs along the x^1 axis and

$$r^2 = \boldsymbol{x}\cdot\boldsymbol{x} = \eta_{ij}x^i x^j, \; i, j = 2, \ldots, D-1$$

These relations between the different fields ensure that half of the supersymmetries will be preserved. This ansatz give us

$$H_{i01} \equiv \partial_i B_{01} = \partial_i G_{00} = -\partial_j(e^A e^{\alpha\phi}),$$

$$B_{01} = -e^E, \qquad A + \alpha\phi = E$$

The solution

The equations of motion for the dilaton and the antisymmetric tensor field both reduce to the same linear equation:

$$\eta^{ij}\partial_i\partial_j \mathrm{e}^{-E} = -2\mu\kappa^2\delta^{(D-2)}(x)\,,$$

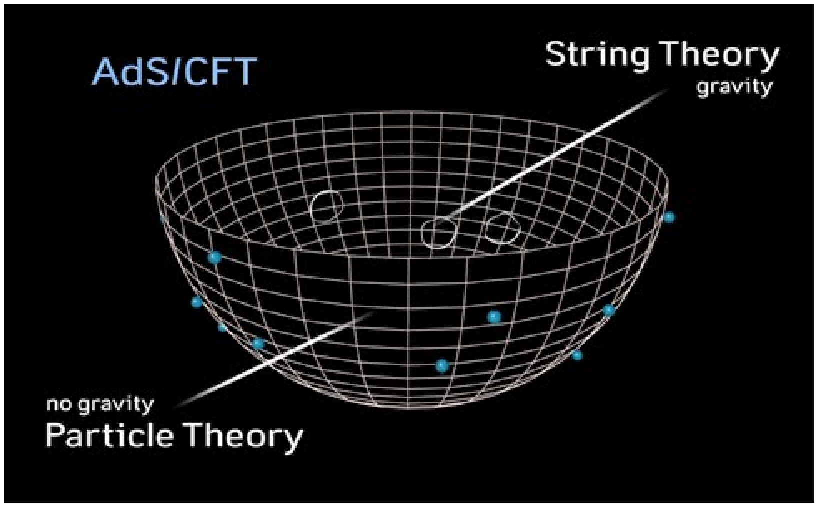
with solution

$$\mathrm{e}^{-E} = \begin{cases} 1 + \dfrac{M}{r^{D-4}}\,, & D > 4\,, \\ 1 - 8G\mu\ln(r)\,, & D = 4\,, \end{cases}$$

Where $M = 2\kappa^2\mu/(D-4)\omega_{D-3}$ and ω_{D-3} is the volume of the sphere $\mathbf{S}^{D-3}$.

$$ds^2 = -dt^2 + (1 - 8G\mu\ln(r))(dr^2 + r^2 d\theta^2)$$

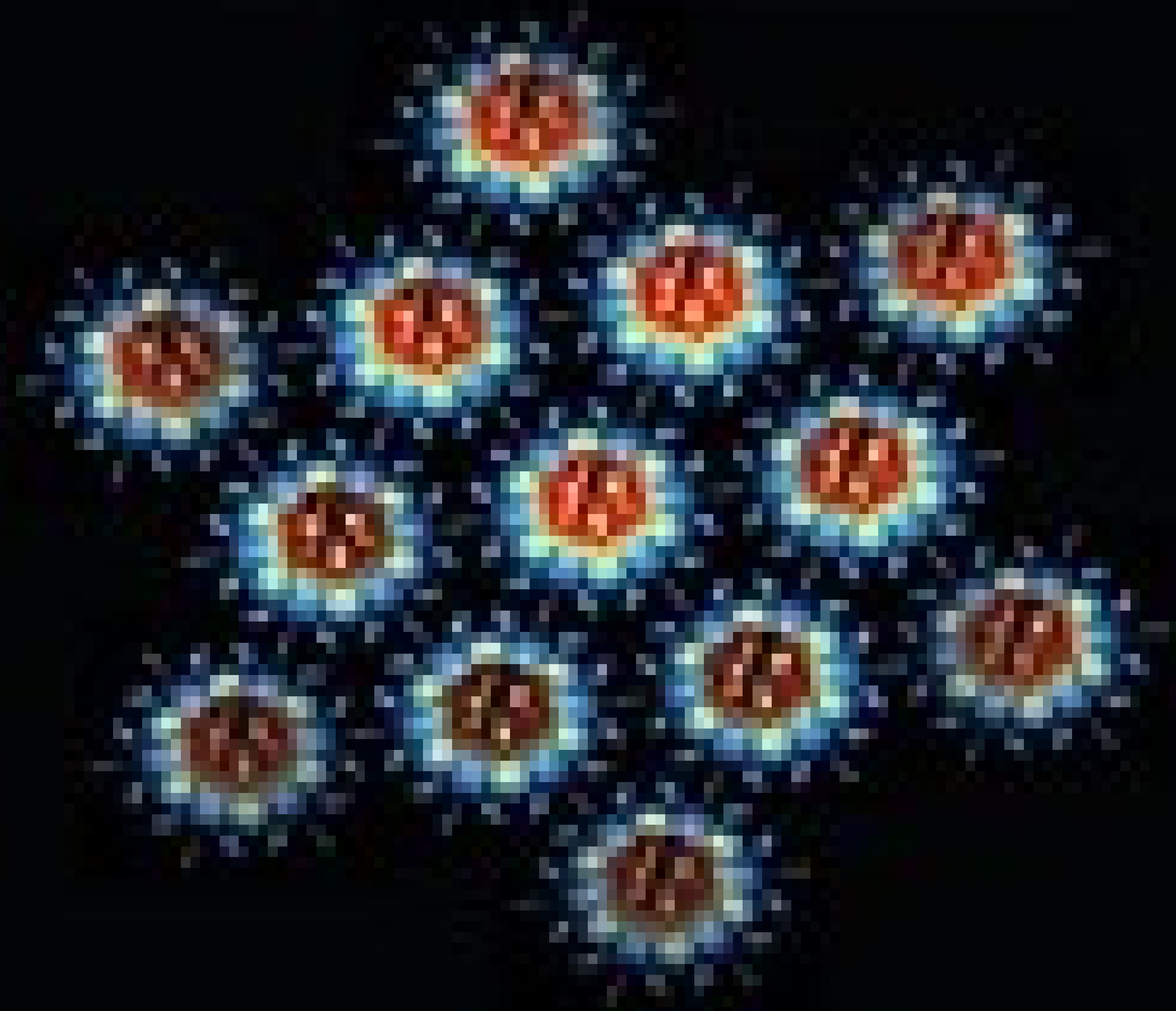
AdS/CFT Correspondence



hep-th/9711200

- The AdS/CFT duality is a concrete example of holography, which suggests that all the information about the interior of some region is actually contained on the boundary.
- In AdS/CFT, a conventional particle theory, called "Conformal Field Theory", on the boundary corresponds to a String Theory in Anti de Sitter space in the interior.
- The string theory or M-theory includes gravity and has one more dimension than the particle theory.

A COSMIC STRING IN



AdS$_4$ SPACE TIME

Motivation

- Inspired by the AdS/CFT correspondence we propose a new duality that allows the study of strongly coupled field theories living in a $2 + 1$ conical space-time.
- Solving the 4-d Einstein equations in the presence of an infinite static string and negative cosmological constant
- We obtain a conical AdS_4 space-time whose boundary is identified with the 2+1 cone found by Deser, Jackiw and t' Hooft.

arXiv:1003.5396

Consider an infinite static string living in a 4-d space-time with metric $g_{\mu\nu}$ in the presence of a negative cosmological constant $\Lambda = -6/L^2$. This system is described by the Einstein Hilbert Nambu-Goto action

$$
\begin{aligned}
S &= S_G + S_{NG} \\
&= \frac{1}{16\pi G_4}\int dt dz dr d\theta \sqrt{-g}\left[R + \frac{6}{L^2}\right] - \mu \int d\sigma^0 d\sigma^1 \sqrt{-\det P[g_{ab}]},
\end{aligned}
$$

$$
P[g_{ab}] = g_{\mu\nu}(X)\,\frac{\partial X^\mu(\sigma)}{\partial \sigma^a}\frac{\partial X^\nu(\sigma)}{\partial \sigma^b},
$$

A variation in the metric implies in

$$\delta S_{NG}^{o.s.} = \frac{1}{2}\int dt\,dz\,dr\,d\theta\sqrt{-g}\,T^{\mu\nu}\delta g_{\mu\nu}\,,$$

where

$$T^{\mu\nu} = -\frac{\mu}{\sqrt{g_{rr}g_{\theta\theta}}}\frac{\delta(r)}{\pi}\begin{pmatrix} (g_{tt})^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (g_{zz})^{-1} \end{pmatrix}$$

is the stress energy tensor of the static string. The variation of the gravity action leads to the Einstein equations of motion

$$R^{\mu\nu} - \frac{R}{2}g^{\mu\nu} - \frac{3}{L^2}g^{\mu\nu} = 8\pi G_4 T^{\mu\nu}$$

The conical AdS4 solution

Assuming that a static rigid string can be described by a static cylindrical metric, we begin with the general form in AdS background

$$ds^2 = e^{\frac{-2z}{L}}\left[-e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + e^{2\psi(r)}d\theta^2\right] + e^{2\lambda(r)}dz^2\,,$$

Then, the equations reduce to

$$\left[\psi'' + \psi'^2\right]e^{\psi} = \left[e^{\psi}\right]'' = -8G_4\mu\,\delta(r)\,,$$

with solution

$$e^{\psi} \;=\; ar + b + \frac{4G_4\mu}{\pi}\int_{-\infty}^{\infty} dk\,\frac{e^{ikr}}{k^2} \;=\; (a - 4G_4\mu)r + b$$

We must have a = 1 and b = 0 in order to recover the Anti-de-Sitter space-time in the absence of the string (μ = 0).

$$ds^2 = e^{-2\frac{z}{L}}\left[-dt^2 + dr^2 + (1 - 4G_4\mu)^2 r^2 d\theta^2\right] + dz^2\,.$$

The 2 + 1 conical boundary

Redefining the transverse radial coordinate

$$r = \frac{\rho^{1-4G_4\mu}}{(1-4G_4\mu)}$$

the metric takes the form

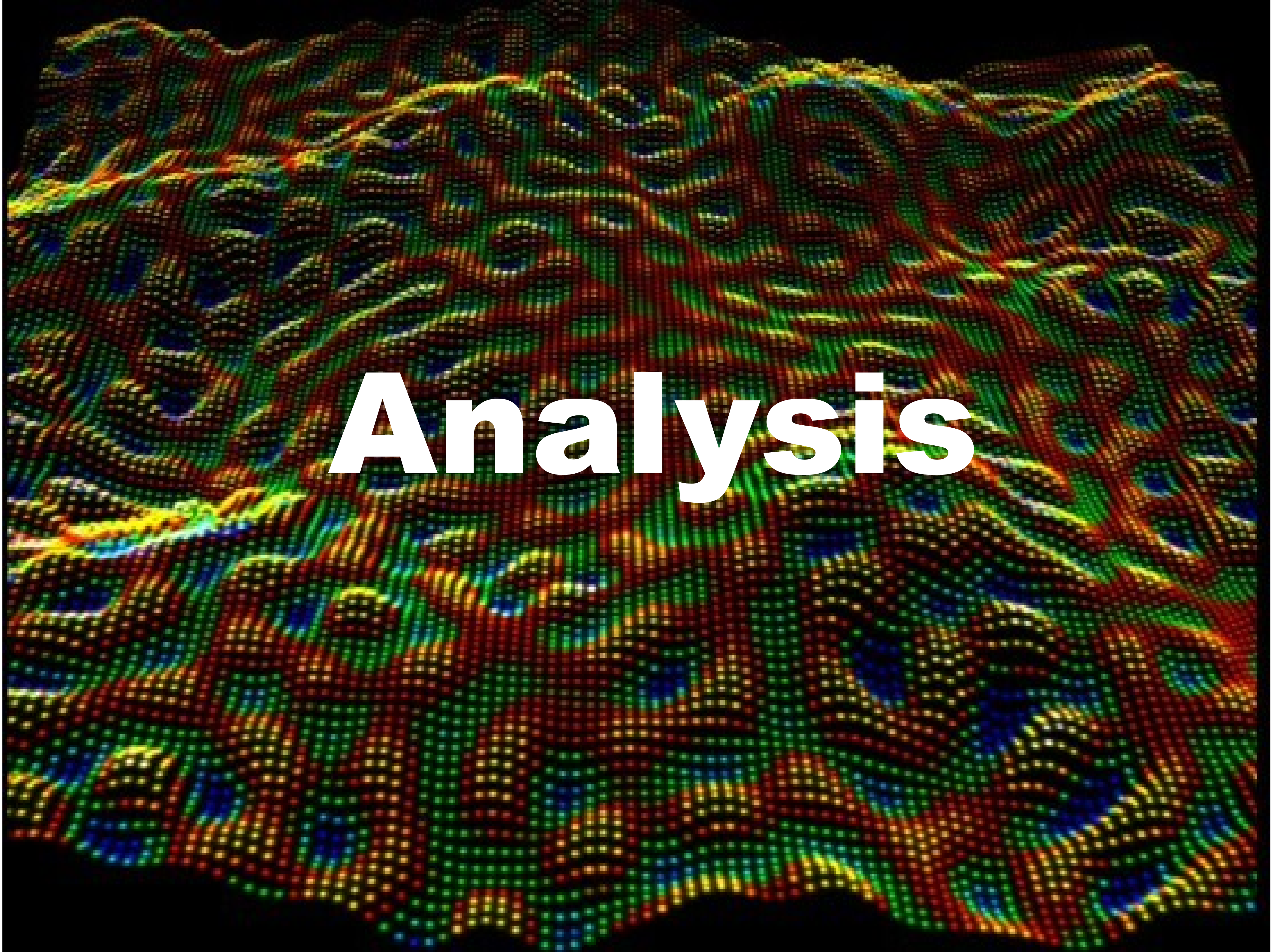
$$ds^2 = e^{-2\frac{z}{L}}\left[-dt^2 + \rho^{-8G_4\mu}(d\rho^2 + \rho^2 d\theta^2)\right] + dz^2 \,.$$

Taking the limit z $\longrightarrow 0$ we get the boundary of 4-d conical AdS space-time :

$$ds^2 = -dt^2 + \rho^{-8G_4\mu}(d\rho^2 + \rho^2 d\theta^2)\,.$$

By Desser, Jackiw and t' Hooft when solving the 2 + 1 Einstein equations with zero cosmological constant in the presence of a point particle of mass M localized at the origin implies the relation

$$M = \frac{G_4}{G_3}\mu\,.$$


Analysis

The Metric Analysis

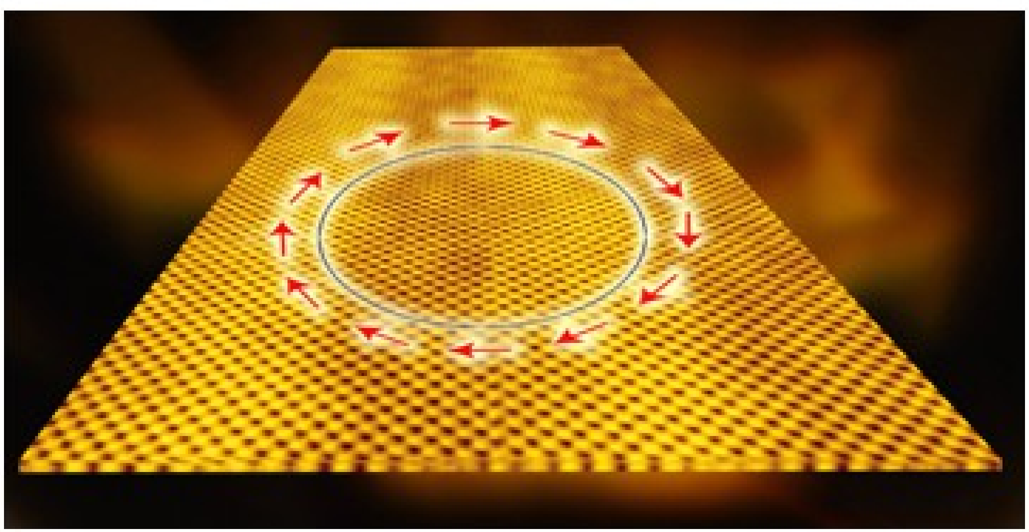
In the limit of small $G_4\mu$ we can approximate the metric by

$$ds^2 = -dt^2 + (1 - 8G_4\mu \ln \rho)\,(d\rho^2 + \rho^2 d\theta^2)\,.$$

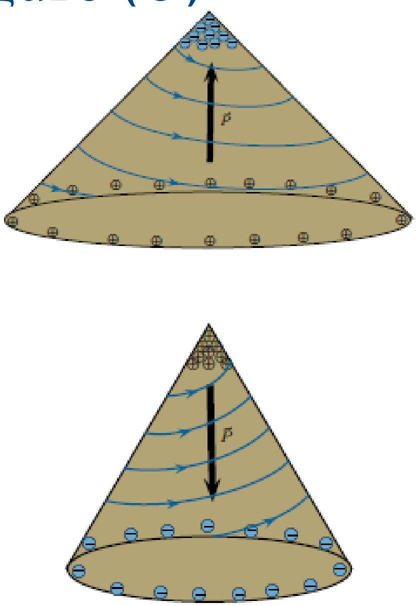
- The logarithmic term is associated with the spatial components of the graviton and is characteristic of 2 + 1 fields.
- This way, solving the 3+1 Einstein equations in the presence of an infinite static string and negative cosmological constant we have found a conical AdS_4 space-time whose boundary can be identified with a 2+1 conical space-time.
- This result suggests a duality between these spaces similar to the usual AdS_4/CFT_3 correspondence.

- In this work we can see that, for some conditions of the Green function, we can describe a local planar topological insulator (a) and it is represented by a Dirac equation in (2+1) that represents the chiral fermions.

- When the material is put into a form of a cone there appears a polarization electrical current as show in the figure (b)



(a)



(b)

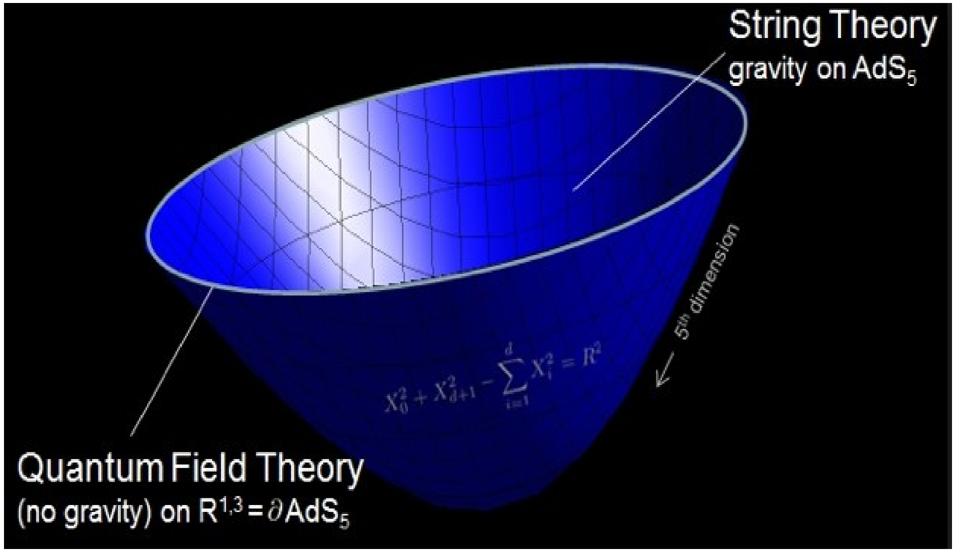
M/String Theory Importance

In theoretical physics, M-theory is an extension of string theory in which the M theory is in 11 dimensions. It is possible, until now, to know only the low energy limit of M-theory. The low energy limit can be described by a supergravity theory in 11 dimensions.

M (11D) → 4D + 7 HD A_{MNO}, g_{MN} $F = dA$

Proponents believe that the 11-dimensional theory unites all five 10 dimensional string theories (10D = 6 HD + 4D) to get the M-theory.

M2 (3,8), M5 (6,5) → Solitonic



Stability M2-defect (3,2,6)

$$ds^2 = e^{2A(y_1,y_2)}(-dt^2 + dx_1^2 + dx_2^2) + e^{2B(y_1,y_2)}(dy_1^2 + dy_2^2) + e^{2C(y_1,y_2)} \sum_{n=1}^{6} dz_n dz_n$$

$$\bar{D}_M \epsilon = 0$$

$$\delta\psi_M = (D_M + A_M^{(1)} + A_M^{(2)})\epsilon = \bar{D}_M \epsilon$$

$$\epsilon = e^{-E/6}\epsilon_0$$

$$D_M \epsilon = \partial_M \epsilon - \frac{1}{4}\omega_M{}^{\hat{A}\hat{B}}\Gamma_{\hat{A}\hat{B}}$$

$$A_M^{(1)} = -\frac{1}{288}\Gamma_M{}^{NOPQ}F_{NOPQ}$$

$$A_M^{(2)} = \frac{1}{36}\Gamma^{OPQ}\delta_M^L F_{LOPQ}$$

$$A = \frac{1}{3}E.$$

$$C = -\frac{1}{6}E.$$

$$B = -\frac{1}{6}E$$

- arXiv:1312.0578

Solutions

$$\delta^{mn}\partial_m\partial_n\, e^{-E(y_1,y_2)} = -16\, G\mu\, \delta^2(y_1,y_2)$$

$$e^{-E(r)} = 1 - 8G\mu \ln(\frac{r}{r_0})$$

$$T^t_t = T^{x_1}_{x_1} = T^{x_2}_{x_2} = 8\, G\mu\, \delta^2(y_1,y_2)e^{\frac{4}{3}E}$$

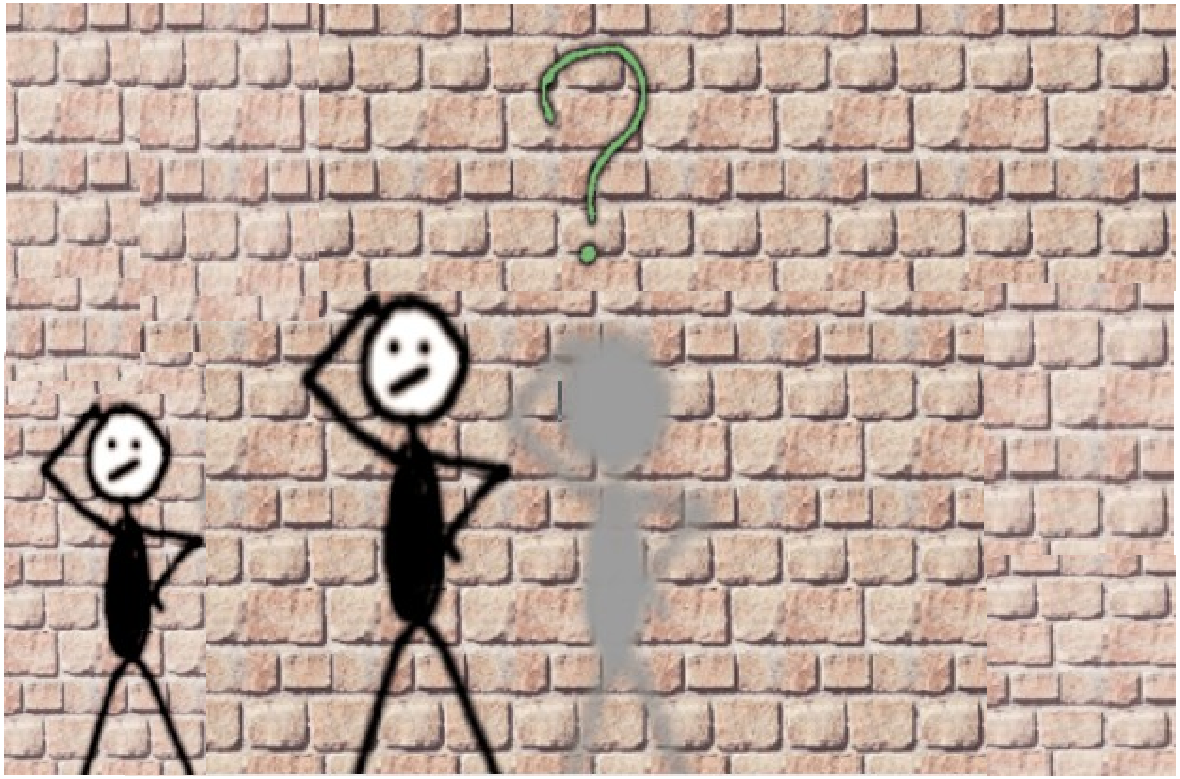
$$T^{y_1}_{y_1} = T^{y_2}_{y_2} = 0$$

$$T^{z_1}_{z_1} = ... = T^{z_6}_{z_6} = 0$$

Conclusions

- Conical defect in 4D give us 3D conical defect that can having application in Graphene;
- The eleven dimensional conical defect with AdS can be obtained with intersecting branes;
- The current in 3D graphene can be analyzed with the probe brane limit. d

Help Me !!!!



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