Fases Geométricas e Violação Espontânea de Simetria Lorentz

Humberto Belich

PROFESSOR ASSOCIADO I - UFES

CBPF, fevereiro de 2015

XII Atividades Formativas de Verão – (Edição 2015)

Componentes:

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Colaboradores:

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J. A. Helayel-Neto. (externo/GFTJLL-CBPF)

Knut Bakke(UFPb), M. Hott, J. Marny(Guará), E. O. Silva(UFMa)

Linhas de Pesquisa

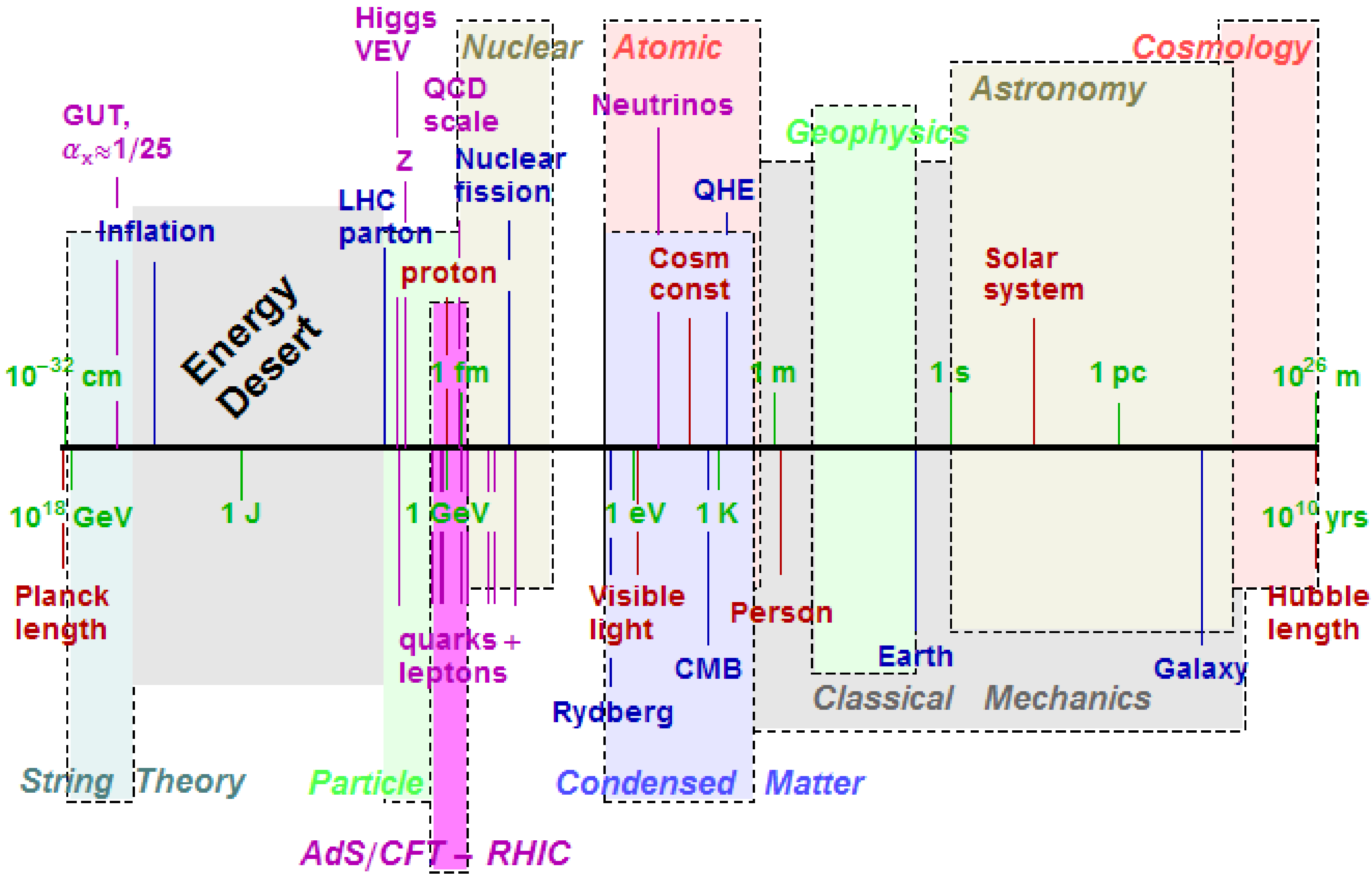
Supercondutividade

Teoria de Campos e Fenômenos Críticos

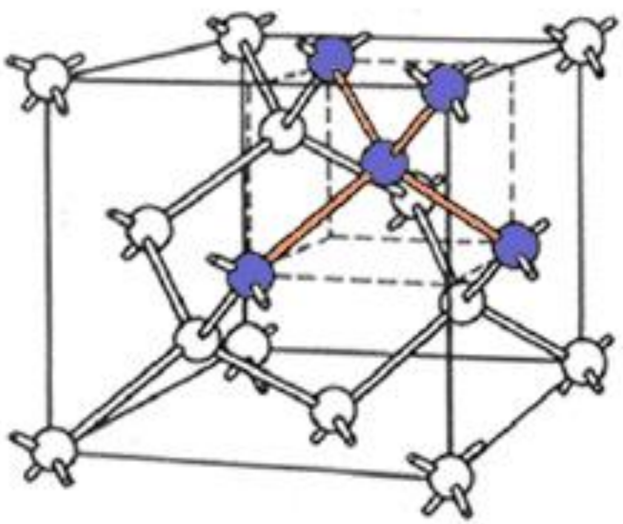
Efeito Casimir em Supercondutores

Materiais e Aplicações (Pressão)

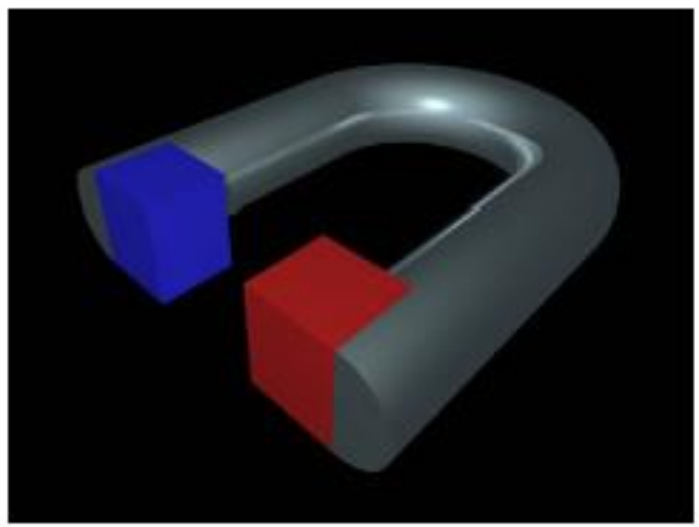
Escalas da Física



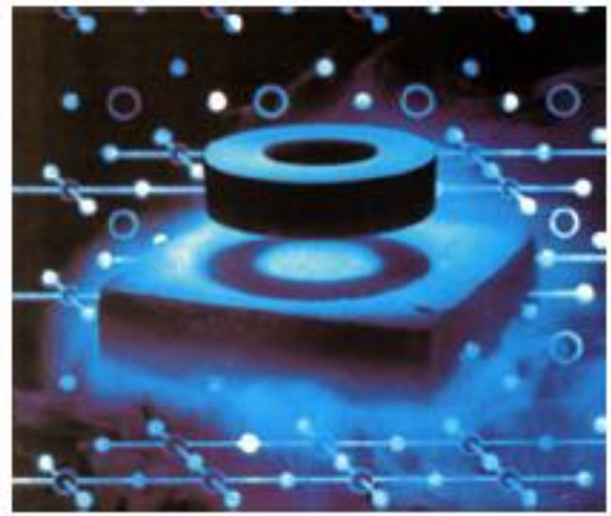
The search for new states of matter



Crystal: Broken translational symmetry



Magnet: Broken rotational symmetry



Superconductor: Broken gauge symmetry

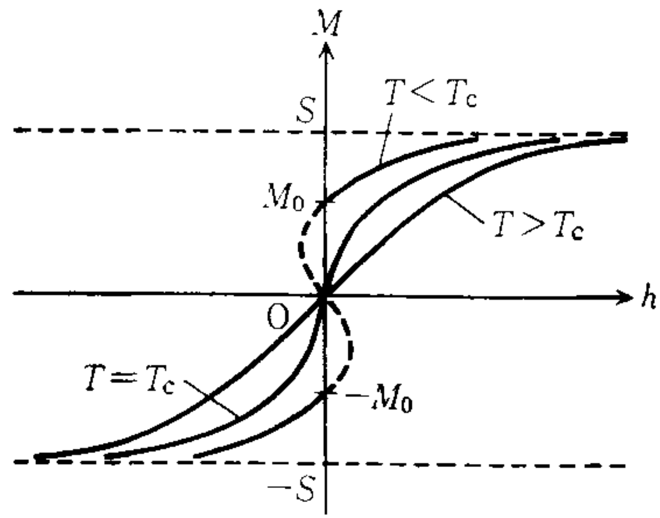
Orders in condensed matter systems

spontaneous symmetry breakdown

$$H = -J \sum_{(i,j)} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \boldsymbol{h} \cdot \sum_i \boldsymbol{S}_i$$

$$Z = \mathrm{tr}\, e^{-\beta H}, \text{ where } \beta = 1/T$$

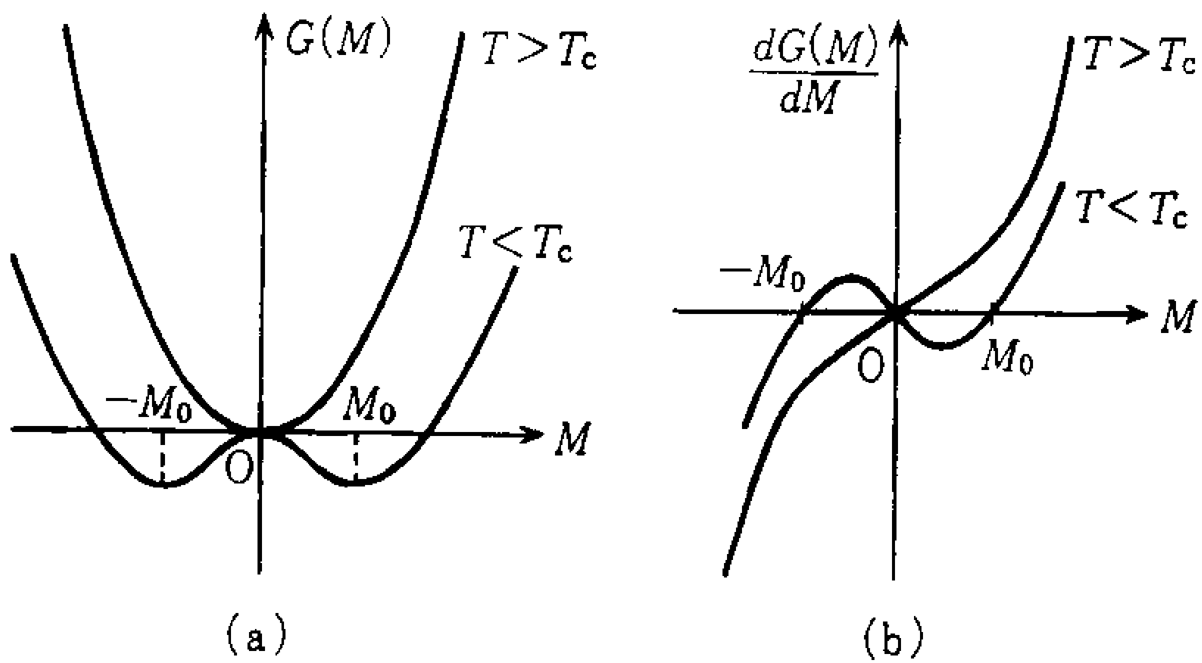
$$\mathrm{M} \equiv \frac{1}{N} \sum_i \langle \boldsymbol{S}_i \rangle = \frac{1}{N\beta} \frac{\partial F}{\partial \boldsymbol{h}}$$



spontaneous magnetization

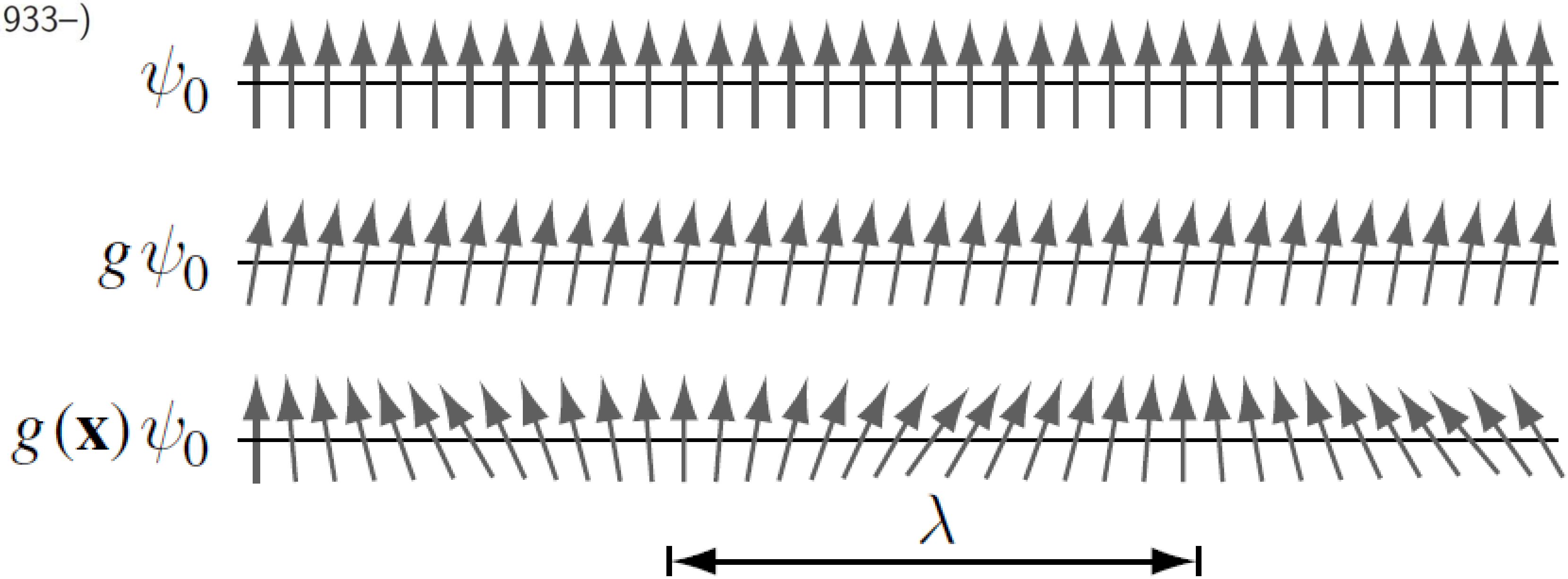
$$G(M) = aM^2 + bM^4 \qquad (b > 0)$$

$$h = \frac{\partial G(M)}{\partial M}$$



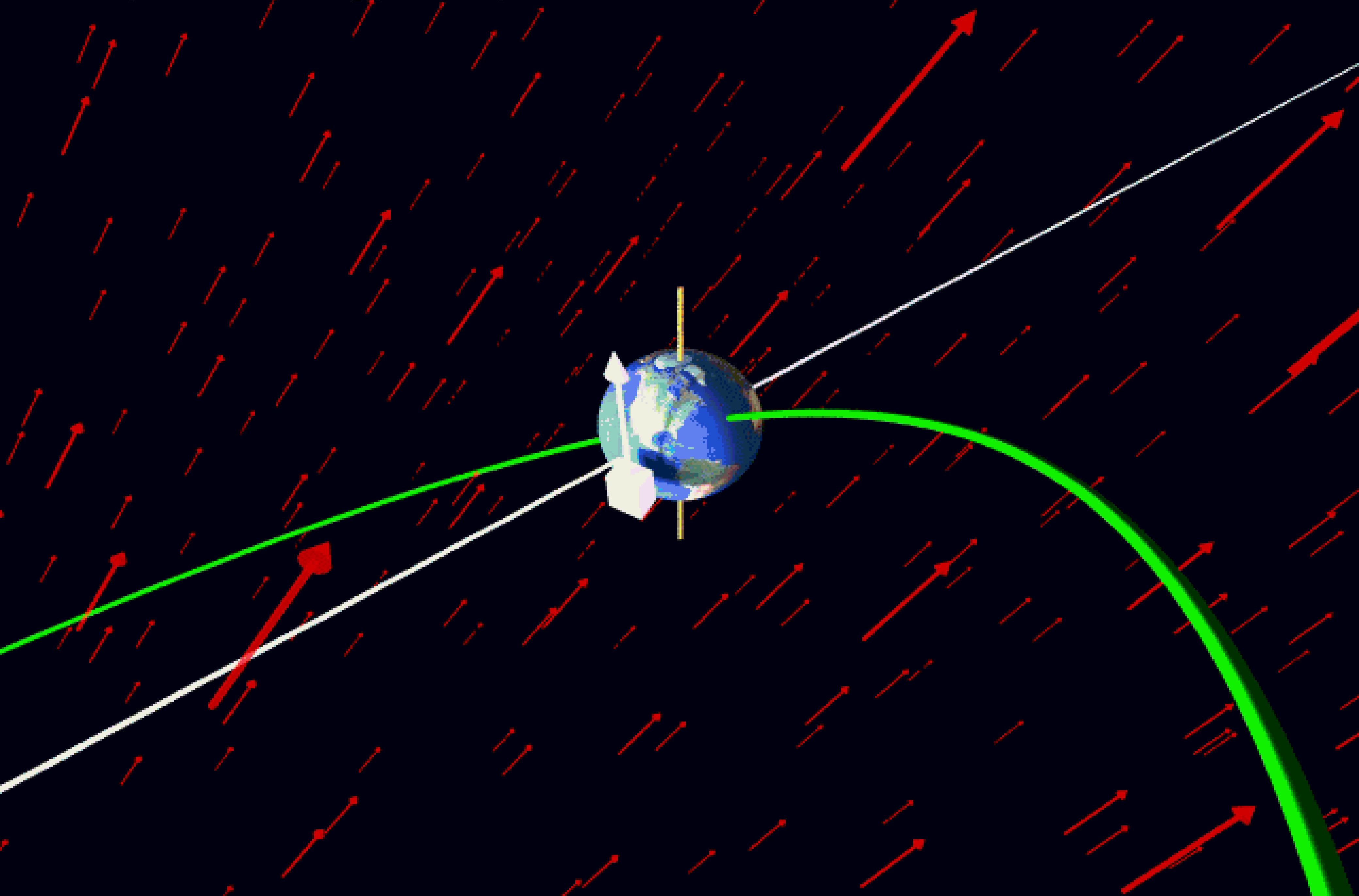
Jeffrey R. Goldstone (1933–)

System	Broken symmetry	Goldstone mode
Magnet	Rotation	Spin wave
Solid	Translation	Phonon
Liquid helium	Global gauge invariance	Phonon
Superconductor	Local gauge invariance	N/A



System	*Broken symmetry*	*Goldstone excitation*
Crystal	Translational	Phonon
Ferromagnet	Rotational	Spin wave
Superfluid	Global gauge	Phonon
Superconductor	Local gauge	(Higgs mode)
Electro-weak[b]	Local gauge	(Higgs mode)
QCD[c]	Chiral	π mesons

Bluhm, Kostelecky, Lane, Russell PRL 2002



Violação Espontânea de Simetria de Lorentz

PHYSICAL REVIEW D VOLUME 39, NUMBER 2 15 JANUARY 1989

Spontaneous breaking of Lorentz symmetry in string theory

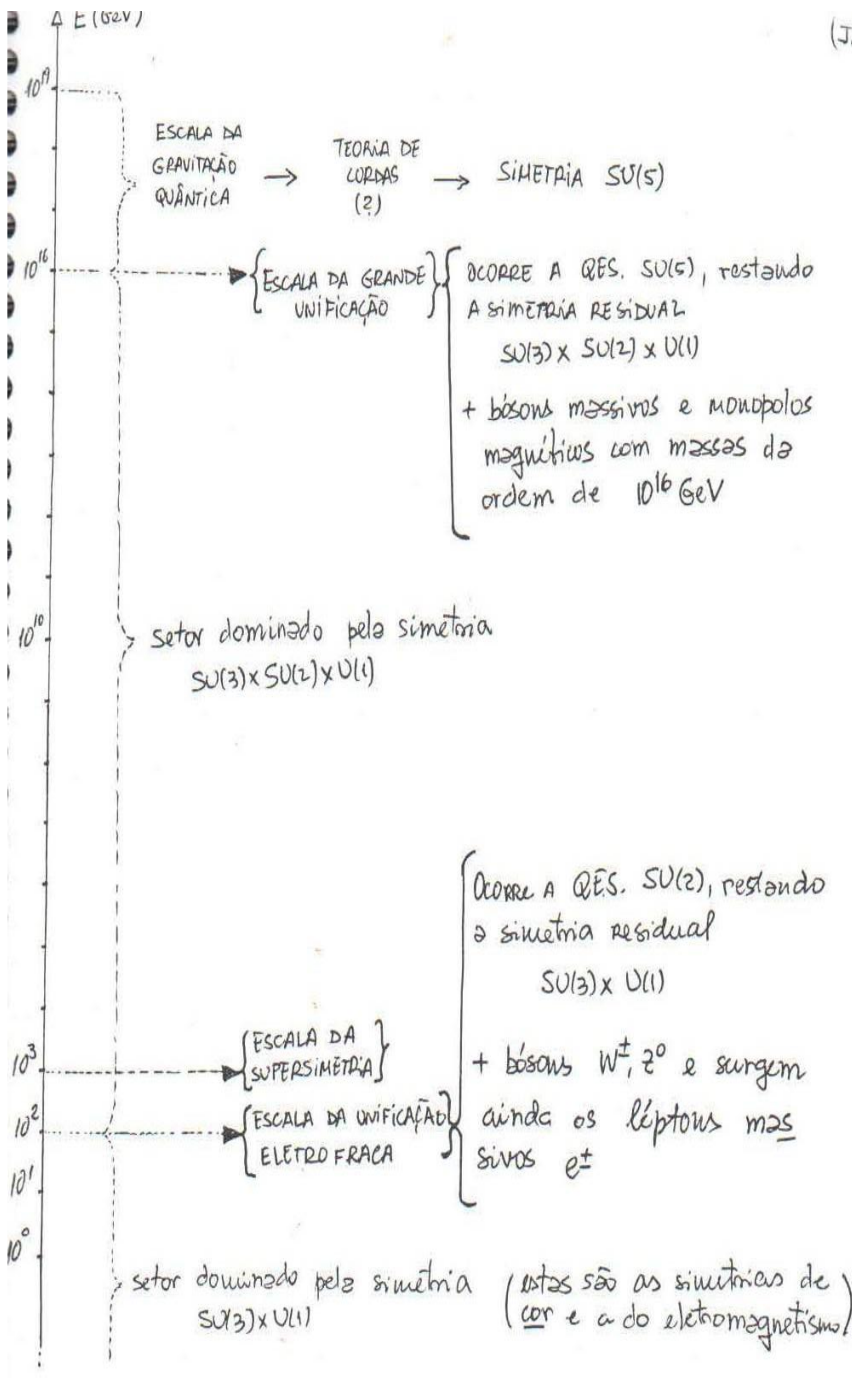
V. Alan Kostelecký
Physics Department, Indiana University, Bloomington, Indiana 47405

Stuart Samuel
Physics Department, City College of New York, New York, New York 10031

(Received 27 June 1988)

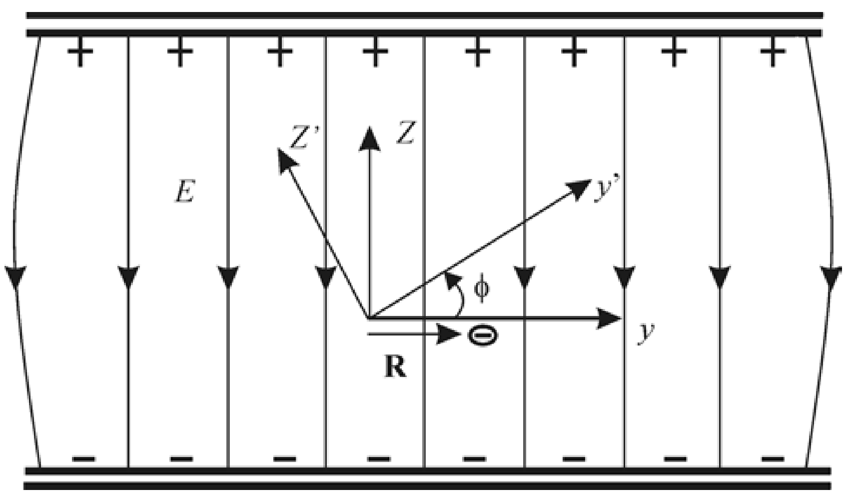
The possibility of spontaneous breakdown of Lorentz symmetry in string theory is explored via covariant string field theory. A potential mechanism is suggested for the Lorentz breaking that may be generic in many string theories.

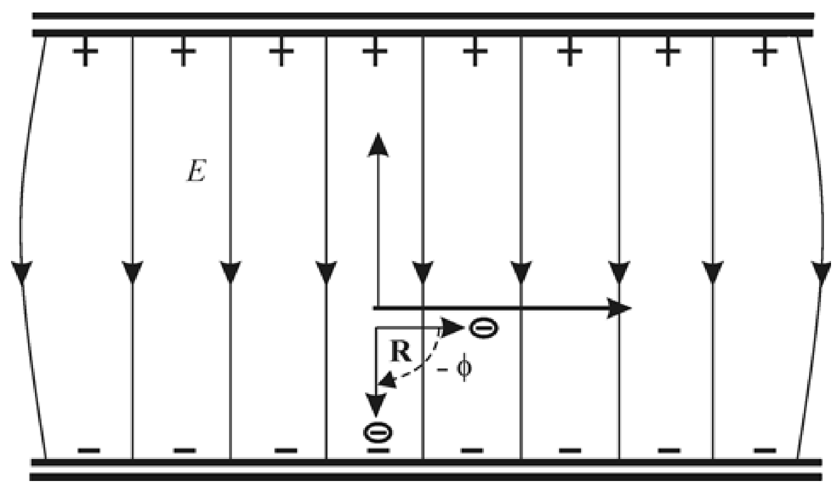
$$\Psi = [\phi(x) + \alpha^{\mu}_{-1} A_{\mu}(x) + \cdots]|0\rangle \, ,$$


E (GeV)
10^{19}
10^{16}
10^{10}
10^{3}
10^{2}
10^{1}
10^{0}
ESCALA DA GRAVITAÇÃO QUÂNTICA → TEORIA DE CORDAS (?) → SIMETRIA SU(5)
ESCALA DA GRANDE UNIFICAÇÃO
OCORRE A QES. SU(5), restando A SIMETRIA RESIDUAL SU(3) x SU(2) x U(1)
+ bósons massivos e monopolos magnéticos com massas da ordem de 10^{16} GeV
setor dominado pela simetria SU(3) x SU(2) x U(1)
ESCALA DA SUPERSIMETRIA
ESCALA DA UNIFICAÇÃO ELETROFRACA
Ocorre A QES. SU(2), restando a simetria residual SU(3) x U(1)
+ bósons $W^{\pm}$, Z^{0} e surgem ainda os léptons massivos $e^{\pm}$
setor dominado pela simetria SU(3) x U(1)
(estas são as simetrias de cor e a do eletromagnetismo)

Violação da simetria de Lorentz

Revista Brasileira de Ensino de Física, v. 29, n. 1, p. 57-64, (2007)

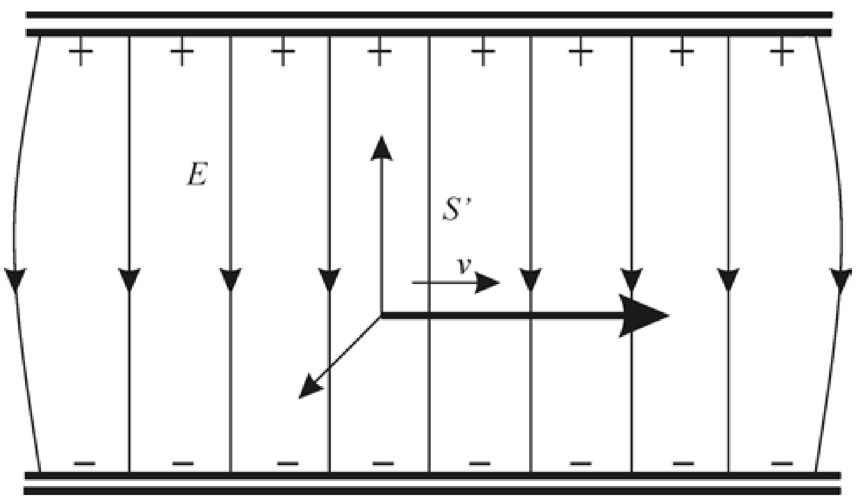


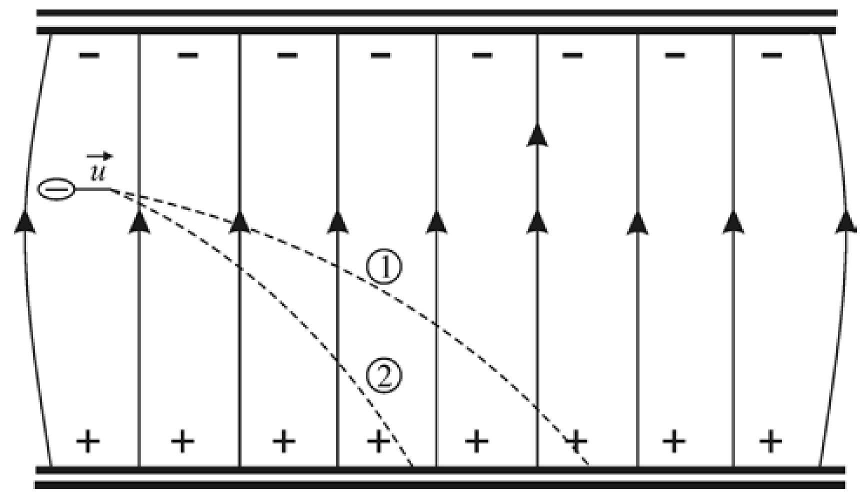


Observable boost

Particle boost

$$\mathbf{E}' = \frac{\mathbf{E}}{\sqrt{1-\frac{v^2}{c^2}}}, \qquad x'(t) = \sqrt{1-\frac{v^2}{c^2}}\,x(t).$$





Violação Espontânea de Simetria de Lorentz

S. M. Carroll, G. B. Field, R. Jackiw; PHYS. REV. D, VOL. 41, N. 4
15 FEBRUARY 1990

$$\mathcal{L}_p = \mathcal{L}_{\rm EM} + \mathcal{L}_{\rm CS}$$

$$\mathcal{L}_{\rm CS} = -\tfrac{1}{2} p_\alpha A_\beta \widetilde{F}^{\alpha\beta} \ ,$$

$$\partial_\mu F^{\mu\nu} = 4\pi J^\nu + p_\mu \widetilde{F}^{\mu\nu}$$

in terms of components,

$$\nabla \cdot \mathbf{E} = 4\pi\rho - \mathbf{p}\cdot\mathbf{B} \ ,$$

$$-\partial_t \mathbf{E} + \nabla \times \mathbf{B} = 4\pi \mathbf{J} - p_0 \mathbf{B} + \mathbf{p} \times \mathbf{E} \ .$$

Preserva invariância de Gauge
Apresenta birrefringência
Viola simetria CPT

Violação Espontânea de Simetria de Lorentz

Extensão do Modelo Padrão com violação de sim. Lorentz - Colladay & Kostelecky [PRD 55,6760 (1997); PRD 58, 116002 (1998).]

Violação Espontânea de Simetria de Lorentz

Extensão do Modelo Padrão com violação de sim. Lorentz - Colladay & Kostelecky [PRD 55,6760 (1997); PRD 58, 116002 (1998).]

SETOR FERMIÔNICO

The general form of the relativistic Lagrangian for a free spin-$\frac{1}{2}$ Dirac fermion ψ of mass m in the standard-model extension is[2]

$$\mathcal{L}=\tfrac{1}{2}i\overline{\psi}(\gamma_\nu+c_{\mu\nu}\gamma^\mu+d_{\mu\nu}\gamma_5\gamma^\mu+e_\nu+if_\nu\gamma_5+\tfrac{1}{2}g_{\lambda\mu\nu}\sigma^{\lambda\mu})\overleftrightarrow{\partial}^\nu\psi$$
$$-\overline{\psi}(m+a_\mu\gamma^\mu+b_\mu\gamma_5\gamma^\mu+\tfrac{1}{2}H_{\mu\nu}\sigma^{\mu\nu})\psi. \qquad (1)$$

Violação Espontânea de Simetria de Lorentz

Extensão do Modelo Padrão com violação de sim. Lorentz - Colladay & Kostelecky [PRD 55,6760 (1997); PRD 58, 116002 (1998).]

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JOURNAL OF MATHEMATICAL PHYSICS VOLUME 40, NUMBER 12 DECEMBER 1999

Nonrelativistic quantum Hamiltonian for Lorentz violation

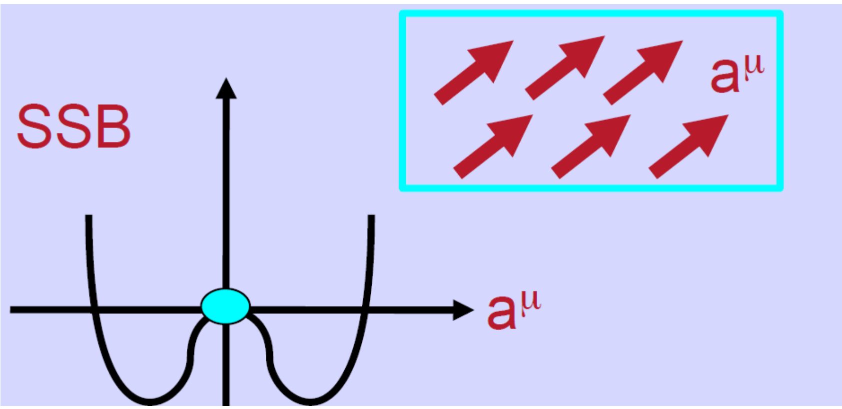
V. Alan Kostelecký and Charles D. Lane
Physics Department, Indiana University, Bloomington, Indiana 47405

(Received 18 May 1999; accepted for publication 30 August 1999)

Fermionic Sector

Non-minimal coupling to the gauge field and background

$$L = i\overline{\Psi}\gamma_{\mu}\partial^{\mu}\Psi - m\overline{\Psi}\Psi$$



$$L = i\overline{\Psi}\gamma_{\mu}\partial^{\mu}\Psi - m\overline{\Psi}\Psi + \overline{\Psi}\gamma_{\mu}a^{\mu}\Psi$$

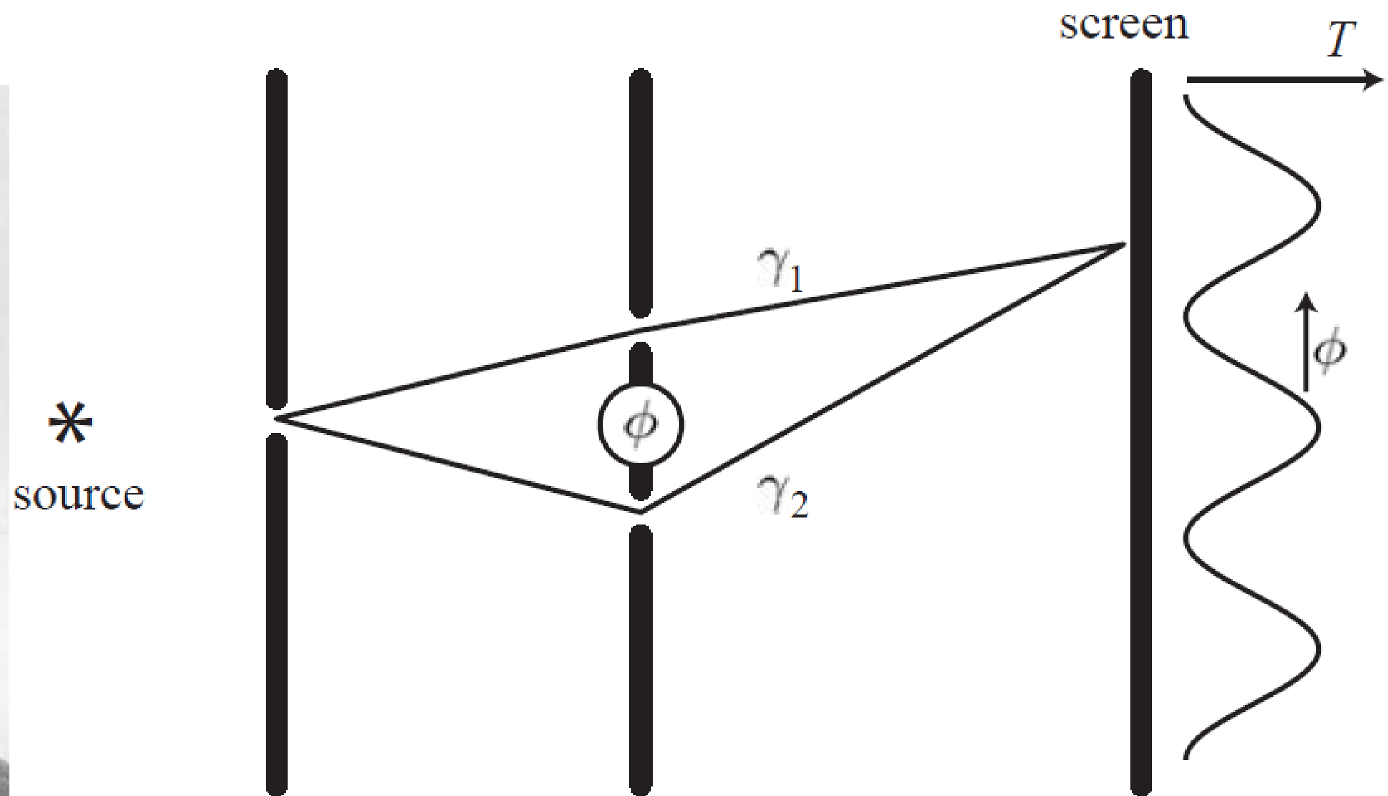
Setor de Gauge

$$\mathcal{L}_{gauge} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}(K_{AF})^{\mu}\epsilon_{\mu\nu\alpha\beta}A^{\nu}F^{\alpha\beta} - \frac{1}{4}(K_F)_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta},$$

$$\frac{1}{4}\epsilon^{\mu\nu\kappa\lambda}(k_{AF})_{\mu}A_{\nu}F_{\kappa\lambda},$$ **CPT ímpar**

$$S = -\frac{1}{4}\int d^4x\, K_{\mu\nu\kappa\lambda}F^{\mu\nu}F^{\kappa\lambda}$$ **CPT par**

Aharonov–Bohm experiment

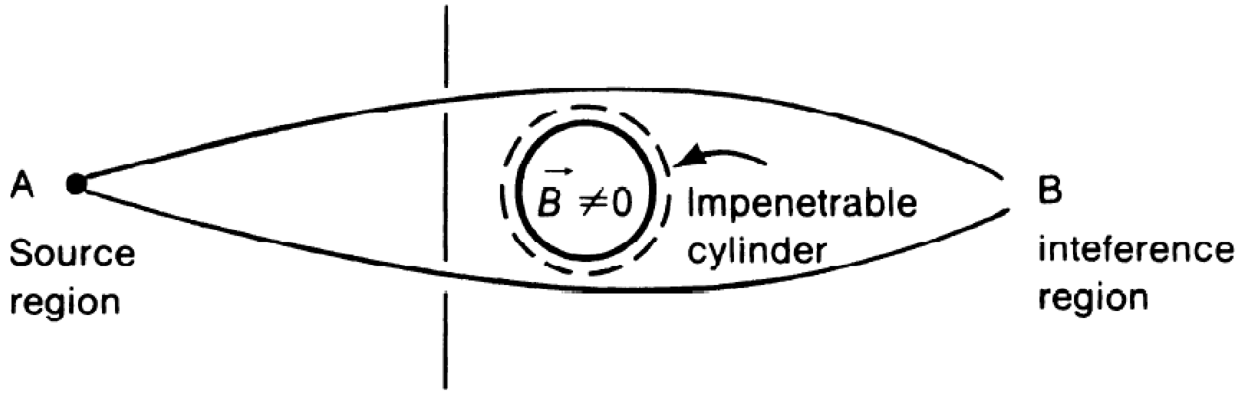


$$\Delta\varphi_{\mathrm{AB}} = -\frac{|e|}{\hbar}\oint \mathbf{A}\,ds$$

The Aharonov-Bohm Effect

$$L^{(0)}_{\text{classical}} = \frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^2 \rightarrow L^{(0)}_{\text{classical}} + \frac{e}{c}\frac{d\mathbf{x}}{dt}\cdot\mathbf{A}$$

$$\prod \exp\left[\frac{iS^{(0)}(n,n-1)}{\hbar}\right] \rightarrow \left\{\prod \exp\left[\frac{iS^{(0)}(n,n-1)}{\hbar}\right]\right\}\exp\left(\frac{ie}{\hbar c}\int_{\mathbf{x}_1}^{\mathbf{x}_N}\mathbf{A}\cdot d\mathbf{s}\right)$$



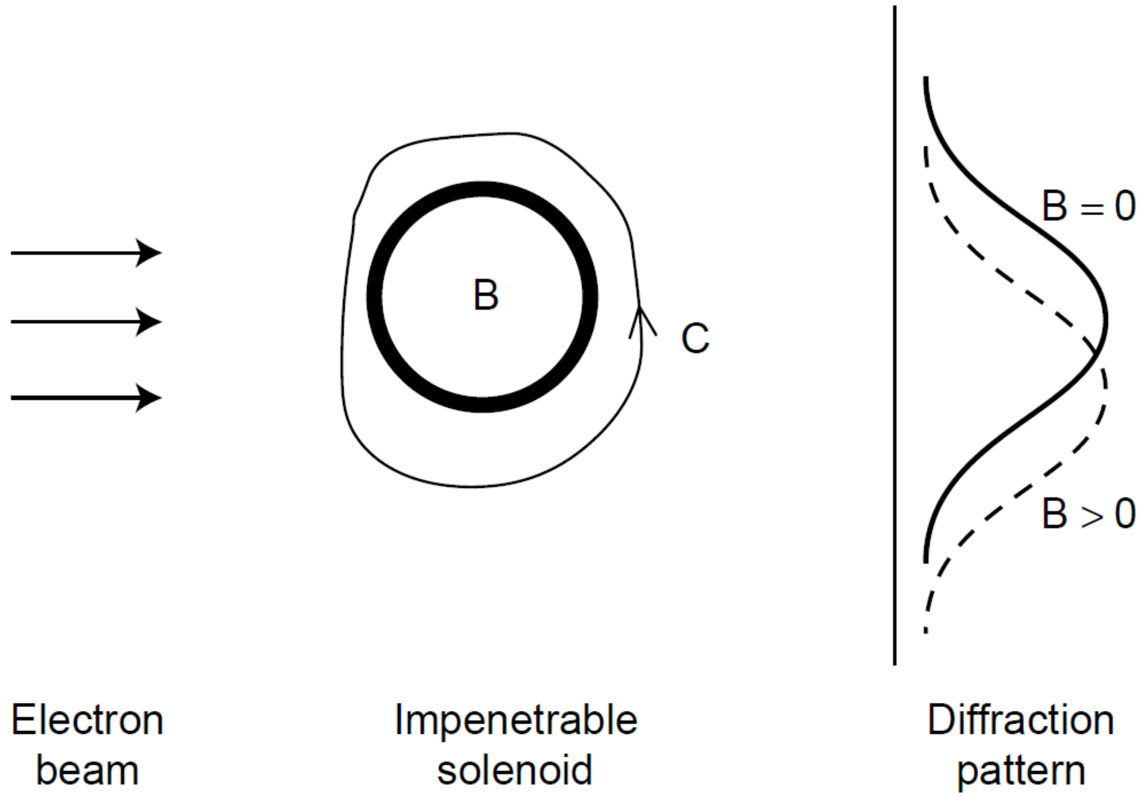
$$\left[\left(\frac{e}{\hbar c}\right)\int_{\mathbf{x}_1}^{\mathbf{x}_N}\mathbf{A}\cdot d\mathbf{s}\right]_{\text{above}} - \left[\left(\frac{e}{\hbar c}\right)\int_{\mathbf{x}_1}^{\mathbf{x}_N}\mathbf{A}\cdot d\mathbf{s}\right]_{\text{below}} = \left(\frac{e}{\hbar c}\right)\oint \mathbf{A}\cdot d\mathbf{s}$$

$$= \left(\frac{e}{\hbar c}\right)\Phi_B,$$

$$\rightarrow \int_{\text{above}} \mathscr{D}[\mathbf{x}(t)] \exp\left[\frac{iS^{(0)}(N,1)}{\hbar}\right] \left\{ \exp\left[\left(\frac{ie}{\hbar c}\right) \int_{\mathbf{x}_1}^{\mathbf{x}_N} \mathbf{A} \cdot d\mathbf{s}\right]_{\text{above}} \right\}$$

$$+ \int_{\text{below}} \mathscr{D}[\mathbf{x}(t)] \exp\left[\frac{iS^{(0)}(N,1)}{\hbar}\right] \left\{ \exp\left[\left(\frac{ie}{\hbar c}\right) \int_{\mathbf{x}_1}^{\mathbf{x}_N} \mathbf{A} \cdot d\mathbf{s}\right]_{\text{below}} \right\}$$

$$\oint \mathbf{A} \cdot d\mathbf{x}$$



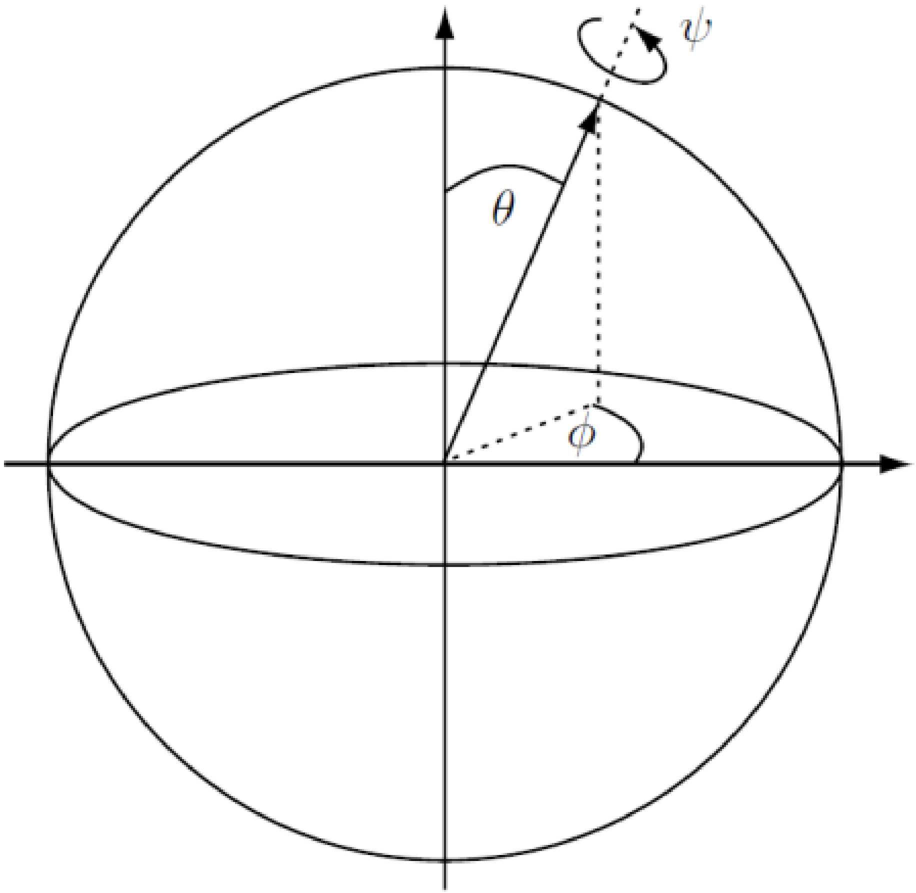
The Aharonov–Bohm phase

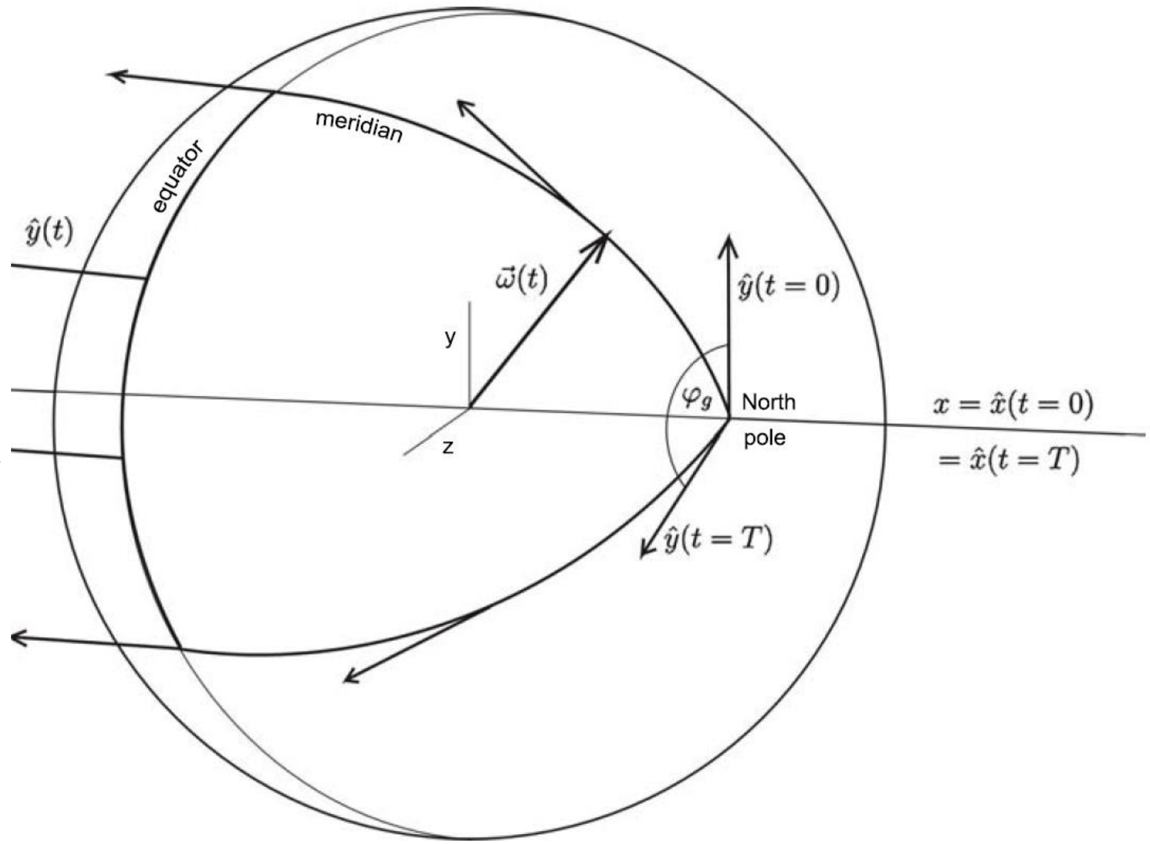
$$\mathbf{p} \longrightarrow \mathbf{p} + |e|\mathbf{A} \qquad \Delta\varphi_{\mathrm{AB}} = -\frac{|e|}{\hbar}\oint \mathbf{A}\,d\mathbf{s}$$

Aharonov–Casher phase

$$\mathbf{p} = m\mathbf{v} + \frac{1}{c}\boldsymbol{\mu} \times \mathbf{E}$$

$$\Delta\varphi_{\mathrm{AC}} = \frac{1}{c}\oint \boldsymbol{\mu} \times \mathbf{E}\,d\mathbf{s}$$


θ
ψ
φ


meridian
equator
$\hat{y}(t)$
$\vec{\omega}(t)$
$\hat{y}(t=0)$
y
z
φ_g
North
pole
$x = \hat{x}(t=0)$
$= \hat{x}(t=T)$
$\hat{y}(t=T)$

Berry phase

Schrödinger equation $H(x(t))|\psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle$

We can choose a basis of eigenstates $|n(x(t))\rangle$

$$H(x(t))|n(x(t))\rangle = E_n(x(t))|n(x(t))\rangle$$

in the n-th energy eigenstate,

$$|\psi(0)\rangle = |n(x(0))\rangle$$

$$|\psi(t)\rangle = e^{i\phi_n}|n(x(t))\rangle$$

$$\phi_n = \theta_n + \gamma_n$$

$$\theta_n(t) = -\frac{1}{\hbar}\int_0^t E_n(\tau)d\tau$$

Berry phase

Schrödinger equation $$H(x(t))|\psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle$$

We can choose a basis of eigenstates $|n(x(t))\rangle$ in the n-th energy eigenstate,

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$$\theta_n(t) = -\frac{1}{\hbar}\int_0^t E_n(\tau)d\tau$$

$$H(x(t))|n(x(t))\rangle = E_n(x(t))|n(x(t))\rangle$$

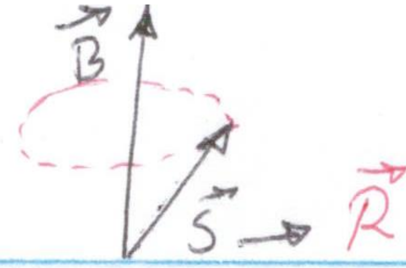
$$\frac{\partial}{\partial t}|n(x)\rangle + i\frac{d}{dt}\gamma_n(t)|n(x)\rangle = 0$$

$$\frac{d}{dt}\gamma_n(t) = i\langle n(x)|\frac{\partial}{\partial t}|n(x)\rangle = i\langle n|\nabla|n\rangle\frac{dx}{dt}$$

when we are given a cyclic evolution,

$$\gamma_n(C) = i\oint_C \langle n(x)|\nabla|n(x)\rangle dx$$

Fases de Berry



$$\hat{H}(\vec{R}(t)), \; |\vec{n}(\vec{R}(t))\rangle$$

$$|\Psi_n(t)\rangle = e^{i\gamma(t) - \frac{i}{\hbar}\int_0^t dt' \, \mathcal{E}_n(R(t'))} \, |n(\vec{R}(t))\rangle \, ;$$

$$i\hbar \frac{d}{dt}|\Psi_n(t)\rangle = \hat{H}(\vec{R}(t))|\Psi_n(t)\rangle$$

$$\frac{d}{dt}|\Psi_n(t)\rangle = \Big(i\frac{d\gamma}{dt}|\Psi_n(t)\rangle - \frac{i}{\hbar}\mathcal{E}_n|\Psi_n(t)\rangle + e^{i\gamma(t) - \frac{i}{\hbar}\int\cdots} \frac{d}{dt}|\vec{n}\rangle \, ;$$

$$i\hbar \frac{d}{dt}|\Psi_n(t)\rangle = \Big(-\hbar\frac{d\gamma}{dt}|\Psi_n(t)\rangle + \mathcal{E}_n|\Psi_n(t)\rangle + e^{i\gamma(t)+\cdots} \, i\hbar\frac{d}{dt}|n\rangle = \mathcal{E}_n|\Psi_n\rangle$$

$$\downarrow$$

$$\langle n|R(t)| \times (\quad)$$

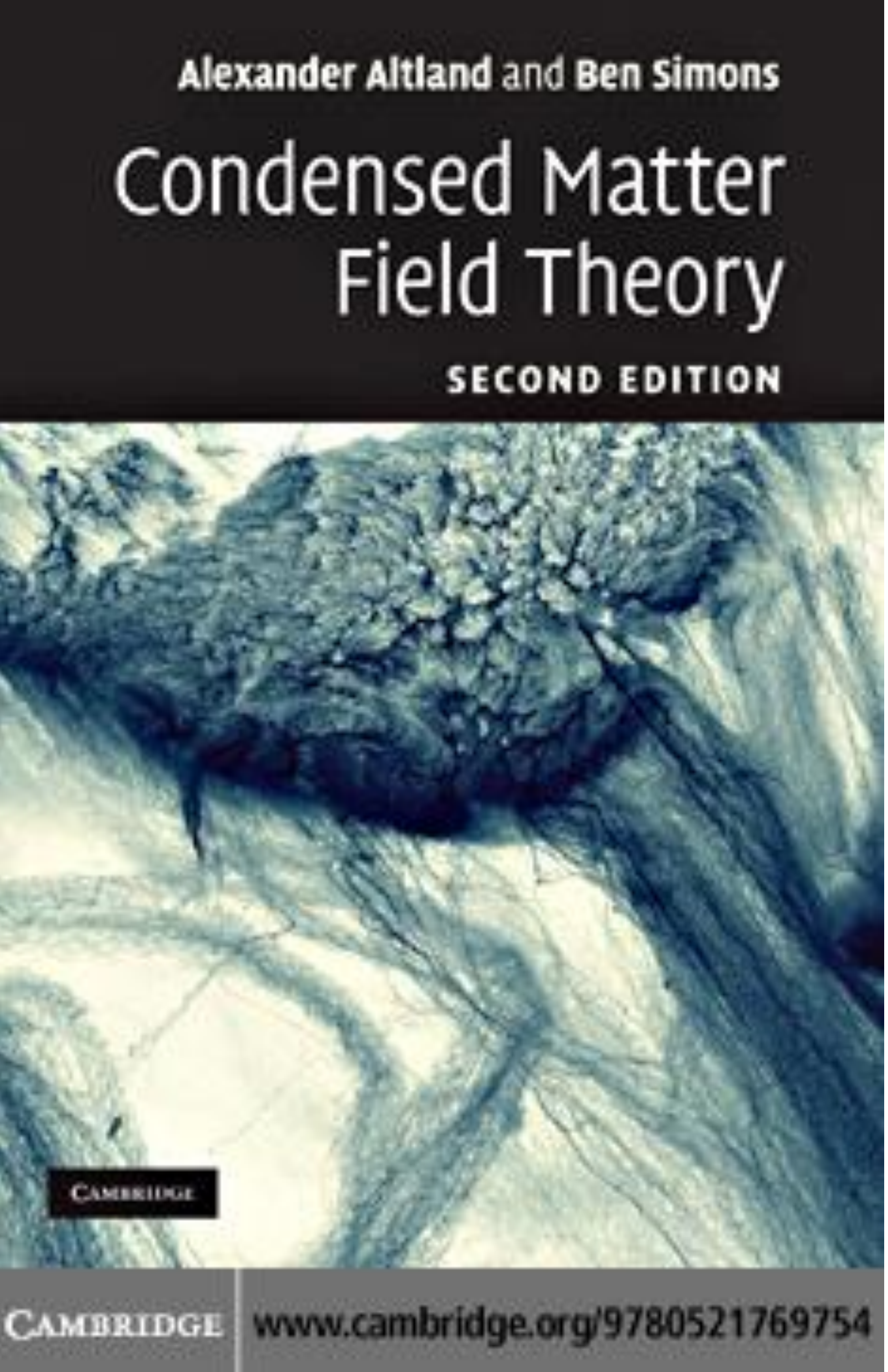
$$\Rightarrow \quad -\hbar \frac{d\gamma}{dt} e^{i\gamma(t) - \frac{i}{\hbar}\int_0^t + \dots} + i\hbar\, e^{i\gamma(t) + \dots} \langle \vec{m}(\vec{R}(t)) | \frac{d}{dt} | \vec{m}(\vec{R}(t)) \rangle = 0$$

$$\Rightarrow \frac{d\gamma}{dt} = i \langle m(\vec{R}(t)) | \frac{d}{dt} | m(\vec{R}(t)) \rangle$$

$$\gamma = \int_0^t i \langle m(\vec{R}(t)) | \frac{d}{dt} | m(R(t)) \rangle \, dt = \int_{R(0)}^{R(t)} i \langle m(\vec{R}(t)) | \frac{\partial}{\partial R^i} | m(\vec{R}(t)) \rangle dR^i$$

$$= \int_{R(0)}^{R(t)} A_i^m(R)\, dR^i ,$$

$$\therefore \boxed{\vec{A}^m = i \langle m(\vec{R}(t)) | \vec{\nabla} | m(\vec{R}(t)) \rangle}$$



Path integral for spin 134

A reminder of finite-dimensional SU(2)-*representation theory*

$$\mathrm{SU}(2) = \{g \in \mathrm{Mat}(2\times 2, \mathbb{C}) | g^\dagger g = \mathbf{1}_2, \det g = 1\},$$

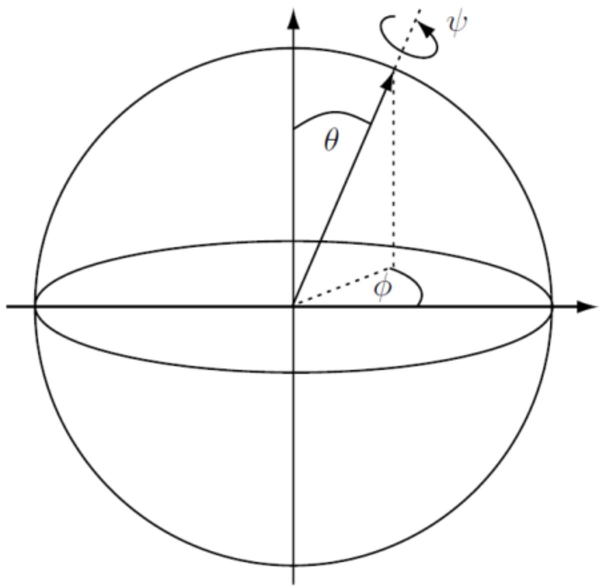
$$[\hat{S}^i, \hat{S}^j] = i\epsilon_{ijk}\hat{S}^k, \qquad \hat{S}^\pm = \hat{S}^x \pm i\hat{S}^y$$

$$[\hat{S}^+, \hat{S}^-] = 2\hat{S}^z, \; [\hat{S}^z, \hat{S}^\pm] = \pm\hat{S}^\pm.$$

the **Euler angle representation**,

$$\mathrm{SU}(2) = \left\{ g(\phi,\theta,\psi) = e^{-i\phi\hat{S}_3} e^{-i\theta\hat{S}_2} e^{-i\psi\hat{S}_3} \,\middle|\, \phi,\psi \in [0,2\pi], \theta \in [0,\pi] \right\} .$$

let us consider a particle of spin *S* subject to the Hamiltonian



$$\hat{H} = \mathbf{B}\cdot\hat{\mathbf{S}},$$

$\mathbf{B}$ is a magnetic field

$$\hat{\mathbf{S}} \equiv (\hat{S}_1, \hat{S}_2, \hat{S}_3)$$

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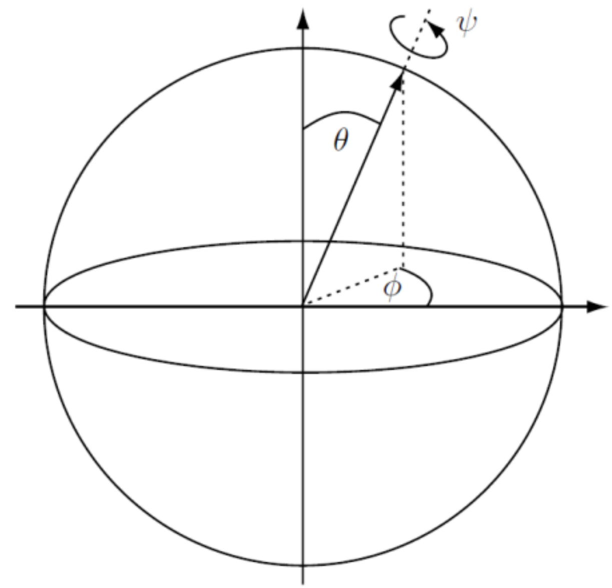
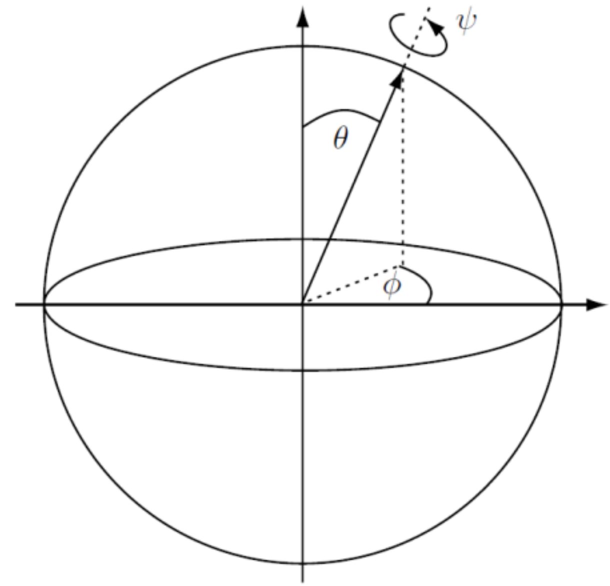


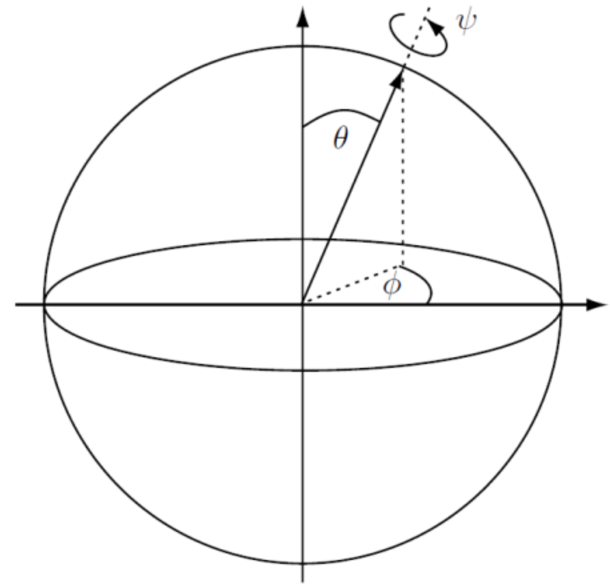
$$\begin{aligned} \langle g_{i+1}|e^{-\epsilon \mathbf{B}\cdot\hat{\mathbf{S}}}|g_i\rangle &\simeq \langle g_{i+1}|g_i\rangle - \epsilon\langle g_{i+1}|\mathbf{B}\cdot\hat{\mathbf{S}}|g_i\rangle \overset{\langle g_i|g_i\rangle=1}{=} 1 - \langle g_i|g_i\rangle + \langle g_{i+1}|g_i\rangle - \epsilon\langle g_{i+1}|\mathbf{B}\cdot\hat{\mathbf{S}}|g_i\rangle \\ &\simeq \exp\Big(\langle g_{i+1}|g_i\rangle - \langle g_i|g_i\rangle - \epsilon\langle g_{i+1}|\mathbf{B}\cdot\hat{\mathbf{S}}|g_i\rangle\Big) , \end{aligned}$$

$$\langle g_{i+1}|e^{-\epsilon \mathbf{B}\cdot\hat{\mathbf{S}}}|g_i\rangle \simeq \langle g_{i+1}|g_i\rangle - \epsilon\langle g_{i+1}|\mathbf{B}\cdot\hat{\mathbf{S}}|g_i\rangle \overset{\langle g_i|g_i\rangle=1}{=} 1 - \langle g_i|g_i\rangle + \langle g_{i+1}|g_i\rangle - \epsilon\langle g_{i+1}|\mathbf{B}\cdot\hat{\mathbf{S}}|g_i\rangle$$
$$\simeq \exp\left(\langle g_{i+1}|g_i\rangle - \langle g_i|g_i\rangle - \epsilon\langle g_{i+1}|\mathbf{B}\cdot\hat{\mathbf{S}}|g_i\rangle\right),$$



$$\mathcal{Z} = \lim_{N\to\infty}\int_{g_N=g_0}\prod_{i=0}^{N} dg_i \exp\left[-\epsilon\sum_{i=0}^{N-1}\left(-\frac{\langle g_{i+1}|g_i\rangle - \langle g_i|g_i\rangle}{\epsilon} + \langle g_{i+1}|\mathbf{B}\cdot\hat{\mathbf{S}}|g_i\rangle\right)\right].$$

$$\begin{aligned}\langle g_{i+1}|e^{-\epsilon \mathbf{B}\cdot\hat{\mathbf{S}}}|g_i\rangle &\simeq \langle g_{i+1}|g_i\rangle - \epsilon\langle g_{i+1}|\mathbf{B}\cdot\hat{\mathbf{S}}|g_i\rangle \overset{\langle g_i|g_i\rangle=1}{=} 1 - \langle g_i|g_i\rangle + \langle g_{i+1}|g_i\rangle - \epsilon\langle g_{i+1}|\mathbf{B}\cdot\hat{\mathbf{S}}|g_i\rangle \\ &\simeq \exp\left(\langle g_{i+1}|g_i\rangle - \langle g_i|g_i\rangle - \epsilon\langle g_{i+1}|\mathbf{B}\cdot\hat{\mathbf{S}}|g_i\rangle\right),\end{aligned}$$



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$$\boxed{\mathcal{Z} = \int Dg\, \exp\left[-\int_0^\beta d\tau\left(-\langle\partial_\tau g|g\rangle + \langle g|\mathbf{B}\cdot\hat{\mathbf{S}}|g\rangle\right)\right],}$$

spin coherent states

$$|\tilde{g}(\phi,\theta,\psi)\rangle \equiv e^{-i\phi\hat{S}_3} e^{-i\theta\hat{S}_2} e^{-i\psi\hat{S}_3}|\uparrow\rangle,$$

http://www.professorglobal.com.br/fisica-pos-graduacao/teoria-de-grupos/videos

Teoria de Grupos (2012 - 2013)

Aula 12. Tensores e representações de SO(3) e SU(2)

$$|\tilde{g}(\phi,\theta,\psi)\rangle \equiv e^{-i\phi\hat{S}_3} e^{-i\theta\hat{S}_2}|\uparrow\rangle e^{-i\psi S}$$

$\phi \in [0, 2\pi)$ and $\theta \in [0, \pi)$

$$n_i \equiv \langle\tilde{g}(\phi,\theta,\psi)|\hat{S}_i|\tilde{g}(\phi,\theta,\psi)\rangle, \quad i = 1, 2, 3.$$

$$e^{-i\phi\hat{S}_i}\hat{S}_j e^{i\phi\hat{S}_i} = e^{-i\phi[\hat{S}_i,\ ]}\ \hat{S}_j = \hat{S}_j\cos\phi + \epsilon_{ijk}\hat{S}_k\sin\phi,$$

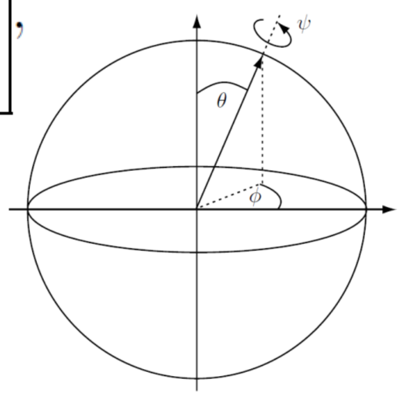
$$\mathbf{n} = S(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta),$$

$$S_B[\phi,\theta] \equiv \int_0^\beta d\tau \ \langle\tilde{g}|\mathbf{B}\cdot\hat{\mathbf{S}}|\tilde{g}\rangle = \int_0^\beta d\tau \ \langle g|\mathbf{B}\cdot\hat{\mathbf{S}}|g\rangle = S\int_0^\beta d\tau \ \mathbf{n}\cdot\mathbf{B} = SB\int_0^\beta d\tau \ \cos\theta.$$

$$\begin{aligned} S_{\mathrm{top}}[\phi,\theta] &\equiv -\int_0^\beta d\tau \ \langle\partial_\tau\tilde{g}|\tilde{g}\rangle = -\int_0^\beta d\tau \ \langle\partial_\tau e^{-iS\psi}g|ge^{-iS\psi}\rangle \\ &= -\int_0^\beta d\tau \left(\langle\partial_\tau g|g\rangle - iS\partial_\tau\psi\langle g|g\rangle\right) = -\int_0^\beta d\tau \ \langle\partial_\tau g|g\rangle, \end{aligned}$$

$$S_{\mathrm{top}}[\phi,\theta] = -\int_0^\beta d\tau \ \langle\partial_\tau g|g\rangle = -iS\int_0^\beta d\tau \ \partial_\tau\phi\cos\theta = iS\int_0^\beta d\tau \ \partial_\tau\phi(1-\cos\theta).$$

$$\boxed{S[\theta,\phi] = S_B[\phi,\theta] + S_{\mathrm{top}}[\phi,\theta] = S\int_0^\beta d\tau \left[B\cos\theta + i(1-\cos\theta)\partial_\tau\phi\right]},$$



The Aharonov–Bohm phase

$$\mathbf{p} \longrightarrow \mathbf{p} + |e|\mathbf{A}$$

$$\Delta\varphi_{\rm AB} = -\frac{|e|}{\hbar}\oint \mathbf{A}d\mathbf{s}$$

The Aharonov–Bohm phase

$$\mathbf{p} \longrightarrow \mathbf{p} + |e|\mathbf{A} \qquad \Delta\varphi_{\mathrm{AB}} = -\frac{|e|}{\hbar}\oint \mathbf{A}d\mathbf{s}$$

Aharonov–Casher phase

$$\mathbf{p} = m\mathbf{v} + \frac{1}{c}\boldsymbol{\mu} \times \mathbf{E} \qquad \Delta\varphi_{\mathrm{AC}} = \frac{1}{c}\oint \boldsymbol{\mu} \times \mathbf{E}d\mathbf{s}$$

Eur. Phys. J. C 41, 421–426 (2005)
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THE EUROPEAN PHYSICAL JOURNAL C

Non-minimal coupling to a Lorentz-violating background and topological implications

H. Belich[1,2,a], T. Costa-Soares[2,3,4,b], M.M. Ferreira Jr.[2,5,c], J.A. Helayël-Neto[2,3,d]

$$(i\gamma^\mu D_\mu - m)\Psi = 0, \qquad D_\mu = \partial_\mu + eA_\mu + igv^\nu \widetilde{F}_{\mu\nu},$$

$$\overrightarrow{\Pi} = \left(\overrightarrow{p} - e\overrightarrow{A} + gv^0\overrightarrow{B} - g\,\overrightarrow{v} \times \overrightarrow{E}\right)$$

$$H = \frac{1}{2m}\overrightarrow{\Pi}^2 + e\varphi - \frac{e}{2m}\overrightarrow{\sigma}\cdot(\overrightarrow{\nabla}\times\overrightarrow{A}) + \frac{1}{2m}gv^0\overrightarrow{\sigma}\cdot(\overrightarrow{\nabla}\times\overrightarrow{B}) + \frac{g}{2m}\overrightarrow{\sigma}\cdot\overrightarrow{\nabla}\times(\overrightarrow{v}\times\overrightarrow{E})$$

Non-minimal coupling to the gauge field and background

$$(i\gamma^{\mu}D_{\mu}-m)\Psi=0,\qquad D_{\mu}=\partial_{\mu}+eA_{\mu}+igv^{\nu}\widetilde{F}_{\mu\nu},$$

$$H=\frac{1}{2m}\vec{\Pi}^{2}+e\varphi-\frac{e}{2m}\vec{\sigma}\cdot(\vec{\nabla}\times\vec{A})+\frac{1}{2m}gv^{0}\vec{\sigma}\cdot(\vec{\nabla}\times\vec{B})+\frac{g}{2m}\vec{\sigma}\cdot\vec{\nabla}\times(\vec{v}\times\vec{E})$$

$$\vec{\Pi}=\left(\vec{p}-e\vec{A}+gv^{0}\vec{B}-g\vec{v}\times\vec{E}\right)$$

$$(\vec{\mu}=g\vec{v}),\qquad \Phi_{AC}=\int(g\vec{v}\times\vec{E})\cdot\vec{dl}$$

$(gv_z)\leq 10^{-14}\ (\mathrm{eV})^{-1}$ PHYSICAL REVIEW D **83,** 125025 (2011)

On the influence of a Coulomb-like potential induced by the Lorentz symmetry breaking effects on the Harmonic Oscillator

$$i\gamma^{\mu}\partial_{\mu}\rightarrow i\gamma^{\mu}\partial_{\mu}-g\,b^{\mu}\,F_{\mu\nu}\left(x\right)\,\gamma^{\nu},$$

Eur. Phys. J. Plus 127, 102 (2012)

$$i\,\gamma^{\mu}\,D_{\mu}\,\Psi+\frac{i}{2}\sum_{k=1}^{3}\gamma^{k}\left[D_{k}\,\ln\left(\frac{h_{1}\,h_{2}\,h_{3}}{h_{k}}\right)\right]\Psi-g\,b^{\mu}\,F_{\mu\nu}\left(x\right)\,\gamma^{\nu}=m\Psi,$$

$$h_{2}=\rho \text{ and } h_{3}=1.$$

$$\text{in cylindrical coordinates}:\; h_{0}=1,\; h_{1}=1,\, h_{2}=\rho \text{ and } h_{3}=1.$$

$$i\frac{\partial\psi}{\partial t}=-\frac{1}{2m}\left[\frac{\partial^{2}}{\partial\rho^{2}}+\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{1}{\rho^{2}}\frac{\partial^{2}}{\partial\varphi^{2}}+\frac{\partial^{2}}{\partial z^{2}}\right]\psi+\frac{1}{2m}\frac{i\sigma^{3}}{\rho^{2}}\frac{\partial\psi}{\partial\varphi}$$
$$+\frac{1}{8m\rho^{2}}\psi-i\frac{gbB_{0}}{m\rho}\frac{\partial\psi}{\partial\varphi}+\frac{\left(gbB_{0}\right)^{2}}{2m}\psi.$$

J. Phys. G: Nucl. Part. Phys. **39** (2012) 085001 (10pp) doi:10.1088/0954-3899/39/8/085001

Abelian geometric phase for a Dirac neutral particle in a Lorentz symmetry violation environment

K Bakke[1] and H Belich[2]

$$i\gamma^{\mu}\partial_{\mu} \rightarrow i\gamma^{\mu}\partial_{\mu} - \frac{g}{2}\eta^{\alpha\beta}F_{\mu\alpha}F_{\beta\nu}\gamma^{\mu}b_{\lambda}\gamma^{\lambda}\gamma^{\nu} - \frac{\nu}{2}\eta^{\alpha\beta}F_{\mu\alpha}F_{\beta\nu}\gamma^{\mu}\gamma^{\nu},$$

Projetos iniciados:

-Supersimetria do setor de Gauge Par.

-Supergravidade e Violação de Sim. de Lorentz.

On the influence of a Coulomb-like potential induced by the Lorentz symmetry breaking effects on the Harmonic Oscillator

$$i\gamma^{\mu}\partial_{\mu} \rightarrow i\gamma^{\mu}\partial_{\mu} - g\, b^{\mu}\, F_{\mu\nu}\,(x)\;\gamma^{\nu},$$

Eur. Phys. J. Plus 127, 102 (2012)

$$i\,\gamma^{\mu}\,D_{\mu}\,\Psi + \frac{i}{2}\sum_{k=1}^{3}\gamma^{k}\left[D_{k}\,\ln\left(\frac{h_{1}\,h_{2}\,h_{3}}{h_{k}}\right)\right]\Psi - g\,b^{\mu}\,F_{\mu\nu}\,(x)\;\gamma^{\nu} = m\Psi,$$

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$$i\,\gamma^{\mu}\,D_{\mu}\,\Psi + \frac{i}{2}\sum_{k=1}^{3}\gamma^{k}\left[D_{k}\,\ln\left(\frac{h_1\,h_2\,h_3}{h_k}\right)\right]\Psi - g\,b^{\mu}\,F_{\mu\nu}\,(x)\,\gamma^{\nu} = m\Psi,$$

in cylindrical coordinates : $h_0 = 1,\ h_1 = 1,\ h_2 = \rho$ and $h_3 = 1.$

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m}\left[\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial z^2}\right]\psi + \frac{1}{2m}\frac{i\sigma^3}{\rho^2}\frac{\partial\psi}{\partial\varphi}$$
$$+ \frac{1}{8m\rho^2}\,\psi - i\frac{gbB_0}{m\rho}\frac{\partial\psi}{\partial\varphi} + \frac{(gbB_0)^2}{2m}\psi.$$

Geometric quantum phases from Lorentz symmetry breaking effects in the cosmic string spacetime

Phys. Rev. D 90, 025026 (2014)

The CPT-even gauge sector of the SME

$$S = -\frac{1}{4}\int d^4x\, K_{abcd}\, F^{ab}\, F^{cd}$$

$$K_{abcd} = K_{[ab][cd]};\;\; K_{abcd} = K_{cdab};\;\; K^{ab}{}_{ab} = 0$$

$$K_{abcd} = \frac{1}{2}\left(\eta_{ac}\,\tilde{\kappa}_{bd} - \eta_{ad}\,\tilde{\kappa}_{bc} + \eta_{bd}\,\tilde{\kappa}_{ac} - \eta_{bc}\tilde{\kappa}_{ad}\right) \qquad \tilde{\kappa}_{ab} = \kappa\left(\xi_a\xi_b - \frac{\eta_{ab}\,\xi^c\xi_c}{4}\right),$$

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$$\mathcal{L}_{\mathrm{modM}} = -\sqrt{g}\,\left(\frac{1}{4}\,F_{\mu\nu}F_{\rho\sigma}\,g^{\mu\rho}g^{\nu\sigma} + \frac{1}{4}\,K^{\mu\nu\rho\lambda}\,F_{\mu\nu}F_{\rho\lambda}\right)$$

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$$\mathcal{L}_{\mathrm{modM}}=-\sqrt{g}\left(1-\frac{1}{2}\kappa\,\xi_\alpha\,\xi^\alpha\right)\frac{1}{4}F^{\mu\nu}\left(x\right)F^{\rho\sigma}\left(x\right)\,\bar{g}_{\mu\rho}\left(x\right)\,\bar{g}_{\nu\sigma}\left(x\right),$$

$$\bar{g}_{\mu\rho}\left(x\right)=g_{\mu\rho}\left(x\right)-\epsilon\,\xi_\mu\,\xi_\rho,\qquad \epsilon=\frac{\kappa}{1+\frac{\kappa}{2}}$$

Locally, the reference frame of the observers can be build via a noncoordinate basis

$$\hat{\theta}^a = e^a{}_\mu (x)\, dx^\mu , \qquad g_{\mu\nu}(x) = e^a{}_\mu (x)\, e^b{}_\nu (x)\, \eta_{ab}$$

inverse of the tetrads

$$dx^\mu = e^\mu{}_a (x)\, \hat{\theta}^a ,$$

the 1-form connection,

$$\omega^a{}_b = \omega_\mu{}^a{}_b (x)\, dx^\mu$$

$$i\gamma^\mu \partial_\mu \psi + i\gamma^\mu \Gamma_\mu (x)\, \psi = m\psi .$$

Cartan's structure equations in the absence of torsion

$$\Gamma_\mu (x) = \frac{i}{4}\, \omega_{\mu ab}(x)\, \Sigma^{ab} \qquad \Sigma^{ab} = \frac{i}{2}\left[\gamma^a , \gamma^b\right] \qquad d\hat{\theta}^a + \omega^a{}_b \wedge \hat{\theta}^b = 0$$

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the effective metric of the cosmic string spacetime under Lorentz symmetry breaking effects

$$d\bar{s}^2 = dt^2 - d\rho^2 - \eta^2\rho^2(1+\epsilon)\, d\varphi^2 - dz^2$$

$$\hat{\Theta}^0 = dt; \;\; \hat{\Theta}^1 = d\rho; \;\; \hat{\Theta}^2 = \eta\rho\sqrt{1+\epsilon}\; d\varphi; \;\; \hat{\Theta}^3 = dz$$

the 1-form connection

$$\omega_{\varphi}{}^{2}{}_{1}(x) = -\omega_{\varphi}{}^{1}{}_{2}(x) = \eta\sqrt{1+\epsilon}.$$

the spinorial connection

$$\Gamma_{\varphi}(x) = -\frac{i}{2}\eta\sqrt{1+\epsilon}\,\Sigma^{3}$$

the Dirac equation in the cosmic string spacetime

$$m\psi = i\gamma^{0}\frac{\partial\psi}{\partial t} + i\gamma^{1}\left(\frac{\partial}{\partial\rho} + \frac{1}{2\rho}\right)\psi + \frac{i\gamma^{2}}{\eta\rho\sqrt{1+\epsilon}}\frac{\partial\psi}{\partial\varphi} + i\gamma^{3}\frac{\partial\psi}{\partial z}.$$

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By applying the Dirac phase factor method

$$\psi=e^{i\phi}\,\psi_{0},\quad m\psi_{0}=i\gamma^{0}\frac{\partial\psi_{0}}{\partial t}+i\gamma^{1}\frac{\partial\psi_{0}}{\partial\rho}+\frac{i\gamma^{2}}{\eta\rho\sqrt{1+\epsilon}}\frac{\partial\psi_{0}}{\partial\varphi}+i\gamma^{3}\frac{\partial\psi_{0}}{\partial z},$$

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the relativistic geometric phase acquired by the wave function of the Dirac particle is

$$\phi = \frac{1}{2}\oint\eta\sqrt{1+\epsilon}\,\Sigma^{3}\,d\varphi = \pi\,\eta\,\sqrt{1+\epsilon}\,\Sigma^{3}.$$

Obrigado